PHAS1245: Mathematical Methods I - Problem Class 1

Week starting Monday 8th October

1. If x_1 and x_2 are the roots of the quadratic equation $ax^2 + bx + c = 0$, show without using the formula for the solutions that

$$x_1 + x_2 = -\frac{b}{a} \quad \text{and} \quad x_1 x_2 = \frac{c}{a}\,.$$

2. Express the following as partial fractions:

$$\frac{x^2+3}{x(x^2+2)} \quad \text{and} \quad \frac{3}{x(3x-1)^2}\,.$$

3. (a) Starting from

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

show that $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$ and

$$\sin A = \frac{2\tan(\frac{A}{2})}{1+\tan^2(\frac{A}{2})} \qquad \cos A = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}\,.$$

(b)Solve the following for values of θ between 0 and 2π

$$\cos 2\theta + 3\sin\theta = 2 \quad \text{and} \quad \sin\theta + 2\cos\theta = 1.$$

(c)Find all the solutions of

$$\sin\theta + \sin 4\theta = \sin 2\theta + \sin 3\theta$$

that lie in the range $-\pi < \theta \leq \pi$. What is the multiplicity of the solution $\theta = 0$?

4. The maximum possible efficiency ϵ of a power plant is given by

$$\epsilon = 1 - \frac{T_C}{T_H}$$

where T_C is the temperature of the cooling water and T_H the maximum steam temperature.

Obtain a formula for the fractional change in efficiency $\frac{\Delta\epsilon}{\epsilon_1}$ where $\Delta\epsilon = \epsilon_2 - \epsilon_1$ if the temperature of the cooling water changes from T_{C1} to T_{C2} where $T_{C2} > T_{C1}$.

In the summer of 2003 the temperature of the cooling water of continental European power plants rose by $15K$. If T_H for a gas plant and for a PWR nuclear plant are $675K$ and $525K$ respectively calculate the fractional change in the maximum possible efficiencies for both type of power plant, given that T_{C1} is normally $290K$.

5. Using the formula for the binomial coefficients from your notes, calculate the expansion of $(a+b)^6$.

6. (A much harder question both conceptually and technically.)

   A car is at the origin of a coordinate system at $t = 0$ and is travelling along the positive z axis with speed v so that after time t its x, y, z coordinates are $(0, 0, vt)$. The car emits a sharp pulse of sound which travels through still air with speed c_s. An observer at coordinates (x, y, z) detects the sound pulse at time t. Explain why the time at which the pulse was emitted is given by $[t]$ where

$$[t] = t - \frac{[|\vec{r}|]}{c_s} \quad \text{where} \quad [|\vec{r}|] = \sqrt{(z - v[t])^2 + x^2 + y^2)}$$

   Obtain an expression for $[t]$.