Taking the Fourier transform of ATLAS

James Monk, Mario Campanelli & Chris Taylor

University College London
Gower Street, London, WC1E 6BT
United Kingdom

August 31, 2009

Abstract

Hadronic final states in hadron-hadron collisions are often studied by clustering final state hadrons into jets, each jet approximately corresponding to a hard parton. The typical size of the jetfinders used for analyses at ATLAS is between 0.4 and 1.0 in $\eta - \phi$. On the other hand, there may be structures of interest in an event that are of a different scale to the jet size. For example, to a first approximation the underlying event is a uniform emission of radiation spanning the entire detector, colour connection effects between hard partons may fill the region between a jet and the proton remnant and hadronisation effects may extend beyond the jets. We consider the possibility of performing a Fourier decomposition on individual events in order to produce a power spectrum of the transverse energy radiated at different angular scales. We attempt to identify correlations in the emission of radiation over distances ranging from the full detector size to approximately 0.2 in $\eta - \phi$. As a demonstration this technique by applying it to events produced from colour singlet exchange, in which there should exist a region of the detector that contains very little radiation that originates from the hardest scattering.

1 Introduction

The study of collective event-wide distributions, with the aim of extracting QCD properties, has been carried on in several generations of lepton and hadron colliders [7]. We believe that within a single event it should be possible to isolate both some global characteristics as well as the smaller scale radiation in order to simultaneously discriminate between various hard scattering and hadronisation models.

This problem of separating objects of different size is not unique to high energy collider physics. In the field of cosmology, for example, a key observation is the angular size of correlations in the temperature and polarisation of the cosmic microwave background (CMB). This is studied by decomposing the image of the CMB into a set of spherical harmonics in order to produce the well known angular power spectrum plot [8]. Fourier transforms are also used in many other fields of physics and signal processing, but have so far remained largely ignored in the analysis of high energy collider physics data. In this note we propose the idea of performing a Fourier decomposition on events at ATLAS with the goal of separating via their different Fourier coefficients the large scale features such as the underlying event from smaller features such as hadronisation, showering and the hard jets.

As a test-case, and because it was the event topology that inspired us to consider using a Fourier transform, we will examine di-jet production through the exchange of a colour singlet object. Such a colour singlet interactions occur in both vector boson exchange or, with much a larger cross section, hard diffraction with forward jets. Such di-jets produced via the exchange of a colour singlet have a feature that there is a region in pseudo-rapidity, η, between the jets with suppressed emission of radiation. The low-activity region between the jets is commonly known as a gap. This jet-gap-jet topology is present in both diffractive and vector boson fusion events and could be used to remove background events produced through the exchange of coloured QCD objects. However, the underlying event, that is multiple interactions between the same pair of protons, typically produces radiation throughout the whole detector, including in the gap region. If a ``clean'' rapidity gap is required, only a small fraction (around 0.1, depending on the model) of the original colour-singlet events (those with a very soft underlying event) can be retained.This could make it difficult or even impossible to distinguish colour connection di-jets (referred to as QCD di-jets from now on) from colour singlet di-jets. Pile-up due to a high proton density in the beam bunch has a similar effect, bringing the efficiency for finding a gap almost to zero. Clearly a better strategy for identifying gaps is needed.

The problem of separating the colour connection effect from the underlying event or pile up is one of separating features of differing physical size in the event. As already stated, the underlying event fills the whole detector with radiation at all η. Colour connection effects are approximately the size of the jet-jet or jet-beamline interval. Hadronisation and showering effects can be expected to be of a similar size (i.e. smaller) to the colour connection effects. The hard jets will be smaller still, with a radius of $R \simeq 0.5$ and there may be jets originating from softer partons with R as small as 0.1.

2 Monte Carlo Event samples and selection

Herwig [5] was used to generate a sample of QCD di-jet events (for the background) and colour singlet exchange events (for the signal). The colour singlet exchange process in Herwig has been modified for ATLAS according to [6]. In order to understand the effect of the underlying event we produced samples of QCD and colour singlet events both with and without the Jimmy [2] underlying event model turned on. This initial study used the Atlfast fast detector simulation [4], however full simulation datasets of events filtered to contain jets widely separated in η have recently been produced and will be used in future studies.

In order to apply an event selection we run the KT jet algorithm [3] using an R parameter of 0.7. Events are chosen in which the two leading jets both have transverse energy, E_T, above 30 GeV and are separated by an η interval of $\Delta\eta > 4$. No requirement is made on the absence or presence of radiation in the η interval between the two leading jets. We shall refer to the harder of the two jets as the hardest jet and the softer of the two jets as the softest jet. Atlfast produces neither calorimeter towers, cells nor topoclusters in its output, so we run the KT algorithm a second time with an R parameter of 0.1 and a minimum E_T cut of 1 GeV to provide the input to the Fourier transform.

3 Fourier decomposition

Unlike the case of the CMB, ATLAS does not have complete 4π coverage of its events; it is bounded and periodic in the ϕ direction but not in the η direction. Further, the cylindrical co-ordinate system of ATLAS together with the additive nature of rapidity under longitudinal boosts is more suited to cylindrical rather than spherical harmonics. The simplest Fourier transform that can be made, and therefore the one that we attempt first, is the discrete one dimensional Fourier transform of the E_T distribution in ϕ. The N coefficients, C_n, of a one dimensional discrete Fourier transform are

$$C_n = \frac{1}{\sqrt{N}} \sum_{l=0}^{N} E_T\left(\phi_l\right) e^{in\phi_l} \tag{1}$$

The ϕ co-ordinate is defined such that the centroid of the hardest jet lies at $\phi = 0$. The positive ϕ direction is defined such that the softer jet lies between $\phi = 0$ and $\phi = \pi$. The calorimeter is divided into a grid of $N = 32$ bins in ϕ and the E_T sum, $E_T\left(\phi_n\right)$, is calculated in each bin from the KT 0.1 clusters. The centre of the $l = 0$ bin is aligned with $\phi = 0$, the centre of the hardest jet. Notice that the input to the Fourier decomposition is 32 real $E_T\left(\phi_n\right)$ and the output is 32 complex coefficients. Performing the Fourier transform therefore seemingly doubles the number of degrees of freedom. However, there is a symmetry between the n^{th} and $\left(N-n\right)^{th}$ coefficient such that

$$C_{N-n} = C_n^{\star} \tag{2}$$

so there are only 16 independent complex coefficients in the output, matching the amount of information in the input distribution.

We use the GNU Scientific Library (GSL) [1], available in the ATLAS software framework as the package `External/AtlasGSL`, to provide fast one dimensional Fourier transform routines. Note that these routines perform best if the number of input data bins can be written as a power of 2, $N = 2^{\hat{N}}$, with $\hat{N}$ an integer. In order to verify that the transformation has been performed correctly we take the output coefficients for a single event, reverse the transformation and overlay the resulting function on the input set of E_T bins. The result in figure 1 shows that the curve produced by the Fourier coefficients matches the input distribution. The match is good for a colour singlet exchange event that contains very little activity other than the di-jets and a QCD di-jet event that includes activity outside of the leading jets.

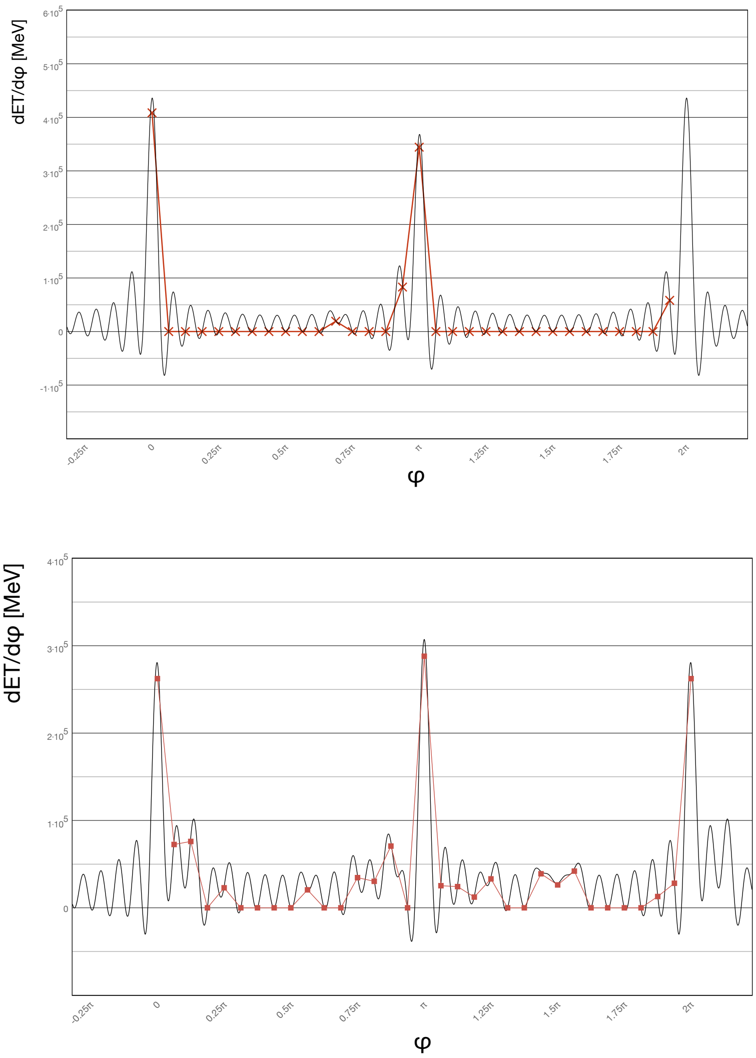

Figure 1: Reverse Fourier transform of the N coefficients (black curve) compared to the input data from a single event (red line). The top plot is a colour singlet exchange event without underlying event and contains almost no activity away from the leading jets. The bottom event is a QCD di-jet event with underlying event and shows activity away from the leading jets. In both cases the small features of radiation between the jets are present in the reverse function. Note that the Black curve has been scaled down by a third in order to overlay on the input E_T.

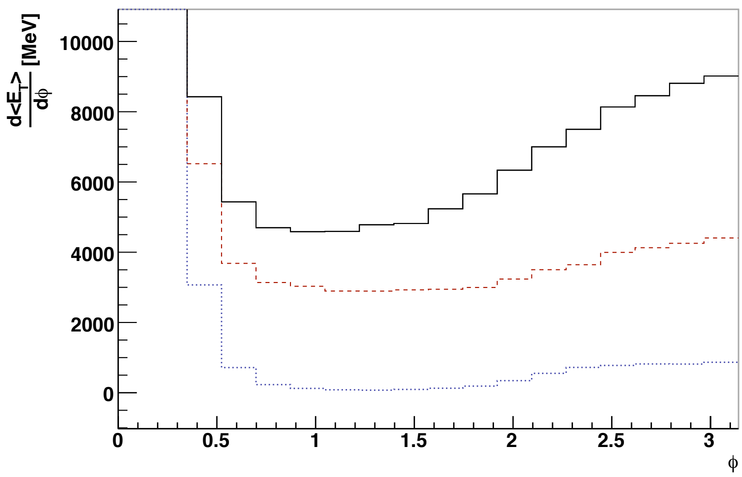

Figure 2: The flow of E_T Vs. ϕ in the half of the rapidity interval that is nearest the hardest jet. Out-of-jet radiation from the softer jet causes a rise towards $\phi = \pi$. The blue dotted line shows di-jet events produced from colour singlet exchange without underlying event, the red dashed line shows the effect of turning the underlying event on and the solid black line shows colour connected QCD di-jet events.

4 Monte Carlo Results

We first demonstrate the difference between colour singlet exchange and colour connected QCD di-jet production by plotting the E_T flow against the distance ϕ from the hardest jet. In figure 2 the η interval between the two leading jets has been divided in two and we plot the ϕ distribution of radiation, weighted by E_T, in the region between the centre of the hardest jet and the middle of the gap. The hardest jet is at $\phi = 0$ and, after an initial fall, there is a rise in E_T towards $\phi = \pi$. The rise is caused by showering and hadronisation from the softer jet on the opposite side of the η interval. The three lines show the effect of colour singlet exchange without underlying event (blue dots), turning the underlying event on (red dashes) and a colour connection between the two leading jets (solid black). Underlying event adds radiation uniformly in ϕ and colour connection effects increase the rise of E_T with ϕ due to an enhancement of showering and hadronisation.

We apply the Fourier transform to a sample of four million colour singlet events without underlying event, which is the most purely di-jet-like sample considered. The average magnitude, real part, imaginary part and phase of the first sixteen coefficients, C_n, are shown in figure 3 for the colour singlet exchange sample without underlying event. Both the magnitude and real part show a suppression of the odd coefficients, especially for the lower frequencies. The n^{th} coefficient corresponds to features in the event of size $R \simeq \pi/n$. Since the phase of each event is chosen so that the hardest jet appears at $\phi = 0$,

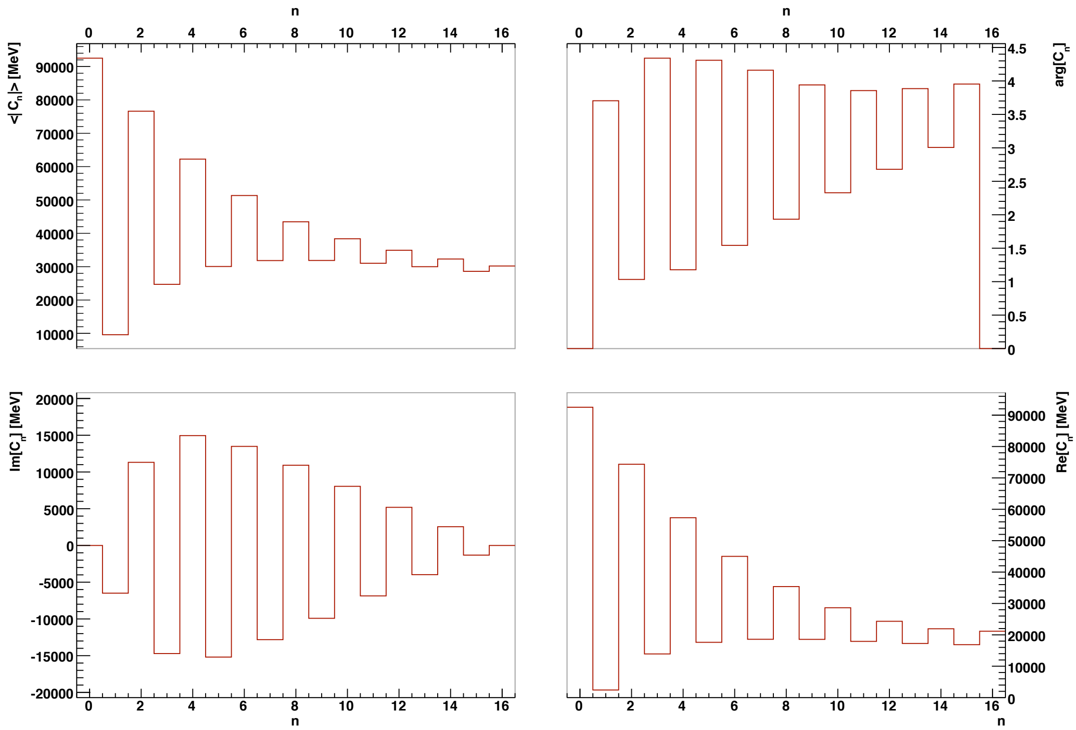

Figure 3: Clockwise from top left: the mean magnitude, phase, real part and imaginary part of the first sixteen coefficients for colour singlet exchange events without underlying event.

the odd coefficients *always* produce a trough at $\phi = \pi$, which is the approximate location of the softer jet. Odd coefficients therefore represent features that deviate from the behaviour of back-to-back di-jets. The small n odd coefficients therefore correspond to non-di-jet like features that are large; in other words radiation between the jets. Since this colour singlet sample contains very little radiation between jets, the odd coefficients are suppressed. This is confirmed in figure 4, in which the magnitudes of the coefficients for colour connected QCD di-jets with underlying event show a smaller suppression of the odd coefficients, indicating that these events are less di-jet like, as expected.

The meaning of the separate real and imaginary or even and odd coefficients can best be interpreted using figure 5, in which the reverse Fourier transform is performed on the average coefficient over four million events for both the colour singlet (no underlying event) and QCD (with underlying event) di-jets. The reverse transformation is performed separately for the real, imaginary, even and odd coefficients. For these di-jet events the real even part is largely responsible for recreating the hardest jet, while the real odd part describes the difference in E_T between the two leading jets. The imaginary part is responsible for the shift of the softer jet away from $\phi = \pi$ and, together with the real

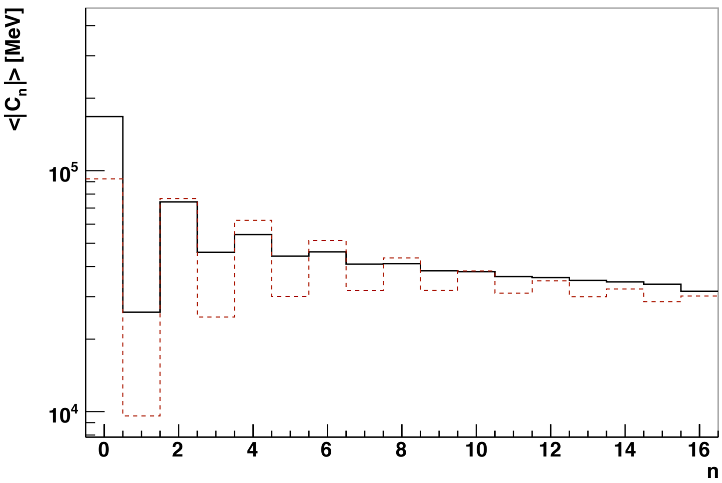

Figure 4: The average magnitude of the first 16 Fourier coefficients for QCD di-jets with underlying event (black solid line). Compared to the colour singlet jets without underlying event (red dashed line) the events with colour flow show less suppression of the small n odd coefficients.

odd part, a broadening of the softer jet.

The imaginary part of the coefficients shows a peak around $n = 4-6$, which corresponds to features of size $R \simeq 0.6$, the typical size of the hadronic jets. The choice of phase means that the imaginary components correspond to radiation away from the centroid of the hardest jet. Such radiation could either be the softer jet or radiation between the jets. Figure 6 shows the distribution of the largest imaginary coefficient in each event, comparing colour singlet events with and without underlying event and QCD di-jets. The colour singlet sample shows a peak around $n = 8$ (again, roughly the size of a jet), which turns into a shoulder once the underlying event is present. The underlying event effects the small n coefficients more than the medium to large n. In the absence of underlying event the QCD di-jets show a peak around $n = 3$, which shows that the larger scale radiation between jets is more significant in that sample. There is a bump in the distribution around $n = 8$, corresponding to the jet-like features that are also present (and more dominant) in the colour singlet sample. Turning on the underlying event favours lower n coefficients and flattens the bump around $n = 8$. Overall, comparing QCD with colour singlet di-jets shows that, even in the presence of underlying event, the colour singlet sample has an excess around $n = 8$, indicating that those events are more back-to-back di-jet like. On the other hand, the QCD sample is dominated more by the lower coefficients that arise from the inter-jet radiation caused by colour connection effects, hadronisation and showering. Note the relative depletion of the 3^{rd} and (to a lesser extent) 5^{th} coefficients in the colour singlet sample with underlying

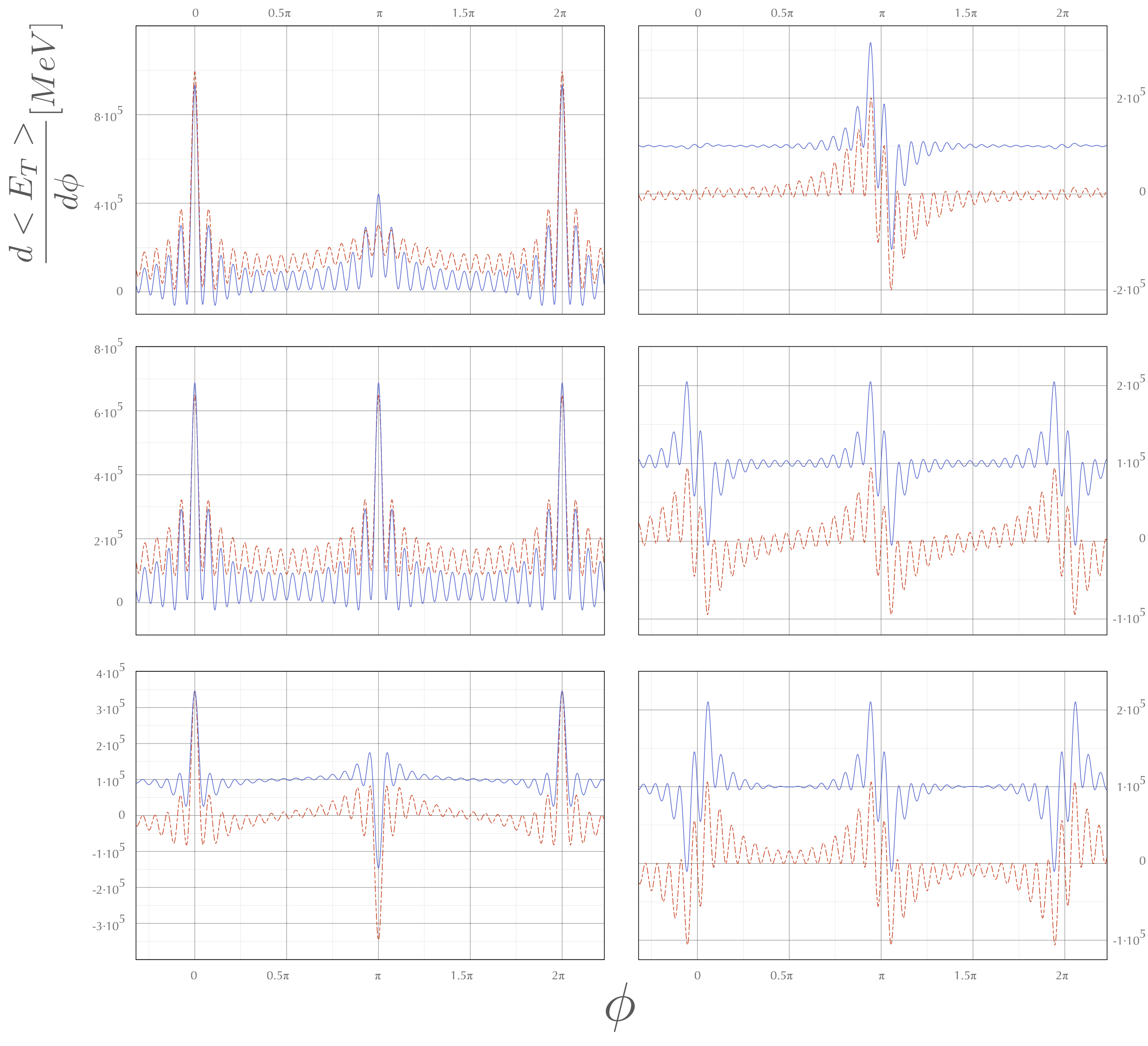

Figure 5: The reverse Fourier transform using the average coefficients over the colour singlet sample without underlying event (blue solid curve) and the QCD di-jets with underlying event (red dashed curve). The left column uses only the real part of the coefficients, the right column only the imaginary. The top row is both even and odd components combined, the middle row shows only the even component and the bottom row only the odd components. For all but the total and real even plots the blue curve has been raised by 10^5 MeV in order to separate it from the red curve.

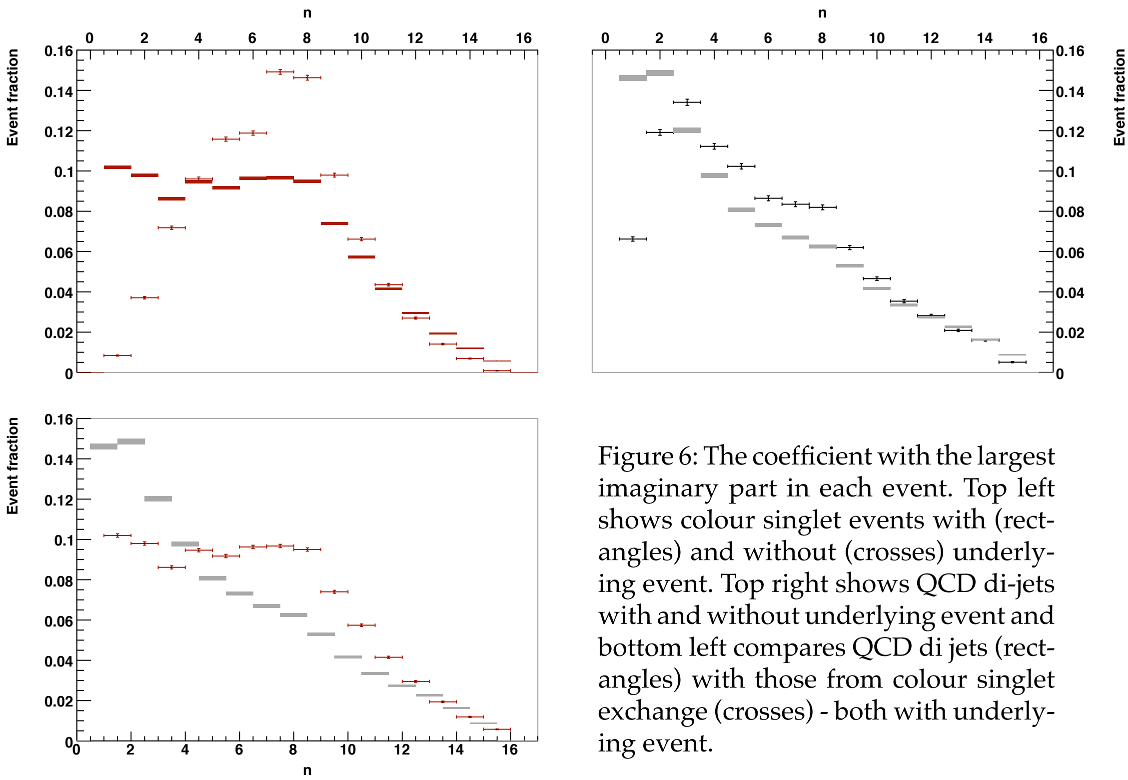

Figure 6: The coefficient with the largest imaginary part in each event. Top left shows colour singlet events with (rectangles) and without (crosses) underlying event. Top right shows QCD di-jets with and without underlying event and bottom left compares QCD di jets (rectangles) with those from colour singlet exchange (crosses) - both with underlying event.

event. The $n = 3$ and $n = 5$ coefficients are expected to correspond to radiation between the jets and are at large enough n that the effect of the underlying event is not completely dominant. Thus the $n = 3, 5$ coefficients may be sensitive to differences in colour connection effects, showering and hadronisation.

5 Conclusion

Fourier analysis is a relatively unexplored tool for high energy physics that, based upon this first simple application, appears to be quite interesting. We believe that a very promising area for its application is the study of the different scales present in fully-hadronic final states in a hadron collider. In the best-case scenario a Fourier decomposition should be able to separate the different effects of hard scattering, showering, hadronisation and the underlying event, each one of them having a distinctive $\eta - \phi$ scale.

Another area in which this technique may prove to be useful is the tuning and understanding of the detector simulation. For example, calorimeter noise if present throughout the calorimeter will appear in the lower frequencies, whereas the higher frequencies are more sensitive to the effective calorimeter granularity, which provides a cut-off on the frequency.

In this note, we show the application of this idea, still in the one-dimensional form, to the analysis of dijet events with large rapidity separation between the main jets. In partiuclar we have shown the effect of the underlying event and specific features of colour singlet and colour octet exchange. Features that are otherwise difficult to extract in a traditional analysis emerge in a quite clear way, showing that the Fourier transform technique could be useful for studying the early collisions at the LHC.

We aim to extend the Fourier transform to a full two dimensional decomposition, and we hope the additional information thus preserved will render the analysis more powerful. We also plan to look for further inspiration in fields such as image processing and compression, which employ similar but more sophisticated techniques such as wavelet analysis.

References

[1] http://www.gnu.org/software/gsl/manual/html_node/.

[2] J. M. Butterworth, Jeffrey R. Forshaw, and M. H. Seymour. Multiparton interactions in photoproduction at HERA. *Z. Phys.*, C72:637--646, 1996.

[3] Matteo Cacciari and Gavin P. Salam. Dispelling the N^3 myth for the k_t jet-finder. *Phys. Lett.*, B641:57--61, 2006.

[4] D Cavalli, D Costanzo, S Dean, M Dührssen, S Hassani, M Heldmann, K Jakobs, A Nairz, A Phillips, S Resconi, E Richter-Was, P Sherwood, L Vacavant, I Vivarelli, J B De Vivie de Régie, and I Wingerter-Seez. Performance of the atlas fast simulation atlfast. Technical Report ATL-PHYS-INT-2007-005. ATL-COM-PHYS-2007-012, CERN, Geneva, Jan 2007.

[5] G. Corcella et al. HERWIG 6.5 release note. 2002.

[6] Brian Cox, Jeffrey R. Forshaw, and Leif Lonnblad. Hard color singlet exchange at the Tevatron. *JHEP*, 10:023, 1999.

[7] Geoffrey C. Fox and Stephen Wolfram. Observables for the Analysis of Event Shapes in e+ e- Annihilation and Other Processes. *Phys. Rev. Lett.*, 41:1581, 1978.

[8] M. R. Nolta et al. Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Angular Power Spectra. *Astrophys. J. Suppl.*, 180:296--305, 2009.