Queen Mary, University of London
Royal Holloway, University of London
University College London
Brunel University

# Intercollegiate Postgraduate Course in Elementary Particle Physics

**Examination Paper 1:**

**Section A: Symmetries and Conservation Laws (40 marks)**

**Section B: Neutrino Physics and Collider Physics (20 marks)**

Tuesday, 24 January, 2006

Time allowed for the Examination: 3 hours.

Answer *ALL* questions.

Books and notes may be consulted.

As each question must be marked by different people, please begin each new numbered question on a separate piece of paper (parts a, b, etc. can be on same sheet).

**Section A (questions 1 and 2): Symmetries and Conservation Laws [40 marks]**

**Question 1 [20 marks]**

**(a)** Explain what SU(2) is. [1 mark]

**(b)** Given that the SU(2) operators can be represented by $\exp(i\frac{\theta}{2}\mathbf{n}\cdot\sigma)$, where $\sigma$ is the vector of Pauli spin matrices, show that the set of operators form a group under the operation "follows". It is sufficient to consider infinitesimal transformations. [3 marks]

**(c)** Explain what is meant by a Lie algebra.
Use your answer to (b) to demonstrate that SU(2) is indeed a Lie algebra. [1 mark]

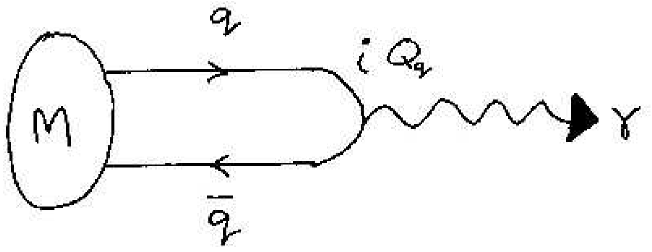
**(d)** When combinations of states of different particles are formed, it is natural to identify multiplets.
Give the characteristics of multiplets, both within the group theory context and for physical combinations of quarks in hadrons. [1 mark]

**(e)** Consider two spin $\frac{1}{2}$ particles. Use Young Tableaux to illustrate the multiplets which can be obtained by combining the single particle states. By inserting quantum numbers inside the boxes of the Tableaux, identify suitable wave-functions for all of the possible states. [2 marks]

**(f)** *This question is a bit harder, you may wish to do it after the next parts.*
Consider the effect of a general SU(2) rotation on the singlet ($j=0, m=0$) state formed from two spin $\frac{1}{2}$ particles. Demonstrate that the state is invariant under all rotations. Contrast this with the transformation of the triplet state ($j=1, m=0$).
*Hints:*
$R(\theta) = \exp(i\theta\mathbf{n}\cdot\mathbf{J})$ and $\mathbf{J} = \frac{1}{2}(\sigma^{\mathbf{1}} + \sigma^{\mathbf{2}})$
$\sigma^{\mathbf{1}}$ is the spin operator for the first particle; $\sigma^{\mathbf{2}}$ is the spin operator for the second particle. These two operators are independent, and hence commute.
You should use the fact that $\exp(i\frac{\theta}{2}\mathbf{n}\cdot\sigma) = \cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\mathbf{n}\cdot\sigma$. You should not consider infinitesimal rotations, but rather finite rotations. [7 marks]

QUESTION 1 CONTINUES ON NEXT PAGE.

**(g)** Consider the decays of a $\pi^0$ (member of SU(2) triplet) and an $\eta$ (SU(2) singlet, *not* SU(3) state, so no $s\bar{s}$ contribution) to a single photon through a decay diagram:




Calculate the ratio of the decay amplitudes.
*Hints:*
If the decay operator is $S$, then the amplitude for meson $M$ to decay to a photon $\gamma$ is

$$< \gamma|S|M >= \Sigma < \gamma|S|q\bar{q} >< q\bar{q}|M >$$

where the sum is over the possible quark flavours and $< \gamma|S|q\bar{q} > \propto Q_q$ is the quark charge. [2 marks]

**(h)** Explain why the above decay is unphysical. (two reasons) [1 mark]

**(i)** Give a *brief* explanation of why there is a minus sign in the wave-function for the $\pi^0$ in its SU(2) description. [2 marks]

**Question 2 [20 marks]**

**(a)** Draw the Unitarity Triangle, labelling its sides and angles. Give an example of a decay mode that can be used to determine each of the angles $\beta$, $\alpha$ and $\gamma$. [2 marks]

**(b)** Draw the Feynman diagram of the $B^0 \to \overline{B}^0$ oscillation and explain how this will give rise to the indirect CP violation observed in the decay $B^0 \to J/\psi K^0_S$. Show how the CP asymmetry $A_{CP}(t)$ is related to the sine of the angle $\beta$ of the Unitarity Triangle. Draw all relevant Feynman diagrams, labelling the CKM matrix elements. [10 marks]

**(c)** Draw the Feynman diagrams for the process $B^- \to K^- \pi^0$, labelling the CKM matrix elements. Using the Wolfenstein parameterization for the CKM matrix $V$, find the dependence of the CP asymmetry $A_{CP}$ in this decay on the parameters of the CKM matrix and on the strong phases. Comment on the conditions required for this process to exhibit a significant asymmetry. Show that $A_{CP}$ is proportional to the area of the unitarity triangle. Use the unitarity conditions to show that there would not exist CP violation for this type of decay if there were only two generations. [8 marks]

## Section B: Neutrino and Hadron Collider Physics [20 marks]

### Question 3 [10 marks]

**(a)** In the salt-phase, SNO detected solar neutrino induced NC and CC events for 391 live days. From the number of detected events, efficiencies and cross-sections given in the table below, compute $\phi_{CC}$ and $\phi_{NC}$, the neutrino fluxes measured by NC and CC processes. [2 marks]

| Process | Number of events | Cross-section ($cm^2$) | Efficiency |
|---|---|---|---|
| CC ($\nu_e d \to e^- pp$) | 2176±78 | $0.6 \times 10^{-42}$ | 1 |
| NC ($\nu_x d \to \nu_x np$) | 2010±85 | $0.4 \times 10^{-42}$ | 0.5 |

**(b)** Using the values of $\phi_{CC}$ and $\phi_{NC}$ found in (a), compute the probability of an electron neutrino to convert into a muon or tau neutrino, P($\nu_e \to \nu_{\mu\tau}$). [2 marks]

**(c)** Assuming oscillation in vacuum and $\sin^2 2\theta = 0.86$, what value of $\Delta m^2$ is suggested by the SNO data? Compare this value with the current best fit to solar neutrino data. Are these values compatible? Justify your answer. [2 marks]

**(d)** Describe briefly the T2K experiment. If you had enough money to build another Cherenkov detector as big as SuperK and the freedom to position it anywhere along the T2K beam line, at what distance from the neutrino beam's origin would you put it in order to maximize the experimental sensitivity to $\theta_{13}$? By how much would the sensitivity be enhanced? Assume no background. [4 marks]

**Question 4 [10 marks]**

**(a)** What is the ratio of the centre-of-mass energy of the Tevatron to that of the Large Hadron Collider? [1 mark]

**(b)** What measurements at the Tevatron can be used to constrain the mass of the Higgs boson ? [1 mark]

**(c)** Consider a 115 GeV Higgs boson at either the Tevatron or LHC. Draw the dominant Feynman diagram for the Higgs production and decay. Explain why, in practice, this mode cannot be used to discover the Higgs boson. [2 marks]

**(d)** Draw the Feynman diagrams corresponding to the most promising signal process for the discovery a 115 GeV Higgs boson at both the Tevatron and at the LHC (one diagram for each). For each case, draw an additional Feynman diagram which would form a large background to such a Higgs signal. [2 marks]

**(e)** Explain how you would identify the Higgs processes in (d), detailing the detector requirements. [1 marks]

**(f)** Suppose no Higgs boson is found at the LHC. Draw a Feynman diagram of the process you would then study to try and elucidate the mechanism of electroweak symmetry breaking. [1 mark]

**(g)** What is the "hierarchy problem" in the context of the Higgs mass and how does the introduction of SUSY help? What other arguments are there for SUSY as a viable theory? [1 marks]

**(h)** What are the typical experimental signatures associated with SUSY particle production at either the Tevatron or the LHC ? [1 marks]