Queen Mary, University of London
Royal Holloway, University of London
University College London
Brunel University

Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 1:

Section A: Symmetries and Conservation Laws (40 marks)

Section B: Neutrino Physics and Supersymmetry (20 marks)

Tuesday, 25 January, 2005

Time allowed for the Examination: 3 hours.

Answer *ALL* questions.

Books and notes may be consulted.

Section A: Symmetries and Conservation Laws [20 marks]

Question 1 [13 marks]

(a) Explain what is meant by the group SO(3). Give a non-trivial example of one of the members (i.e., not the identity). [2 marks]

(b) The members of the group SO(3) can be written in the form $\exp(\alpha G)$, where α is a number and G is one of the generators for the group. In the context of a vector-space consisting of real vectors (rather than wave functions), derive the properties of the generators. How many independent generators are there? Identify a suitable independent set of generators. [4 marks]

(c) By exponentiating one of the generators, demonstrate how a typical member of SO(3) (such as the one you identified in part (a)) can be constructed. [3 marks]

(d) Give the definition of the structure constants and adjoint of a group. How many adjoint matrices are there for SO(3)? Obtain one of the adjoint matrices corresponding to your choice of generators (you may assume cyclic symmetry). Using symmetry, write the other adjoint matrices. [4 marks]

Question 2 [7 marks]

(a) In the $\mathrm{SU}(3)_{\text{colour}} \otimes \mathrm{SU}(3)_{\text{flavour}} \otimes \mathrm{SU}(2)_{\text{spin}}$ quark model, explain the *observed* mesons states and their multiplicities, using Young Tableaux (there is no need to give wave functions or quantum numbers). [5 marks]

(b) In the Hamiltonian for the meson masses, there is a term $\kappa \mathbf{S}_1 \cdot \mathbf{S}_2$, where $\mathbf{S}_{1,2}$ are the quark spins. Explain what effect this would have on the masses. What other effects are present and can account for the mass splittings? [2 marks]

Question 3 [20 marks]

(a) Draw the Unitarity Triangle, labelling its sides and angles. Give an example of a decay mode that can be used to determine the angles α and γ. [2 marks]

(b) Draw the Feynman diagram for $\mathrm{B}^0 \to \overline{\mathrm{B}}^0$ oscillation and explain how this leads to the indirect CP violation observed in the decay channel $\mathrm{B}^0 \to \mathrm{J}/\psi\, \mathrm{K}^0_\mathrm{S}$. Show how the CP asymmetry $A_{CP}(t)$ is related to the sine of the angle β of the Unitarity Triangle. Draw all the relevant Feynman diagrams, labelling the CKM matrix elements. [10 marks]

(c) Draw the Feynman diagrams for the process $\mathrm{B}^- \to \mathrm{K}^-\pi^0$, labelling the CKM matrix elements. Assume that the penguin amplitude is dominated by the top quark loop. Using the Wolfenstein parametrization for the CKM matrix V, find the dependence of the CP asymmetry A_{CP} in this channel on the parameters of the CKM matrix. Show that A_{CP} is proportional to the area of the unitarity triangle. Use the unitarity conditions to show that there would exist no CP violation for this type of decay if there were only two generations. [8 marks]

Section B: Neutrino Physics and Supersymmetry

Question 4 [6 marks] Describe briefly the Kamland experiment and state the main experimental results. Use these to estimate numerically the probability for an electron antineutrino to remain the same by the time it reaches the detector. Using the fact that detected reactor neutrinos have energies of approximately 5 MeV, what can you deduce about Δm^2 and $\sin^2 2\theta$ by applying the two-neutrino oscillation formula?

Question 5 [8 marks] Elastic scattering of a neutrino by an electron is the main reaction for solar-neutrino detection in water Cherenkov detectors.

(a) 22,000 events were observed by Super-K in 1500 days (and nights) in 22 ktons of water. Given an average neutrino cross section of $10^{-44}\,\mathrm{cm}^2$, what is the observed neutrino flux in $\mathrm{cm}^{-2}\mathrm{s}^{-1}$?

(b) Assuming oscillations in vacuum and a maximal mixing angle of $\theta = \pi/4$ what value of Δm^2 does Super-K suggest? Assume that the total neutrino flux from the Sun is $6.6 \times 10^{10}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at the earth's surface.

(c) Is the obtained value of Δm^2 compatible with the current best fit to solar neutrino data? Justify your answer.

Question 6 [6 marks]

(a) How many (and which) new fermions does the introduction of SUSY bring in?

(b) How would you go about setting an absolute lower limit on the mass of the Wino using the LEP-1 data (Z pole running)? Ditto for the LSP.