Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 2: The Standard Model

Wednesday, 31 January 2007

Time allowed for Examination: 3 hours

Answer **ALL** questions

Books and notes may be consulted

The Standard Model

Question 1 (4 marks)

At a collider, two high energy particles, A and B with energies E_A and E_B, which are much greater than their rest masses, collide head on. Derive the expression for the centre-of-mass energy. **[1]**

Using this expression, what would be the centre-of-mass energy of a proposed future facility ("LHeC") which will collide 7 TeV protons with 70 GeV electrons? **[1]**

Now consider particle B (the proton) to be at rest. Derive the formula for the centre-of-mass energy of such a fixed-target experiment. **[1]**

What electron beam energy would be required in the fixed-target experiment in order to achieve the same centre-of-mass energy as in the proposed LHeC facility? **[1]**

Question 2 (6 marks)

Draw the simplest Feynman diagrams for $e^-\mu^- \to e^-\mu^-$ scattering and e^+e^- annihilation to $\mu^+\mu^-$. **[2]**

Using the amplitude for $e^-(k) + \mu^-(p) \to e^-(k') + \mu^-(p')$,

$$|T_{\text{fi}}|^2 = \frac{8e^4}{q^4}\left[(k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p')\right],$$

show that the cross section for $e^+ + e^- \to \mu^+ + \mu^-$ is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}2e^4\left[\frac{t^2+u^2}{s^2}\right]$$

[2]

Starting from $\sigma = 4\pi\alpha^2/3s$ and explicitly showing your working, demonstrate that the total cross section can be written in approximate form as:

$$\sigma \sim \frac{20\text{ (nb)}}{E^2_{\text{beam}}\text{ (in GeV}^2)},$$

where E_{beam} is the beam energy (of both the e^+ and e^-).
($\alpha = \frac{1}{137.036}$, $(1\text{ GeV})^{-2} = 0.389$ mb) **[2]**

Question 3 (7 marks)

The amplitude for the decay $\pi^-(q) \to \mu^-(p) + \bar{\nu_\mu}(k)$ is given by:

$$|T_{\text{fi}}|^2 = \frac{G_F^2}{2}f_\pi^2\cos^2\theta_c m_\mu^2 \text{Tr}\left[(\not{p}+m_\mu)(1-\gamma^5)\not{k}(1+\gamma^5)\right]$$

Use Trace theorems to show this simplifies to

$$|T_{\text{fi}}|^2 = 4G_F^2 f_\pi^2 \cos^2\theta_c m_\mu^2 (p \cdot k)$$

[4]

The ratio of decay rates:

$$R = \frac{\Gamma(K^- \to e^- + \bar{\nu_e})}{\Gamma(K^- \to \mu^- + \bar{\nu_\mu})}$$

can be written in terms of the particle masses. Use this relation to give the value to 2 decimal places showing that the rate is close to that measured from experiment, $\sim 2.44 \times 10^{-5}$.
($m_e = 0.511$ MeV, $m_\mu = 105.7$ MeV, $m_K = 493.7$ MeV) **[3]**

Question 4 (6 marks)

For a 2→2 scattering process, $A + B \to C + D$ write down the Mandelstam variables in terms of the 4-momenta of the particles. **[2]**

In pp collisions, a final state high-energy ("direct") photon can be produced by either $q\bar{q}$ annihilation or the Compton process (initial state: gq). Draw the four Feynman diagrams for the production of direct photons, labelling each as either a s-, t- or u-channel process. **[4]**

Question 5 (3 marks)

Starting from the Dirac equation in terms of α and β matrices:

$$E\psi = (\alpha_i p_i + \beta m)\psi$$

derive its covariant form. **[3]**

Question 6 (5 marks)

For $\sqrt{s} = 35$ GeV, what would you expect the value of

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

to be when considering only the EM coupling? **[2]**

Draw a higher-order diagram (i.e. consideration of the strong force) which would affect this value. **[1]**

Briefly describe how such higher-order diagrams led to the discovery of the gluon. **[2]**

Question 7 (4 marks)

The Klein-Gordon equation can be written as:

$$i(\Box^2 + m^2)\psi = -iV\psi$$

Hence obtain the propagator for a spinless particle in terms of the mass, m, and 4-momentum, p. **[2]**

Now include spin and using the Dirac equation:

$$-i(\not{p} - m)\psi = -iV\psi$$

obtain the form of the propagator for an electron **[2]**

Question 8 (5 marks)

The cross section for the Bhabha process, $e^+ + e^- \to e^+ + e^-$, can be written as (modulo some constants):

$$\frac{d\sigma}{d\Omega} \sim \frac{t^2 + u^2}{s^2} + \frac{s^2 + u^2}{t^2} + \frac{2u^2}{st}$$

Draw the two Feynman diagrams for this process and identify each with the corresponding term in the cross section stating whether the reaction is s-, t- or u-channel. **[4]**

What is the origin of the third term in the cross section? **[1]**

Question 9 (6 marks)

The measured relative rate:

$$R = \frac{\Gamma(D^0 \to K^+ + \pi^-)}{\Gamma(D^0 \to K^- + \pi^+)}$$

is 3.8×10^{-3}. Draw the diagrams of the decays and briefly explain the difference in rate of the two channels. **[3]**

Using the CKM matrix,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97383 & 0.2272 & 0.004 \\ 0.2271 & 0.97296 & 0.042 \\ 0.008 & 0.042 & 0.9991 \end{pmatrix},$$

predict this rate to within a factor of 2. (NB. remember the decay rate is $\sim |T_{\rm fi}|^2$.) **[3]**

Question 10 (7 marks)

Starting from the neutral current Lagrangian,

$$\begin{aligned}\mathcal{L}_{\rm NC} &= \left(g_W J^3_\mu \sin\theta_W + g' J^{\rm EM}_\mu \cos\theta_W - g' J^3_\mu \cos\theta_W\right) A^\mu \\ &+ \left(g_W J^3_\mu \cos\theta_W - g' J^{\rm EM}_\mu \sin\theta_W + g' J^3_\mu \sin\theta_W\right) Z^\mu\end{aligned}$$

determine the relationship between the couplings g_W and g' and show that they can be replaced by e and θ_W. **[4]**

Hence derive the vector and vector-axial couplings, c_V and c_A of the Z^0 to leptons and quarks. **[3]**

Question 11 (8 marks)

The variables x, y and Q^2 define the kinematics of deep inelastic scattering. Write these variables in terms of the four vectors of the initial particles. **[2]**

The DGLAP equations are:

$$\frac{dQ_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[Q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + G(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right]$$

$$\frac{dG(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[Q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right) + G(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right]$$

Explain the functions Q_i and G and the four P_{ii} functions. **[4]**

Hence briefly explain in terms of the different processes why the structure function, F_2, depends on Q^2. **[2]**

Question 12 (8 marks)

Explain the meaning of a global and a local gauge transformation. **[2]**

For the Dirac equation to be covariant under a local gauge transformation, we replace ∂^μ by $D^\mu = \partial^\mu + iqA^\mu$. Demonstrate that $D^\mu\psi$ transforms in the same was as ψ, *i.e.* $(D^\mu\psi)' = e^{iq\alpha(x)}D^\mu\psi$. **[2]**

Explain the four terms in the Lagrangian of QED:

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi + e\bar{\psi}\gamma_m u A^\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

[4]

Question 13 (6 marks)

What property of the EM interaction means that photons do not self-couple? **[1]**

Draw a Feynman diagram of a process at the LHC in which three gluons couple at one vertex. **[2]**

Explain briefly why the QCD coupling, α_s, has a different behaviour with the scale, Q^2, compared to that of the QED coupling, α. **[3]**

Question 14 (6 marks)

Outline the main steps in the Weinberg-Salam formulation of SU(2)×U(1) which yields the mass of the W boson to be $vg_W/2$ where v is the value of the field at which the Higgs potential is a minimum. **[3]**

Use the above and $M_Z = \frac{1}{2}v(g_W^2 + g'^2)^{1/2}$ to relate M_Z and M_W in terms of the Weinberg angle. **[3]**

Question 15 (6 marks)

From the Lagrangian

$$\frac{1}{8}\left[g_W^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2(g'B_\mu - g_W W_\mu^3)(g'B^\mu - g_W W_3^\mu)\right]$$

obtain the WWH and WWHH couplings. **[2]**

Hence derive the ZZH and ZZHH couplings. (Simplify your answer to remove any dependency on v.) **[4]**