Queen Mary, University of London
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Intercollegiate Postgraduate Course in Elementary Particle Physics

Examination Paper 2: The Standard Model

Thursday, 17 January, 2002.

Time allowed for the Examination: 3 hours.

Answer *three* out of the five questions. Books and notes may be consulted.

Question 1

1. The four momenta (k, p) and (k', p') describe the initial and final state kinematics of $e^- \mu^-$ scattering.

Draw the Feynman diagram for this process, and hence write down the first order matrix element.

2. The cross section for electron - muon scattering, in the limit that masses tend to zero, is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\hat{s}} \left[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right]$$

where $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual Mandelstam variables for $e^- \mu^-$ scattering and α is the fine structure constant. From the above obtain the differential cross section for e^+e^- to $\mu^+\mu^-$ annihilation and show that the angular distribution is of the form $(1 + cos^2\theta)$.

3. Use simple helicity argument to explain this angular distribution given the rotation matrix element $d^1_{\pm 1} = 1/2(1 \pm cos\theta)$ and show that the total cross section σ is given by

$$\sigma = \frac{4\pi}{3s}\alpha^2.$$

4. The ratio of the total e^+e^- annihilation cross section to the $e^+e^- \to \mu^+\mu^-$ cross section is called R. Explain the processes that contribute to the variation of R between threshold and 200 Gev total energy.

5. Define, in terms of the usual four vectors, the scaling variables x and y for deep inelastic lepton - nucleon scattering (DIS). Give simple interpretations of the variables x and y in terms of the parton model and the lepton parton CM frame.

6. From the result in part 2, obtain the quark parton result for deep inelastic scattering, explaining the quantities you introduce.

Question 2

1. Draw Feynman diagrams and write down the matrix elements for the following charged current weak processes, assuming a pointlike weak interaction and the Cabibbo hypothesis.

$\nu_\mu + d \to u + \mu^-$,

$\nu_\mu + d \to c + \mu^-$ and

$\pi^+ \to \mu^+ + \nu_\mu$.

2. Determine an expression for the decay rate of the π^+, and obtain an expression for the ratio of the K^+ and π^+ decay rates.

3. Obtain the non relativistic form of the charged current, identify the *Fermi* and *Gamov-Teller* transitions and explain why the beta decay $^{13}N \to ^{13}C$ is *superallowed*.

4. Explain the GIM mechanism in the context of K^0 to $\mu^+\mu^-$ decay.

5. Show that the Kobayashi - Maskawa matrix must contain four parameters, one of which being a phase angle, to relate the weak and the mass eigenstates for three generations of quark states.

6. Assuming without proof that CP exchanges the space-time parts of the W^+ and W^- couplings, show that the phase angle allows CP violation in the Standard Model.

Question 3

1. Starting from the charged weak current expressed in the form

$$J^{+\mu} = \frac{g_w}{\sqrt{2}}\overline{u_\nu}\gamma^\mu \frac{1}{2}(1-\gamma^5)u_e$$

show that the weak charged current may be written as

$$J^{+\mu} = \frac{g_w}{\sqrt{2}}\overline{\lambda_l}\gamma^\mu \tau^+ \lambda_l$$

where λ_l are lepton weak isospin doublets, and τ^+ is the weak isospin raising operator.

Show that g_w is related to the Fermi constant G_F by

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_w^2}$$

2. Explain what is meant by weak hypercharge Y where $Y = 2Q - 2T^3$.

3. Starting from the interaction term

$$g_w J^{\mu 3} W_\mu^3 + \frac{g^{'}}{2} J_Y^\mu B_\mu$$

and making the massless W^3 and B fields superpositions of the photon and Z^0 fields show that the neutral current coupling is given by

$$\frac{g_w}{cos\theta_w}\overline{u_f}\gamma^\mu \frac{1}{2}(c_V^f - c_A^f\gamma^5)u_f$$

where $c_V^f = T_f^3 - 2sin^2\theta_w$ and $c_A^f = T_f^3$. In the above θ_w is the Weinberg angle, T^3 is the third component of the weak isopin and the suffix f signifies a fermion.

4. Explain the main weak processes involved in the collapse of the iron core to a neutron star and the emission of neutrinos, in a supernova explosion.

Question 4

1. Starting from the Maxwell equation show that the spin one (vector) massless field A^μ coupling to a current J^μ satisfies the partial differential equation

$$\frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = J^\mu$$

2. Modify the above to apply for a massive spin one particle of mass M and show that the Lorentz condition $\partial_\nu A^\nu = 0$ is deduced.

3. For a massive spin one particle of mass M, energy p_0 which propagates along the z axis with momentum p_z show that

$$\begin{aligned}
\epsilon_1^\mu &= (0,1,0,0) \\
\epsilon_2^\mu &= (0,0,1,0) \\
\epsilon_3^\mu &= \frac{1}{M}(p_z,0,0,p_0)
\end{aligned}$$

is a suitable set of polarization vectors.

4. Hence obtain the sum over the polarization states,

$$\sum_{\lambda=1,2,3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}.$$

5. Write down the propagator for a massive spin one particle and show how this may be modified for an unstable spin one particle whose full width Γ is much less than the mass M.

6. From the Feynman diagram write down the matrix element for W^+ decay to $e^+\nu$. If the partial width for this decay mode is Γ_i what would you expect the total width to be, giving reasons for your answer?

7. Draw the Feynman diagrams for the neutral and charged current deep inelastic processes at the HERA electron proton collider.

What would be the effect if the electron beam were longitudinally polarized?

Question 5

1. Explain the structure of the $SU(2) \times U(1)$ electroweak theory in the massless limit, explaining as you do so, the concepts of weak isospin, weak hypercharge and local gauge invariance.

2. Write down the covariant derivative D^μ for the above interaction and obtain the transformation for the W_i^μ field under local gauge transformation. (You may assume without proof that $D^\mu \psi$ transforms in the same way as ψ itself.)

3. Explain how the tri and quadrilinear W couplings arise.

4. Show that the *Goldstone boson* arises when a global $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, is spontaneously broken.

5. Demonstrate that by means of a suitable symmetry breaking of a local $U(1)$ gauge symmetry of a complex scalar field, which describes charged scalar particles, that the EM field A^μ acquires mass.

6. Write down Weinberg's choice of the scalar field in $SU(2) \times U(1)$ electroweak theory before and after symmetry breaking.

7. Explain why an electron mass term cannot occur in the $SU(2) \times U(1)$ Lagrangian and explain how fermion masses may be generated in the Weinberg-Salam model.