Queen Mary, University of London
Royal Holloway, University of London
University College London
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Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 1:

Section A: Symmetries and Conservation Laws (40 marks)

Section B: LEP and Linear Collider (20 marks)

Wednesday, 28 January, 2004.

Time allowed for the Examination: 3 hours.

Answer *ALL* questions.

Books and notes may be consulted.

Section A: Symmetries and Conservation Laws

Question 1 [20 marks]

Consider the 3-flavour (u, d, s) Quark Model.

(a) Use Young Tableaux to identify the possible combinations of 3 quarks in the $\mathrm{SU}(6)_{flavour\&spin}$ Model. Give the multiplicities for all of the combinations, and calculate explicitly the multiplicity for the Tableau corresponding to the *observed* Baryon states. [2 marks]

(b) In the $\mathrm{SU}(3)_{flavour} \otimes \mathrm{SU}(2)_{spin} \otimes \mathrm{SU}(3)_{colour}$ Model, explain how suitable wave-functions for the *observed* Baryon states are obtained and account for the multiplicities of the various multiplets. [5 marks]

(c) The proton wave-function ($S_z = +\frac{1}{2}$) in terms of flavour and spin is

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{6}}\{(ud + du)u - 2uud\} \cdot \frac{1}{\sqrt{6}}\{(\uparrow\downarrow + \downarrow\uparrow)\uparrow - 2\uparrow\uparrow\downarrow\} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}(ud - du)u \cdot \frac{1}{\sqrt{2}}\{(\uparrow\downarrow - \downarrow\uparrow)\uparrow$$

Apply the Isospin lowering operator

$$I_- = I_-^1 + I_-^2 + I_-^3$$

(superscripts refer to 1st, 2nd and 3rd quarks) to this wave-function. Identify the resultant state and express its wave-function in a form to make the similarity with that of the proton obvious.

[4 marks]

(d) The Magnetic Moment operator for a single quark is proportional to the product of the charge and spin operators: $Q\mathbf{S}$. When considering a Baryon, since the wave-function is symmetrised with respect to all three quarks, it will suffice to consider only the third quark (and multiply the result by 3). Find the ratios of the magnetic moments of the Proton and Neutron, by considering the Matrix Elements for the operator $Q^3\sigma_z^3$ (the superscript '3' indicates that these operators only act on the third quark).
(*Hints:* Q only operates on the flavour states and σ_z only operates on the spin states. The charge-flavour parts and spin parts of the calculation factorise. Work with the flavour ($\phi_{M,S}$ and $\phi_{M,A}$) and spin ($\chi_{M,S}$ and $\chi_{M,A}$) wave-functions and avoid evaluating expressions until really necessary. Use the symmetries identified in part (c).) [7 marks]

(e) In some GUT Models, $p \to \pi^0 e^+$. Suppose a proton is polarised with its spin upwards, then the angular distribution of the positrons will depend on the amplitudes for the different helicity states. What would be the angular distribution of the right-handed positrons ? Various rotations matrices which you can find in the PDG are reproduced below. [2 marks]

Some Rotation Matrices:

$$d^1_{0,0} = \cos\theta \qquad d^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}} = \cos\frac{\theta}{2} \qquad d^{\frac{1}{2}}_{\frac{1}{2},\frac{-1}{2}} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

Question 2 [20 marks]

(a) Write down the six unitarity conditions for the 3×3 CKM matrix. Sketch the six triangles labelling their sides and calculate their areas to lowest order in the Wolfenstein parameterization of the CKM matrix. [4 marks]

(b) Show that the CP asymmetry $A_{\mathrm{CP}}(t)$ in the decay channel $B^0 \to J/\psi K^0_{\mathrm{S}}$ is related to the sine of the angle β of the unitarity triangle by

$$A_{\mathrm{CP}}(t) \propto \sin 2\beta \, .$$

Begin by drawing all the relevant Feynman diagrams, labelling the CKM matrix elements.

Outline the experimental procedure for making a measurement of this asymmetry at a B factory experiment like BaBar. [6 marks]

(c) Draw the Feynman diagrams for the process $B^- \to K^- \pi^0$, labelling the CKM matrix elements. (Assume that the penguin amplitude is dominated by the top quark loop.)

Using the Wolfenstein parameterization for the CKM matrix V, find the dependence of the CP asymmetry A_{CP} in this channel on the parameters of the CKM matrix.

Show that A_{CP} is proportional to the area of the unitarity triangle. Use the unitarity conditions to show that there would exist no CP violation for this type of decay if there were only two generations. [10 marks]

Section B: LEP and Linear Collider

Question 3 [20 marks]

Suppose a new theory predicts a pair of Higgs bosons:

H_1 has mass 110 GeV and direct standard model couplings to W bosons, u-type quarks and charged leptons only.

H_2 has mass 118 GeV and direct standard model couplings to Z bosons, d-type quarks and neutrinos only.

Both of them have loop couplings to gamma-gamma and gluon-gluon, with the particles in the loops determined by the direct couplings.

Explain why these two particles would not have been detected in the LEP experiments.

How would each of them be detected at LHC and at a Linear e^+e^- Collider with 200 to 800 GeV in the centre-of-mass? Specify the production mechanisms, the decay channels and the estimated relative abundances of accessible signals.

Explain whether and how the signals from the two particles might be separated from one another at either or both colliders? How precisely might the masses and accessible branching ratios be measured at each collider?