Queen Mary, University of London
Royal Holloway, University of London
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Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 2: The Standard Model

Thursday, 27 January, 2005

Time allowed for the Examination: 3 hours.

Answer *ALL* of the 18 questions (60 marks total).

Books and notes may be consulted.

1 Explain how the E. M. interaction may be considered to arise from the requirement of local gauge invariance, basing your explanation on a wave function satisfying the Dirac equation. [3 marks]

2 The four momenta (k, p) and (k', p') describe the initial and final state kinematics of $e^- \mu^-$ scattering. Draw the Feynman diagram for this process, and hence write down the first order matrix element. [3 marks]

3 How does R vary from low energies to energies below the Z^0 resonance? Give quantitative answers where possible. [4 marks]

4 Define, in terms of the usual four vectors, the scaling variables x and y for deep inelastic lepton-nucleon scattering (DIS). Give simple interpretations of the variables x and y in terms of the parton model and the lepton parton CM frame. [4 marks]

5 Write down the Lagrangian for QCD. [4 marks]

6 Starting from the charged weak current expressed in the form

$$J^{+\mu} = \frac{g_w}{\sqrt{2}} \overline{u_\nu} \gamma^\mu \frac{1}{2}(1 - \gamma^5) u_e$$

show that the weak charged current may be written as

$$J^{+\mu} = \frac{g_w}{\sqrt{2}} \overline{\lambda_l} \gamma^\mu \tau^+ \lambda_l$$

where λ_l are chiral, weak isospin doublets, and τ^+ is the weak isospin raising operator. [2 marks]

7 Show that g_w is related to the Fermi constant G_F by [2 marks]

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_w^2} \ .$$

8 For a massive spin one particle of mass M, energy p_0 which propagates along the z axis with momentum p_z show that

$$\epsilon_1^\mu = (0,1,0,0)$$

$$\epsilon_2^\mu = (0,0,1,0)$$

$$\epsilon_3^\mu = \frac{1}{M}(p_z,0,0,p_0)$$

is a suitable set of polarization vectors. [2 marks]

9 Hence or otherwise, obtain the sum over the polarization states, [2 marks]

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

10 Write down the propagator for a massive spin one particle and show how this may be modified for an unstable spin one particle whose full width Γ is much less than the mass M. [4 marks]

11 From the Feynman diagram, write down the matrix element for e^+e^- annihilation at the Z^0 resonance. [4 marks]

12 Explain the structure of the $SU(2) \times U(1)$ electroweak theory in the massless limit, explaining as you do so, the concepts of weak isospin, weak hypercharge and local gauge invariance. [4 marks]

13 Write down the covariant derivative D^μ for the above interaction and obtain the transformation for the W_i^μ field under local gauge transformation. (You may assume without proof that $D^\mu\psi$ transforms in the same way as ψ itself.) [4 marks]

14 Demonstrate that by means of a suitable symmetry breaking of a local $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, that the EM field A^μ acquires mass. [4 marks]

15 Write down Weinberg's choice of the scalar field in $SU(2) \times U(1)$ electroweak theory before and after symmetry breaking. [2 marks]

16 Obtain the relation $M_Z = M_W / \cos\theta_W$. [4 marks]

17 Draw the Feynman diarams of processes that become important at energies above about 161 GeV in e^+e^- annihilation. [4 marks]

18 Explain what is meant by 'double β decay' and 'neutrinoless double β decay'. Explain the 'see saw' mechanism and obtain the mass eigenvalues. [4 marks]