Queen Mary & Westfield College
Royal Holloway UoL
University College

Intercollegiate Postgraduate Course in Elementary Particle Physics

Paper 1: Monday, 6 April 1998

Time allowed for the Examination: 3 hours

Attempt THREE *questions - One from each section*

Section A: Symmetries and Conservation Laws
Section B: Electroweak Interactions part 1
Section C: LEP Physics

Section A : Symmetries and Conservation Laws

Question 1

(a) State the four group axioms. **2 marks**

(b) What is meant by the "fundamental" and "adjoint" representations of a Lie group. What are their relevance to gauge theories in particle physics? Give examples for $SU(2)$ and $SU(3)$. **6 marks**

(c) Explain what is meant by the term "generator" of a unitary group. **2 marks**

(d) The generators of the group $SU(2)$ are the Pauli spin matrices:

$$\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Derive the structure constants of this group. **3 marks**

(e) In QCD it is postulated that all observable hadrons are colour singlets. Using Young's tableaux show that $q\bar{q}$ and qqq combinations may be colour singlets but qq and $qq\bar{q}$ may **not**. Are any other colour singlet states possible? **7 marks**

Question 2

(a) Describe how B meson decays can be used to measure the magnitudes of two of the CKM matrix elements. **4 marks**

(b) Draw the unitarity triangle, labelling its sides and angles. What is its area in terms of the Wolfenstein parameters **3 marks**

(c) Explain, by means of appropriate diagrams, how a neutral meson such as B^0 can oscillate into its anti-particle $\bar{\mathrm{B}}^0$. Assuming, the weak eigenstates have equal lifetimes, how is the oscillation frequency related to their masses? What are the particular experimental challenges of making a measurement of $\bar{\mathrm{B}}_s^0$ oscillations. **8 marks**

(d) Explain how the presence of B meson mixing gives rise to indirect CP violation. Give an example of a process that may be used to measure one of the angles of the unitarity triangle. What are the experimental difficulties of such a measurement? **5 marks**

Section B : Electroweak Interactions part 1

Question 3

1. If the Dirac equation is written in the form

$$H\psi = (\alpha.\mathbf{p} + \beta m)\psi$$

show that $\alpha_1, \alpha_2, \alpha_3$ and β all anticommute with each other and that $\alpha_i^2 = \beta^2 = I$ where I is the unit matrix.

2. Obtain the covariant form of the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

and show that $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$.

3. Show that

$$Tr(\not{a}\ \not{b}) = 4(a.b)$$

and

$$Tr(\not{a}\ \not{b}\ \not{c}\ \not{d}) = 4[(a.b)(c.d) - (a.c)(b.d) + (a.d)(b.c)].$$

4. The four momenta (k, p) and (k', p') describe the initial and final state kinematics of $e^-\mu^-$ scattering.

Write down the first order matrix element for this process.

5. The cross section for electron - muon scattering, in the limit that masses tend to zero, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}}\left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $e^-\ \mu^-$ scattering and α is the fine structure constant. From the above obtain the differential cross section for e^+e^- to $\mu^+\mu^-$ annihilation and show that the angular distribution is of the form $(1 + cos^2\theta)$.

Use simple helicity argument to explain this angular distribution given the rotation matrix element $d^1_{\pm 1} = 1/2(1 \pm cos\theta)$.

6. The ratio of the total e^+e^- annihilation cross section to the $e^+e^- \rightarrow \mu^+\mu^-$ cross section is called R. Explain in as much detail as you can how R varies between threshold and 150 Gev total energy.

Question 4

1. The spin one (vector) massless field A^μ coupling to a current J^μ satisfies the partial differential equation

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$$

Modify the above to apply for a massive spin one particle of mass M and show that the Lorentz condition $\partial_\nu A^\nu$ is deduced.

2. For a massive spin one particle of mass M, energy p_0 which propagates along the z axis with momentum p_z show that

$$\begin{aligned} \epsilon_1^\mu &= (0,1,0,0) \\ \epsilon_2^\mu &= (0,0,1,0) \\ \epsilon_3^\mu &= \frac{1}{M}(p_z,0,0,p_0) \end{aligned}$$

is a suitable set of polarization vectors.

3. Hence obtain the sum over the polarization states,

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

4. Write down the propagator for a massive spin one particle and show how this may be modified for an unstable spin one particle whose full width Γ is much less than the mass M.

5. Write down the matrix element for W^+ decay to $e^+\nu$. If the partial width for this decay mode is Γ_i what would you expect the total width to be giving reasons for your answer?

Section C : LEP Physics

Question 5

(a) Briefly describe Bhabha scattering. **3 marks**

(b) Discuss the design criteria for a low angle luminosity monitor for a LEP experiment, which makes use of Bhabha scattering. **7 marks**

(c) Explain the development of events in which a Z-boson decays to a $q\overline{q}$ pair, and discuss the selection of hadronic events in LEP. **7 marks**

(d) Draw the Feynman diagrams for the first order strong correction, and for the most important part of the second order strong corrections, to the $q\overline{q}$ decay. **3 marks**

Question 6

(a) Outline an argument which explains why the existence of a Higgs field, which gives masses to the massive particles of the standard model, results in the existence of a Higgs particle, whose coupling to a standard model fermion is equal to the fermion mass divided by the Higgs vacuum expectation value (vev). **4 marks**

(b) Why must the Higgs field be *scalar*, whereas all the other standard model fields are either vector or fermion fields? What other *scalar* particles could we hope to discover at LEP2 in popular extensions of the standard model? **3 marks**

(c) Using simple dimensional arguments, or otherwise, estimate the cross section in *pb* for *direct* production of a single Higgs particle of mass 100 GeV/c^2 at LEP2 and hence explain why this process is not relevant to LEP2 searches. **3 marks**

(d) Draw the two Feynman diagrams responsible for standard model Higgs production at LEP2. Indicate which one is dominant for lower Higgs masses and hence provide a "rule of thumb" as to the maximum Higgs mass detectable for a given value of $\sqrt{s}$. **3 marks**

(e) For the dominant diagram in (c), write down an approximate expression for the matrix element and indicate, without detailed evaluation, how you would turn this matrix element into an expression for a cross section. **3 marks**

(f) Which physics backgrounds would you expect to be important at LEP2 for the discovery of a Higgs of mass 100 GeV/c^2? What experimental techniques can be used to reduce these backgrounds? **4 marks**

[You are given that $1GeV^{-2} = 3.9 \times 10^8 pb$, the Higgs vev is 250 GeV, the mass of the electron= 0.51 MeV/c^2, the mass of the Z^0 is 91 GeV/c^2 and $\sin^2\theta_w = 0.23$]