Queen Mary, UoL
Royal Holloway UoL
University College
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Intercollegiate Postgraduate Course in Elementary Particle Physics

Paper 1: Wednesday, 17 January 2001

Time allowed for the Examination: 3 hours

Attempt THREE *questions - One from section A and Two from Section B*

Section A: Symmetries and Conservation Laws
Section B: Electroweak and Strong Interactions

Section A : Symmetries, Conservation Laws

Question 1

(a) Outline an argument in the context of non-relativistic quantum mechanics which shows that a unitary symmetry leads to a conserved quantity. Derive any important results that your argument relies on. Your argument should include the formalism for transformations of states and operators, the definition of symmetry, the existence of generators and the conditions for constants of motion. **16 marks**

(b) What are meant by the fundamental and regular representations of a unitary group? What is their relevance to the particle content of the Standard model? **4 marks**

Question 2

(a) Write down the Sakharov conditions for obtaining a baryon asymmetry in the universe, and explain briefly why each is necessary. **3 marks**

(b). Consider a decay $i \to f$ which has more than one amplitude. Each amplitude will have, in general, a phase which can be factored into two parts, one coming from weak interaction effects (the CKM matrix), Φ_i, and the other from strong interaction effects (the rest), Δ_i, so that each amplitude has the form:

$$A_i = |A_i| \mathrm{e}^{i\Phi_i} \mathrm{e}^{i\Delta_i}.$$

Write down the total amplitude, A_T, for the decay when there are two different amplitudes contributing, and obtain an expression for $|A_T|^2$. **4 marks**

(c). State how each of the contributions to the phase, $\overline{\Phi}_i$ and $\overline{\Delta}_i$ for the charge conjugate decay are related to Φ_i and Δ_i within the Standard Model. By finding the squared amplitude for the conjugate decay, obtain an expression for the CP violating asymmetry,

$$\mathcal{A}_{CP} = \frac{\Gamma(i \to f) - \Gamma(\overline{i} \to \overline{f})}{\Gamma(i \to f) + \Gamma(\overline{i} \to \overline{f})},$$

and determine the conditions on the amplitudes for CP violation to occur. **5 marks**

(d). The decay $B^- \to K^- \pi^0$ can proceed through both tree and penguin processes in the Standard model, and the interference between them could lead to this mode exhibiting CP violation. Draw Feynman diagrams for both of these processes, and mark on them the appropriate CKM matrix elements, assuming that the penguin amplitude is dominated by the case where the top quark propagates in the internal loop. **4 marks**

(e) The CKM matrix can be written in the Wolfenstein parameterisation:

$$V \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Find the dependence of $\mathcal{A}_{CP}$ on the parametrs of the CKM matrix and comment on this mode's potential for the observation of direct CP violation. **4 marks**

Section B : Electroweak and Strong Interactions

Question 3

(a) The four momenta (k, p) and (k', p') describe the initial and final state kinematics of $e^- \mu^-$ scattering.

Write down the first order matrix element for this process.

(b) The cross section for electron - muon scattering, in the limit that masses tend to zero, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}} \left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $e^- \ \mu^-$ scattering and α is the fine structure constant. From the above obtain the differential cross section for e^+e^- to $\mu^+\mu^-$ annihilation and show that the angular distribution is of the form $(1 + cos^2\theta)$.

(c) Use simple helicity argument to explain this angular distribution given the rotation matrix element $d^1_{\pm 1} = 1/2(1 \pm cos\theta)$ and show that the total cross section σ is given by

$$\sigma = \frac{4\pi}{3s}\alpha^2.$$

(d) The ratio of the total e^+e^- annihilation cross section to the $e^+e^- \to \mu^+\mu^-$ cross section is called R. Explain in as much detail as you can how R varies between threshold and 150 GeV total energy.

(e) Obtain an expression for the differential cross section for $\mu^+\mu^-$ production by the Drell-Yan process in $p - p$ collisions at a collider explaining the quantities you introduce.

Question 4

(a) The spin one (vector) massless field A^μ coupling to a current J^μ satisfies the partial differential equation

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$$

Modify the above to apply for a massive spin one particle of mass M and show that the Lorentz condition $\partial_\nu A^\nu$ is deduced.

(b) For a massive spin one particle of mass M, energy p_0 which propagates along the z axis with momentum p_z show that

$$\begin{aligned}
\epsilon_1^\mu &= (0,1,0,0) \\
\epsilon_2^\mu &= (0,0,1,0) \\
\epsilon_3^\mu &= \frac{1}{M}(p_z,0,0,p_0)
\end{aligned}$$

is a suitable set of polarization vectors.

(c) Hence obtain the sum over the polarization states,

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

(d) Write down the propagator for a massive spin one particle and show how this may be modified for an unstable spin one particle whose full width Γ is much less than the mass M.

(e) Write down the matrix element for W^+ decay to $e^+\nu$. If the partial width for this decay mode is Γ_i what would you expect the total width to be giving reasons for your answer?

(f) Explain the GIM mechanism in the context of K^0 to $\mu^+\mu^-$ decay.

(g) Explain which neutrino interactions are important in scintillator, light water and heavy water detectors which are designed to observe solar neutrinos, commenting on angular distributions.

Question 5

(a) Explain the structure of the $SU(2) \times U(1)$ electroweak theory in the massless limit, explaining as you do so, the concepts of weak isospin, weak hypercharge and local gauge invariance.

(b) Write down the covariant derivative D^μ for the above interaction and obtain the transformation for the W_i^μ field under local gauge transformation. (You may assume without proof that $D^\mu \psi$ transforms in the same way as ψ itself.)

(c) Starting from the interaction term

$$g_w J^{\mu 3} W_\mu^3 + \frac{g^{'}}{2} J_Y^\mu B_\mu$$

and making the massless W^3 and B fields superpositions of the photon and Z^0 fields obtain the form of the neutral current coupling.

(d) Demonstrate that by means of a suitable symmetry breaking of a local $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, that the EM field A^μ acquires mass.

(e) Explain why an electron mass term cannot occur in the $SU(2) \times U(1)$ Lagrangian and explain how fermion masses may be generated in the Weinberg-Salam model.