## PATH LENGTH CORRECTIONS AND ZERO RECONSTRUCTION IN THE MINOS DETECTORS

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> > October 2003

### Abstract

Cosmic ray muons are used to perform strip-to-strip calibration in the MINOS detectors. A path length correction is applied to remove the dependency of the strip light output on the angle of the incident muon. Also, since cosmic ray muons typically produce  $\sim 2-6$  photoelectrons per strip-end in MINOS, an accurate zero reconstruction is necessary in order to properly compare strip-ends with different light outputs.

This note documents attempts to understand these factors, particularly in relation to measurements from the Calibration Detector, and presents a method to correct for them in data.

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## 1 Introduction

The MINOS detectors have been designed to achieve a relative energy calibration of 2% and an absolute energy calibration of 5%. The relative calibration can be accomplished by comparing the response of stopping muons of known energies in each detector. The absolute calibration is determined from measurements with the Calibration Detector (CalDet) which is involved in a series of test-beams at the CERN PS.

However, before the relative and absolute calibrations can be computed and applied, the response of all strip-ends within each detector must be normalized. This is known as a strip-to-strip calibration and is the process of accounting for variations in the response of each readout channel. Differences in scintillator light output, WLS fibre light collection efficiency, cable and connector transmission efficiency as well as phototube gain and quantum efficiency are collectively accounted for using cosmic ray muon energy deposits in each strip. This results in the determination of the characteristic light output of each strip-end for a muon crossing the strip. These values are then used to produce calibration constants which convert all hits to some universal scale. Only after this stage, can the relative and absolute calibration procedures be carried out.

In order to perform a strip-to-strip calibration, it is important to remove dependencies related to the position, angle and time at which the cosmic ray muon enters the detector. Factors which must be corrected for include: light attenuation in the read-out fibres, differences in light output due to differing path lengths in scintillator and gain drifts in the PMTs. Further, since the light level for cosmic ray muons in MINOS is typically  $\sim 4$ photoelectrons per strip-end, the Poisson probability for obtaining zero photoelectrons at the photocathode is an important effect which must be corrected for. Failure to do this will cause a bias in the observed strip-end light output towards higher values.

This note documents work done to understand how light output changes with angle and how this in turn affects the number of zeros produced. An approach for simultaneously performing path length corrections and zero reconstruction is presented and the approach is tested and validated using a simple MC model of a scintillator strip.

## 2 Tools and Method

For the purposes of performing path length corrections and zero reconstruction each scintillator strip-end is considered separately. Each strip is assumed to be a homogeneous block with rectangular cross-section and the fibre is assumed to have infinitesimally small cross-section. The collection efficiency of light by the fibre is assumed to have no transverse dependence (i.e. across the 4.1 cm width) on the muon crossing position.

### 2.1 Simple MC Model

A simple model has been used to simulate the response of a scintillator strip when a muon passes through it. The strip is described in three dimensions, having the usual 4.1 cm by 1 cm MINOS cross-section but with varying length. Muons are randomly generated at a suitably large surface to ensure the entire strip can be uniformly illuminated for the particular muon angular distribution to be investigated. For each track passing through the strip, the actual path length is calculated and the energy deposited is randomly sampled from an appropriate Landau distribution.

This energy is then attenuated based on the distance of the muon crossing from the read-out end of the strip to simulate the effect of the wavelength shifting (WLS) fibre. The resulting quantity is converted to some number of initial photons at the photocathode (which can be varied in order to vary the characteristic light output of the strip) and is passed to a program which simulates the photomultiplication and ADC conversion for a pixel on a Hamamatsu M16 phototube [1].

In this way a strip-end muon energy spectrum in ADC counts can be quickly and easily simulated for different strip light outputs and different muon angular distributions. Each of these stages will now be discussed in more detail.

### 2.1.1 Landau Dependence

A plane of scintillator described in GEANT3 has been used to simulate the energy deposited by a muon as a function of path length. The thickness of the scintillator plane was varied between 0.02 cm and 20 cm and muons of 4 GeV were fired perpendicularly through the plane. The resulting distributions of energy deposited in the scintillator have been fitted with the ROOT Landau function. An example of one of these fits is shown in Figure 1.



Energy Deposited by 4 GeV Muons passing through 1cm of Scintillator

Figure 1: The energy deposited by 4 GeV muons passing through 1 cm of scintillator from GEANT3.

The large  $\chi^2/dof$  value for the fit is due to deviations in the tail of the distribution. Since the Landau function describes energy loss rather than energy deposited, the shape of the distribution is not perfectly described. The variations in the Landau peak and width with path length are shown in Figure 2.



Figure 2: The Landau peaks and widths obtained by fitting to GEANT3 distributions of energy deposited for various muon path lengths in scintillator.

The points shown in Figure 2 have been used in the simple model of the scintillator strip to generate energy deposited with path length. Between these points the Landau parameters are obtained by interpolation. The straight line fits included in Figure 2 illustrate that the Landau peak does not scale linearly with path length. This can be seen more clearly from Figure 3 which shows the Landau peak divided by path length versus path length. This behavior is described by the Landau theory and the light output is expected to have a weak logarithmic dependence as observed.



### Landau Peak/Path-length versus Path-length

Figure 3: The Landau peak divided by the path length in scintillator versus path length for 4 GeV muons.

These simulations of energy loss with path length do not include the effects of having steel and air surrounding the scintillator plane. Also, the scintillator plane is described as a solid block rather than as an array of strips. However, the asymmetry in the response due to the long Landau tail is present and it is the effect of this on the ADC distributions which was considered most important, rather than the specific values of the Landau parameters.

### 2.1.2 Attenuation in Fibres

Most of the work on path length corrections has been geared towards calibration at the CalDet. Therefore, a typical response map measured at CalDet of light output as a function of position along the 1 m length of a strip has been used to simulate attenuation in the WLS fibre. The map used is shown in Figure 4.



Mean Light Output with Position along Strip (example distribution from CalDet)

Figure 4: Mean light output from a CalDet scintillator strip as a function of position along the 1 m length of the strip. (The x values are plotted in units of strip, i.e. multiply by 4.1 to get centimeters.)

In this figure the light is read-out on the left hand side of the strip and so the gentle slope in the middle of the distribution is attributed to attenuation in the WLS fibre. The dramatic reduction of light at the ends of the strip results from the fact that the strip-ends are not coated with any reflective material. Light is therefore absorbed in the casing of the scintillator module.

The response pattern is fitted with a high order polynomial function as shown in the figure and this is used to simulate the variations in light output along the strip.

#### 2.1.3 Phototube Simulation

A simulation of the Hamamatsu M16 phototubes [1] has been used to produce the final ADC distribution. The simulation accounts for the photomultiplication at each of the dynode stages and takes high voltage and pedestal width as inputs. In these studies a high voltage of 750 V and a pedestal width of 4 ADC counts has been used.

Also at this stage of the simulation, the conversion of energy in MeV to an initial number of photons at the phototube is required. This scaling is determined by the characteristic light output of the strip. The characteristic value is the amount of light corresponding to the peak of the Landau distribution for an orthogonal muon crossing (i.e. 1 cm path length) at the mid-point along the strip length. The characteristic value is tunable so that the response of strips with different light outputs can be investigated.



Fit to simulated muon energy spectrum

Figure 5: A fit to a simulated energy spectrum for muons traveling orthogonally through the mid-point of a strip (no WLS fibre attenuation included).

An example of the resulting ADC distribution at a strip-end for a set of orthogonally traveling muons without attenuation is shown in Figure 5. The distribution is fitted using a function described in [2]. The light output of the strip in this example was set to be  $\sim 3.4$  photoelectrons, thus the effect of the landau tail can be seen by comparison with the mean of the distribution of  $\sim 4$  photoelectrons, (n.b. the spectrum is  $\sim 99\%$  truncated in this example).

### 2.2 Theoretical Corrections

The resolution of the MINOS detectors is not adequate for measuring exactly where on the strip surface a muon enters and leaves. Hence, it is impossible on a hit by hit basis to calculate the path length through a particular scintillator strip. However, with enough muons in the detector, one can assume that each strip is subject to a uniform distribution of entry and exit points across its surface. In this case, an average path length can be calculated for a particular track angle. It is this approach that was adopted in [3] and it has been used for this work.

#### 2.2.1 Path Length Correction

The case where each strip is assumed to have finite dimensions in y and z (4.1 cm and 1 cm respectively in MINOS) but infinite x dimensions was first considered by Lee and Seun [3]. In this scenario, the average path length,  $\langle ds \rangle_y$ , is calculated by integrating over all possible muon entry points. The integration taking place can be understood from Figure 6 and the formulation is shown below:

$$\langle ds \rangle_{y} = \frac{\int_{0}^{\Delta y + \delta y} ds(y) dy}{\int_{0}^{\Delta y + \delta y} dy} \langle ds \rangle_{y} = \frac{\int_{\delta y}^{\Delta y} \frac{ds}{dz} \Delta z dy + 2 \int_{\Delta y}^{\Delta y + \delta y} \frac{ds}{dz} \frac{dz}{dy} y dy}{\Delta y + \delta y} \langle ds \rangle_{y} = \frac{ds}{dz} \left( \frac{\Delta y}{\frac{\Delta y}{\Delta z} + \left| \frac{dy}{dz} \right|} \right)$$
(1)

where x is position along the length of the strip, y is position along the width of the strip and z is position along the axis of the detector; hence,  $ds^2 = dx^2 + dy^2 + dz^2$ .  $\delta y$  is the maximum y position beyond the edge of the strip for which the track will still pass through the strip.  $\Delta y = 4.1cm$  is the width of a scintillator strip and  $\Delta z = 1cm$  is the thickness.

Applying this correction to each hit based on the track angles dx/dz and dy/dz, both the increased light output due to extra path length, as well as the reduced light output due to muons which pass through more than one strip in a plane ("corner clipping"), is accounted for.

While this approach has been shown to work well in [3] it does not attempt to account for the finite x dimension, (which becomes more important as the track angle ds/dz, with respect to the axis of the detector increases). At the CalDet and Near Detector one may wish to remove complications related to corner clipping in both the x and y dimensions by simply demanding that the hits are not close to the edges of the detector. However in the Far Detector, the cosmic ray muon rate is very low to begin with and so a further reduction in statistics may be unacceptable. Thus the approach outlined above has been generalized to the case where all three strip dimensions are finite.

To calculate the average path length,  $\langle ds \rangle_{xy}$ , given three finite dimensions the following formulation can be used:



Figure 6: A schematic diagram illustrating the dependence of path length through a scintillator strip on the angle of the track.  $\delta s(y)$  is the actual path length in the strip.

$$\langle ds \rangle_{xy} = \left( \int_{\delta x}^{\Delta x} dx \left[ \int_{\delta y}^{\Delta y} \delta s dy + 2 \int_{0}^{\delta y} \delta s(y) dy \right] \right. \\ \left. + 2 \int_{0}^{\delta x} dx \left[ \int_{\delta y(x)}^{\Delta y} \delta s(x) dy + 2 \int_{0}^{\delta y(x)} \delta s(y) dy \right] \right) \\ \left. \left. / \int_{\delta x}^{\Delta x} dx \left[ \int_{\delta y}^{\Delta y} dy + 2 \int_{0}^{\delta y} dy \right] + 2 \int_{0}^{\delta x} dx \left[ \int_{\delta y(x)}^{\Delta y} dy + 2 \int_{0}^{\delta y(x)} dy \right] \right. \\ \left. \langle ds \rangle_{xy} = \frac{ds}{dz} \left( \frac{\Delta x \Delta y \Delta z}{\Delta x \Delta y + \left| \frac{dx}{dz} \right| \Delta z \Delta y + \left| \frac{dy}{dz} \right| \Delta z \Delta x} \right)$$
 (2)

where  $\delta x = \frac{dx}{dz} \Delta z$ ,  $\delta y = \frac{dy}{dz} \Delta z$  and  $\delta y(x) = \frac{dy}{dx}x$  and correspond to maximum distances beyond the edges of the strip for which the muon will still intercept the strip. Also  $\delta s = \frac{ds}{dz} \Delta z$ ,  $\delta s(x) = \frac{ds}{dx}x$  and  $\delta s(y) = \frac{ds}{dy}y$ .  $\Delta x$  is the length of the strip, which in MINOS varies considerably depending on the detector.

It is also worth noting that by dividing equation 2 through by  $\Delta x$  and then taking the limit as  $\Delta x \to \infty$ , equation 1 is recovered as expected.

#### 2.2.2 Zero Reconstruction

Using this simple description of a scintillator strip, the probability of obtaining zero photoelectrons at the PMT can be considered. The Poisson probability of obtaining a zero, P(0), for a particular muon crossing is given by:

$$P(0) = e^{-\lambda\delta s} \tag{3}$$

where  $\lambda$  is the light output of the strip for a  $\Delta z$  path length ( $\Delta z = 1 cm$  in MINOS); and  $\delta s$  is the actual path length of the muon in the strip.

Thus, a similar integration can be performed for the average probability of getting zero photoelectrons,  $\langle P(0) \rangle_{xy}$ , by replacing  $\delta s$  in equation 2 with the appropriate P(0). Doing this yields the following:

$$\langle P(0) \rangle_{xy} = \left[ e^{-\lambda\delta s} \left( \Delta y - \left| \frac{dy}{dz} \right| \Delta z \right) \left( \Delta x - \left| \frac{dx}{dz} \right| \Delta z \right) \right. \\ \left. + \frac{2}{\lambda \frac{ds}{dz}} \left[ \left( \left| \frac{dy}{dz} \right| \Delta x + \left| \frac{dx}{dz} \right| \Delta y \right) \left( 1 - e^{-\lambda\delta s} \right) \right. \\ \left. + \left| \frac{dy}{dz} \right| \left| \frac{dx}{dz} \right| \Delta z \left( 1 + 2e^{-\lambda\delta s} \right) \right] \right. \\ \left. - \frac{6}{\lambda^2 \left( \frac{ds}{dz} \right)^2} \left| \frac{dy}{dz} \right| \left| \frac{dx}{dz} \right| \left( 1 - e^{-\lambda\delta s} \right) \right] \right. \\ \left. \left. / \left( \Delta x \Delta y + \left| \frac{dx}{dz} \right| \Delta z \Delta y + \left| \frac{dy}{dz} \right| \Delta z \Delta x \right) \right.$$
 (4)

where the symbols are the same as in equation 2.

Again, taking the limit as  $\Delta x \to \infty$  yields the appropriate average zero probability,  $\langle P(0) \rangle_{y}$ , for corner clipping in only the y direction. This is given by:

$$\left\langle P(0)\right\rangle_{y} = \frac{e^{-\lambda\delta s} \left(\Delta y - \left|\frac{dy}{dz}\right| \Delta z\right) + \frac{2\left|\frac{dy}{dz}\right|}{\lambda\frac{ds}{dz}} \left(1 - e^{-\lambda\delta s}\right)}{\Delta y + \left|\frac{dy}{dz}\right| \Delta z}$$
(5)

These equations allow the average probability of obtaining a zero to be computed for a particular set of angles. To check consistency, consider a muon passing parallel to the axis of the detector: corner clipping is no longer possible and, as can be seen from equation 5, the usual Poisson probability of obtaining zero is recovered, (since  $\frac{dy}{dz} \rightarrow 0$ ,  $\frac{ds}{dz} \rightarrow 1$  and  $\langle P(0) \rangle_y \rightarrow e^{-\lambda \delta s} = e^{-\lambda \Delta z}$ ).

### 2.2.3 Dynode and Sparsification Thresholds

The trigger for a channel to be read out during normal cosmic ray muon data taking is a dynode signal above some threshold. This threshold is typically set to be 0.2-0.3 photoelectrons and a trigger causes all channels associated with a single phototube to be read out. Each channel is then subject to a sparsification threshold to suppress pedestal hits in the data stream. The sparsification threshold is set so as to be  $\sim$ 3-4 sigma from the pedestal mean. Typically this corresponds to 0.1-0.2 photoelectrons. For cosmic ray muons only a single pixel is expected to be hit per phototube and so the actual threshold on the observed charge is approximately given by the more stringent of the dynode and sparsification values.

The effect of this threshold is to suppress some fraction of the single photoelectron peak (and to a much lesser extent the multi photoelectron peaks). By knowing the gain of the phototube pixel, the width of the single photoelectron peak and the sparsification/dynode threshold in ADC counts, an estimate of the missing fraction of single photoelectrons can be made and hence corrected for. To do this one must first know the probability of getting a single photoelectron as a function of muon angle.

The average probability of getting a single photoelectron  $\langle P(1) \rangle_{xy}$  is obtained by solving a similar set of integrals as in equations 2 and 4. The solution involves many terms, most of which do not contribute greatly, and is given here only for completeness:

$$\langle P(1) \rangle_{xy} = \left[ \frac{ds}{dz} \lambda \Delta z e^{-\lambda \delta s} \left( \Delta y - \left| \frac{dy}{dz} \right| \Delta z \right) \left( \Delta x - \left| \frac{dx}{dz} \right| \Delta z \right) \right. \\ \left. 2 \left[ \frac{\left( 1 - e^{-\lambda \delta s} \right)}{\lambda \frac{ds}{dz}} - \Delta z e^{-\lambda \delta s} \right] \left( \Delta x \left| \frac{dy}{dz} \right| + \Delta y \left| \frac{dx}{dz} \right| \right) \right. \\ \left. 4 \left( \Delta z \right)^2 \left| \frac{dy}{dz} \right| \left| \frac{dx}{dz} \right| e^{-\lambda \delta s} + 2 \frac{\Delta z \left| \frac{dy}{dz} \right| \left| \frac{dx}{dz} \right|}{\lambda \frac{ds}{dz}} \left( 1 + 5 e^{-\lambda \delta s} \right) \right. \\ \left. - 12 \frac{\left| \frac{dy}{dz} \right| \left| \frac{dx}{dz} \right|}{\left( \lambda \frac{ds}{dz} \right)^2} \left( 1 - e^{-\lambda \delta s} \right) \right] \right. \\ \left. \left. \left( \Delta x \Delta y + \left| \frac{dx}{dz} \right| \Delta z \Delta y + \left| \frac{dy}{dz} \right| \Delta z \Delta x \right) \right)$$

$$(6)$$

where the symbols are the same as in equation 2.

The solution,  $\langle P(1) \rangle_{u}$ , obtained in the limit of  $\Delta x \to \infty$ , is shown below:

$$\langle P(1) \rangle_{y} = \left[ \frac{ds}{dz} \lambda \Delta z e^{-\lambda \delta s} \left( \Delta y - \left| \frac{dy}{dz} \right| \Delta z \right) \right. \\ \left. + 2 \left| \frac{dy}{dz} \right| \left( \frac{\left( 1 - e^{-\lambda \delta s} \right)}{\lambda \frac{ds}{dz}} - \Delta z e^{-\lambda \delta s} \right) \right] \right. \\ \left. \left. / \left( \Delta y + \left| \frac{dy}{dz} \right| \Delta z \right) \right.$$

$$(7)$$

For most strip-ends in MINOS this is an almost negligible correction. However, for channels with very low light output or very low gain it may be an important contribution.

## 3 Application

### 3.1 Simulating Data

This work was prompted by difficulties encountered when performing strip-to-strip calibration at the CalDet. The problems stemmed from the fact that the CalDet has x and y rather than u and v oriented strips. With u and v views, the angular distributions of cosmic ray muons that the different sets of strips "see" is the same. However, at the CalDet, in the local strip coordinate system the vertical and horizontal strips see very different angular distributions. It is this asymmetry that the calibration procedures were not accounting for.

Figure 7 shows the observed angular distributions in (x, y, z) coordinates at CalDet. The distributions show the ratios dx/dz and dy/dz; thus, the angle in the horizontal plane peaks at  $\sim 0^{\circ}$  and the angle in the vertical plane peaks at  $\sim \pm 45^{\circ}$ . The cut-off at |dx/dz|,  $|dy/dz| \sim 3$  arises from a cut on the minimum number of hit planes in an event for a track to be reconstructed.



#### PDFs for Cosmic Ray Muon Angles at CalDet

Figure 7: Cosmic ray muon track angles measured at CalDet. The angles shown are dx/dz and dy/dz, i.e. horizontal and vertical angles with respect to the "beam-line".

A two-dimensional histogram, (in order to retain correlations between dx/dz and dy/dz) of the measured angular distributions is randomly sampled and passed to the strip simulation described above. In this way an ADC spectrum can be generated for a variety of strip light outputs and for each strip orientation. The length of the simulated strips has been set to 1 m as is the case at CalDet. Figure 8 shows the energy spectra generated using 5,000,000 muons for horizontal and vertical strips for a light output of 4 photoelectrons. The light output with track angle ds/dz is also shown in Figure 9.

Note that the light output with ds/dz is not one-to-one, i.e. a track with ds/dz = 2 does not necessarily produce twice as much light as a track with ds/dz = 1. This



Figure 8: The energy spectra generated for horizontal and vertical strips for cosmic ray muons. The light output of each strip is set to be 4 photoelectrons.



Mean Sparsified Response with ds/dz for CalDet CR Muons (Simulated)

Figure 9: The variation of mean light output with track angle for horizontal and vertical strips for cosmic ray muons.

highlights the effect of corner clipping, most importantly in the 4.1 cm strip dimension. These figures also illustrate the asymmetries that are introduced by the cosmic ray muon sample at CalDet.

### 3.2 Applying Corrections

### 3.2.1 Generic CalDet Strip

The technique for performing the path length correction and zero reconstruction is as follows: For each track hit with gain drift corrected and attenuation corrected charge p observed, calculate  $\langle ds \rangle_{xy}$ ,  $\langle P(0) \rangle_{xy}$  and  $\langle P(1) \rangle_{xy}$  (which depend on the track angles and the characteristic light output of the strip). Then fill the strip-end histogram using the following values and weights:

- Value: p, Weight:  $\left(1 \langle P(0) \rangle_{xy} \langle P(1) \rangle_{xy} * f\right)$
- Value: 0, Weight:  $\langle P(0) \rangle_{xy}$
- Value: t/2, Weight:  $\langle P(1) \rangle_{xy} * f$

where t is the path length corrected dynode/sparsification threshold and f is the fraction of the single photoelectron peak below t.

As indicated, to account for the the sparsified single photoelectron hits, some information about the single photoelectron peak is required. Figure 10 shows the peak that is generated by the phototube simulation. From this, the fraction, f, of the peak below the the threshold, t, can be estimated as shown from the annotations on the figure. In real data this can also be attempted by obtaining information from fits to low light level Light Injection data [4]. The fits return the position and the width of the single photoelectron peak; therefore by knowing the sparsification threshold, t, an estimate of f can also be made.



Figure 10: The single photoelectron peak from the phototube simulation. The annotations illustrate the fraction of the peak that is below the sparsification threshold in the strip simulations.

As has been seen from section 2.2, the zero and single photoelectron probabilities depend on the light level; therefore any strip-to-strip calibration procedure which attempts to account for zeros in this way must be iterative. For the purposes of the strip simulation however, the light output is put in by hand, and so this procedure for applying corrections can be tested.

Figure 11 shows the corrected light output with track angle. It can be seen that the horizontal and vertical planes have been corrected such they behave in the same way with ds/dz. However, for a perfect path length correction, a flat response with ds/dz is expected. In the simulation however, the response is seen to increase by  $\sim 5\%$  over the range of track angles plotted.



Path-length Corrected and Zero Reconstructed Response with ds/dz (simulated)

Figure 11: Corrected light output with track angle for cosmic ray muons in CalDet strips. The effect of the logarithmic Landau dependence with path length can be seen.

The origin of this discrepancy is the logarithmic Landau dependence of the energy deposited in scintillator with path length. By applying a correction relative to a 1cm path length, based on the inverse of Figure 3 using the calculated average path length,  $\langle ds \rangle_{xy}$ , (since the truth is not known) this effect can approximately be removed. Figures 12 and 13 show the fully corrected distributions analogous to Figures 8 and 9 above.

The light output with track angle is now seen to be flat and the response of the horizontal and vertical strips lie on top of each other. The corrected energy spectra have different shapes, but the means agree to better than 0.1%, (as expected since the characteristic light output of the strips was set to be the same). Note that the corrected response of the strips for orthogonally traveling (ds/dz = 1) muons is ~5 photoelectrons, rather than 4 photoelectrons. This again is a result of the Landau tail and is the same effect illustrated in Figure 5, (n.b. the Landau correction applied only removed variations with respect to a 1 cm path length).

Figure 14 compares the result in Figure 12 with the path length corrected (and Landau corrected) response when perfect zero reconstruction is used, i.e. from the truth information. The canvas is zoomed into the low end of the energy spectra. It can be



Corrected Energy Spectra for Cosmic Ray Muons at CalDet (simulated)

Figure 12: Corrected energy spectra for simulated cosmic ray muons at CalDet. The histograms are truncated but the displayed means are not. It can be seen that the means agree to better than 0.1% after applying the corrections.



Corrected Response with Track Angle for CR Muons at CalDet

Figure 13: Corrected response with track angle for cosmic ray muons at CalDet including the correction for the logarithmic Landau dependence.

seen that the hit-by-hit re-weighting approach predicts the expected number of zeros well. The deviations that can be seen result from the single photoelectron hits which, when combined with the pedestal, occasionally produce negative values. In the re-weighted histogram, this will not be reproduced. However the difference in the means between the perfectly zero reconstructed and the re-weighted histograms is only  $\sim 0.2\%$  (not shown).



Corrected Energy Spectra for CR Muons at CalDet

Figure 14: Zoom into the low end of the corrected muon energy spectra. The dashed lines show the number of zeros from MC truth and the solid show the results of using the re-weighting approach.

With these tools in place and a procedure for carrying out an accurate path length correction and zero reconstruction, other studies have been carried out. These are described below.

### 3.2.2 Different Light Levels

Samples of 500,000 cosmic ray muons were generated in the CalDet strips for a range of light levels. Figure 15 shows the fractional difference between the uncorrected means for horizontal and vertical strips for light levels ranging between 1 and 8 photoelectrons in half-integer steps. The difference is ~12%. After applying corrections, as can be seen in Figure 16, these differences become less than ~0.5% except for at the lowest light level where the difference is ~0.8%.

### 3.2.3 Different Angular Distribution

In order to verify that this technique is valid at larger angles, a measured angular distribution for Far Detector cosmic ray muons was applied. The distributions were truncated such that both du/dz and dv/dz were allowed to vary between  $0 - 85^{\circ}$ . The strip lengths in the simulation were set to be 8 m to mimic the Far Detector strips and



Fractional Difference between Uncorrected Means for Horizontal and Vertical Strips

Figure 15: The fractional difference between the uncorrected, sparsified means for horizontal and vertical strips.



Figure 16: The fractional difference between the path length corrected and zero reconstructed distribution means for horizontal and vertical strips.

the characteristic light output was set to be 4 photoelectrons. Attenuation in the WLS fibres was not included in this simulation.

In this case no horizontal/vertical asymmetry is introduced since the strips see the same angular distribution in u and v. It is therefore adequate to look at only one strip type. Figure 17 shows the uncorrected and corrected response with ds/dz. After the corrections are applied the response is flat as expected. The mean corrected response is observed to be  $\sim 5$  photoelectrons. This is the same as the value obtained using the CalDet angle cosmic ray angular distributions, indicating that there is no correlation between light output and muon angular distribution once the corrections have been applied.



Response with ds/dz for 8m Strips and FarDet CR Muon Angular Distribution

Figure 17: Uncorrected and corrected response with ds/dz. The strip is exposed to a measured Far Detector cosmic ray muon angular distribution with dx/dz and dy/dz varying between 0 and 85°.

#### 3.2.4 Truncated Means

When dealing with distributions with a Landau tail it is often advantageous to use a truncated mean rather than the true mean such that a few large values do not affect the measurement. To investigate this for the CalDet distributions, the mean corrected response with track angle is plotted again. But this time, a 90% truncated mean is taken at each bin of ds/dz rather than the true mean. Figure 18 shows the result.

It can be seen that this introduces an asymmetry between the horizontal and vertical strips at the  $\sim 2\%$  level. To understand this, consider again Figure 12; the differences in the shape of the spectra dictates that the missing entries will contribute different amounts to the true mean in the two cases. This effect will not be important at the Far or Near Detectors however, since the shapes of the energy spectra should be identical in both the u and v strips.



Corrected Response with 90% Truncation vs Track Angle

Figure 18: Corrected light output with track angle, truncated for each bin of ds/dz, for cosmic ray muons in CalDet strips. A 90% truncation is used here.

#### 3.3Comparison with Data

The work presented in the previous sections deals purely with the expected response given some basic characteristics of the MINOS detectors. As a sanity check, some comparison plots are shown for real data. Figures 19 and 20 are sets of example strip-end energy spectra for horizontal and vertical strips from the CalDet. In each plot, the data is shown in black with error bars, and for comparison, four MC spectra for different light levels are superimposed. It can be seen that the MC generally reproduces the shape of the data.

On closer inspection of the spectra however, it can be seen that often the data has proportionally more entries in the single photoelectron peak than the MC. This is particularly evident when all strip-ends are plotted together. In order to compare the response of the the strip-ends in the detector to the MC, a sample of muons are generated with the CalDet angular distribution and over the CalDet measured range of light levels. The spectra are shown for data and MC for horizontal and vertical strips separately in Figure 21. There is a striking difference in the number of single photoelectrons in the data compared to the MC. Furthermore, there is a larger difference in the total number of hits in the horizontal and vertical strips in the data than in the MC.

Looking at the mean uncorrected light output versus track angle, shown in Figure 22. the difference in the distributions for horizontal and vertical strips is qualitatively similar in both data and MC. Quantitatively however, the data and MC show significant differences. These differences are likely to be related to the discrepancy in the number of single photoelectrons shown in Figure 21

Two possible explanations for the difference between the data and MC are discussed below.



Figure 19: Example uncorrected strip-end energy spectra for horizontal strips from the CalDet CR muon data. Simulated spectra for 2, 3, 4, 5 photoelectrons are shown for comparison.



Figure 20: Example uncorrected strip-end energy spectra for vertical strips from the CalDet CR muon data. Simulated spectra for 2, 3, 4, 5 photoelectrons are shown for comparison.



Sparsified Response for CalDet Cosmic Ray Muons (All Strip-Ends)

Figure 21: Sparsified, uncorrected response for all CalDet strip-ends to cosmic ray muons for real data and MC. Proportionally more entries are seen in the single photoelectron peak in the data than in the MC.



Figure 22: Mean uncorrected light output with track angle for real CalDet data. All strip-ends are included in the distribution.

#### **3.3.1** Transverse Dependence

Measurements of the transverse dependence of light output in MINOS scintillator strips [5] [7], have shown that the response drops by  $\sim 20\%$  with distance from the fibre, (transverse is the 4.1 cm strip dimension). This has been put into the strip simulation by calculating the distance in the y-z plane of the track from the fibre at 0.1 cm steps along the muon path, (where y is the 4.1 cm dimension and z is the 1 cm dimension). At each step, a weight is assigned to that section of the path which is given by:

$$1 - 0.2 \frac{d}{d_{max}}$$

where d is average distance from fibre for that section and  $d_{max}$  is the maximum possible distance from the fibre. The energy deposited and number of ADC counts is then simulated as usual, but the final answer is obtained by multiplying by the ratio of weighted path length to actual path length.

This procedure has some effect on the overall shape of the distribution as can be seen from Figure 23, but the number of single photoelectrons in the simulated sample is still suppressed compared to the data. The absolute value of the means is reduced, but this does not seem to affect the ability of the path length correction and zero reconstruction procedure to equalize the response of the strips as shown in Figure 24.

Sparsified Response for CalDet Strips with 20% Transverse Dependence on Light Output



Figure 23: Simulated strip-end response, including a 20% transverse dependence on light output, for CR muons in CalDet strips.

### 3.3.2 Cross-talk

Cross-talk is the process whereby light or charge on a single pixel leaks into neighbouring pixels on a phototube. Cross-talk may take a number of forms, for example:



Mean Response with ds/dz (20%Transverse Dependence on Light Output)

Figure 24: Simulated strip-end response, including a 20% transverse dependence on light output, with track angle for CR muons in CalDet strips. The uncorrected and corrected response is shown.

photons incident at the photocathode may scatter and be multiplied by a different channel; or electrons produced during multiplication may pass from one channel to another due to imperfect dynode focusing.

In most cases, the effect of cross-talk is largest to nearest-neighbour pixels. Therefore, to minimize confusion (in tracking for example), a particular pattern is used when placing fibres on the faces of the phototubes. In this way, cross-talk hits are spatially separate from real signal hits in the detector. Unfortunately, in some instances, it is unavoidable that nearby strip-ends are also nearby on a phototube face. It is these occurrences that can cause pure cross-talk hits to be included along a track.

Correctly quantifying this in the simple strip simulation is a difficult task, since cross-talk depends on many factors. However, it is known that cross-talk due to muons typically produces a low light level spectrum with a mean of a fraction of a photoelectron,  $(\sim 1-2\%)$  of the source hit [6]). Therefore, the inclusion of these hits in the muon stripend spectra will contribute mainly to the single photoelectron peak. A full detector simulation is required to study this properly.

### 3.4 Applying the Corrections to Real Data

The approach for finding calibration constants in the data is necessarily iterative since the zero reconstruction relies on knowing the light level. Typically in an approach such as this, calibration would be applied to the data assuming some constant, and the mean of the final histogram would be then be used as the next best guess. However, in this case, the iteration would not tend towards the true value. Any guess that decreases the calibration constant causes the number of zeros to increase and so the next guess always becomes lower still. The result is that all constants tend to zero.

Instead, the iteration can be carried out by performing the calibration with some value, then dividing the final strip-end histogram mean by that same value. If the calibration is correct then this value should be unity; if not, then the constant can be changed slightly and the procedure carried out again. To make the next best guess for the calibration constant, consider the following:

$$\overline{x} = \frac{\sum_{i=0}^{N} (1 - \beta_i(\lambda)) n_i}{N\lambda}$$
(8)

where  $\overline{x}$  is the mean of the corrected histogram, in muon energy units (MEUs), when using the calibration constant  $\lambda$  in photoelectrons; N is the number of entries in the histogram;  $n_i$  is hit *i* in photoelectrons.  $\beta_i$  is the calculated probability that hit *i* is not seen, (from  $\langle P(0) \rangle_{ry}$ ).

Similarly, we can also calculate the true mean in MEUs expected, given that the correct calibration constant is used:

$$\hat{x} = \frac{\sum_{i=0}^{N} (1 - \beta_i(\hat{\lambda})) n_i}{N\hat{\lambda}}$$
(9)

where  $\hat{x}$  is the true mean in MEUs and  $\hat{\lambda}$  is the true calibration constant for the stripend. The true mean, as shown in the previous sections, is ~1.25 for the simulated data. Knowing what this value should be is the key to the iteration.

For both  $\overline{x}$  and  $\hat{x}$ , only the true zeros are included in the means; the sparsified single photoelectrons are ignored.

Now,

$$\frac{\overline{x}}{\hat{x}} = \frac{\hat{\lambda} \sum_{i=0}^{N} (1 - \beta_i(\lambda)) n_i}{\lambda \sum_{i=0}^{N} (1 - \beta_i(\hat{\lambda})) n_i}$$
(10)

and using the approximation:

$$\left(\frac{d\langle P(0)\rangle_{xy}}{d\lambda}\right)_{i} \approx \left(\frac{\Delta\langle P(0)\rangle_{xy}}{\Delta\lambda}\right)_{i} = \frac{\beta_{i}(\hat{\lambda}) - \beta_{i}(\lambda)}{\hat{\lambda} - \lambda}$$
(11)

where  $d \langle P(0) \rangle_{xy}$  is obtained by differentiating equation 4 with respect to  $\lambda$ .

Then, the best guess for  $\hat{\lambda}$  is given by:

$$\hat{\lambda} \approx \lambda \frac{(\overline{x} + \alpha)}{(\hat{x} + \alpha)}$$
(12)

where  $\alpha = \frac{1}{N} \sum_{i=0}^{N} n_i \left( \frac{d \langle P(0) \rangle_{xy}}{d \lambda} \right)_i$ .

One final issue in this approach is related to the Landau tail. A strip-end with a light output of 4 photoelectrons produces a corrected distribution with a mean of  $\sim 5$  photoelectrons in the simulation. Therefore, an iterative procedure should ideally

approach 1.25 rather than unity. In the data, this will also be the case, however there is no guarantee that the ratio 4 : 5 will be correct, (since it is based on the GEANT simulations described in section 2.1.1).

One possible way to measure the correct ratio in the data is to select out very straight muons, (as they do not require a path length correction). The distributions can then be fitted using the function shown in Figure 5 to extract the Landau peak and width.

## 4 Conclusions

The path length correction and zero reconstruction method proposed removes variations in response due to the geometry of a scintillator strip in the simulated CalDet data to better than 0.5% for strip light levels above 1 photoelectron. A qualitative comparison with CalDet data indicates that there are other effects present that this approach does not attempt to account for. However, it is not clear at this stage how fully these effects need to be understood in order to perform adequate strip-to-strip calibration in MINOS<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Strip-to-strip calibration for the 2002 and 2003 CalDet data has been attempted with cosmic ray muons based on the path length correction and zero reconstruction procedure described here. A full description of the techniques used to make the calibration constants and an analysis of the quality of the constants is given in [7].

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