Queen Mary & Westfield College
Royal Holloway UoL
University College

Intercollegiate Postgraduate Course in Elementary Particle Physics

Paper 1: Wednesday, 24 March 1999

Time allowed for the Examination: 3 hours

Attempt THREE *questions - One from each section*

Section A: Symmetries and Conservation Laws
Section B: Electroweak Interactions part 1
Section C: LEP 2 Physics

Section A : Symmetries and Conservation Laws

Question 1

(a) The Standard Model of particle physics is believed to have an exact gauge symmetry $U(1) \times SU(2)_L \times SU(3)$. Explain what these symbols mean. How are the observed patterns of fundamental fermions and bosons explained in this picture. Explain carefully any group theory terminology that you use. **10 marks**

(b) The strong interaction has an approximate symmetry in which the light quarks u, d and s form a triplet corresponding to the fundamental representation of a $SU(3)$ flavour group. How does this explain the observed light mesons and baryons? How can the heavier quarks c and b be accommodated? **10 marks**

Question 2

(a) Describe how B meson decays can be used to measure the magnitudes of two of the CKM matrix elements. What difficulties do these measurements present? **5 marks**

(b) Explain how a neutral meson such as B^0 can oscillate into its anti-particle $\bar{\mathrm{B}}^0$. How is the oscillation frequency related to their masses? What are the particular experimental challenges of making a measurement of $\bar{\mathrm{B}}_s^0$ oscillations. **5 marks**

(c) Explain how the presence of B meson mixing gives rise to indirect CP violation, which can be directly related to the phase parameter in the CKM matrix. Give examples of some decay channels that can be used. What are the experimental difficulties of such measurements? **10 marks**

Section B : Electroweak Interactions part 1

Question 3

(a) By demanding that solutions of the Dirac equation be invariant under the local space - time transformation $\psi^{'}(x,t) = exp(iq\chi(x,t)\psi(x,t)$ show that a 4-vector field A^{μ}, satisfying the gauge transformation $A^{\mu^{'}} = A^{\mu} - \partial^{\mu}\chi$ must be introduced, where $\chi(x,t)$ is a scalar function.

(b) The four momenta (k,p) and $(k^{'},p^{'})$ describe the initial and final state kinematics of $e^{-}\mu^{-}$ scattering.

Write down the first order matrix element for this process.

Show that the cross section for electron - muon scattering, in the limit that masses tend to zero, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $e^{-}\ \mu^{-}$ scattering and α is the fine structure constant.

(c) Show that annihilation cross section for $e^{+}e^{-}$ to $\mu^{+}\mu^{-}$ is given by

$$\sigma = \frac{4\pi\alpha^2}{3s}.$$

(d) The ratio of the total $e^{+}e^{-}$ annihilation cross section to the $e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}$ cross section is called R. Explain what you would expect R to be between the charm and bottom pair thresholds.

(e) The Universe is filled with a black body radiation spectrum at 3 K. Explain why you would not expect to detect high energy photons above an energy threshold from deep in the Universe. Evaluate this threshold energy.

Question 4

(a) Draw Feynman graphs and write down the matrix elements for the following charged current weak processes, assuming a pointlike weak interaction.

$\nu_\mu + d \to u + \mu^-$,

$\nu_\mu + d \to c + \mu^-$ and

$\pi^+ \to \mu^+ + \nu_\mu$.

(b) Determine an expression for the decay rate of the π^+, and obtain an expression for the ratio of the K^+ and π^+ decay rates.

(c) The massive, free, spin one particle field satisfies the Lorentz condition. If a massive spin one particle of mass M and energy p_0 is propagating along the z axis with momentum p_z show that

$$\begin{aligned} \epsilon_1^\mu &= (0,1,0,0) \\ \epsilon_2^\mu &= (0,0,1,0) \\ \epsilon_3^\mu &= \frac{1}{M}(p_z,0,0,p_0) \end{aligned}$$

is a suitable set of polarization vectors.

Hence obtain the sum over the polarization states,

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

(d) Write down the matrix element for W^+ decay to $e^+\nu$ and evaluate the "matrix element squared", summed over the spin states and averaged over the initial spins, as far as the trace calculation.

(e) Draw the Feynman graphs for the production of W bosons in $p-p$ and $p-\overline{p}$ colliders.

Section C : LEP 2 Physics

Question 5

(a) The Weizsäcker-Williams distribution of initial state radiation (ISR) has the form:

$$\frac{d^2\sigma}{dz dp_T^2} = \frac{\alpha}{\pi}\frac{1}{p_T^2}\frac{1+z^2}{1-z}\sigma_0\left(s\sqrt{z}\right)$$

Describe the meaning of the various terms in this equation and draw the Feynman diagrams relevant to this process. [3]

(b) Write down the mathematical form of the Z^0 propagator in the zero width approximation. Indicate and justify briefly how a finite width Γ can be included in the propagator term. [3]

(c) Sketch the form of the single Z^0 production cross section as a function of $\sqrt{s}$ at an e^+e^- collider from $0 < \sqrt{s} < 200$ GeV. Discuss how the presence of ISR can result in an appreciable Z^0 background at LEP2 and describe suitable experimental cuts that could be applied to reduce this background. [4]

(d) Write down the *flux factor* and the *differential phase space* associated with the process $e^+e^- \rightarrow f_1 + f_2 + \cdots f_n$ when the centre of mass energy is $\sqrt{s}$ and where f_i is the i^{th} final state particle . Hence show that if $\mathcal{M}$ is the matrix element for pair production of particles of mass m, then the differential cross section $\frac{d\sigma}{d\Omega}$ has the form:

$$\frac{d\sigma}{d\Omega} = \sqrt{(1-4\frac{m^2}{s})}\frac{|\mathcal{M}|^2}{64\pi^2 s}$$

[5]

(e) Describe the experimental techniques that can be applied to measure the production cross sections, including reduction of the relevant backgrounds, for **one** of the following production processes: **either** (i) $h^0 Z^0$ where h^0 is the higgs boson **or** (ii) $\tilde{e}^+\tilde{e}^-$ where $\tilde{e}^-$ is the supersymmetric parter of the electron. [5]