PHAS1245: Mathematical Methods I - Problem Sheet 1

(Solutions to be handed in at the lecture on Tuesday 9th October 2007)

Staple your answer sheets together and put **your name** and your **tutor's name** on your script (or Dr. Konstantinidis, if you have no tutor in the P&A department).

1. Solve the following by completing the square: **[2]**

$$2x^2 - 3x - 5 = 0\,.$$

2. Find the real root(s) of the equation

$$2^{2x} + 3(2^x) - 4 = 0$$

(Hint - make a substitution and first solve the 'hidden' quadratic). **[6]**

3. A particle of energy E and mass m decays into two massless particles of energies E_1 and E_2 respectively. The angle between the trajectories of the two decay particles is ϕ. In such a decay

$$E = E_1 + E_2 \quad \text{and} \quad \sin\frac{\phi}{2} = \frac{m}{2\sqrt{E_1 E_2}}\,.$$

Show that E_1 satisfies the quadratic equation **[3]**

$$E_1^2 - EE_1 + \frac{m^2}{4\,\sin^2\frac{\phi}{2}} = 0\,.$$

Using any method show that **[3]**

$$E_1 = \frac{E}{2} \pm \frac{1}{2}\sqrt{E^2 - \frac{m^2}{\sin^2\frac{\phi}{2}}}\,.$$

At what value of $\sin\frac{\phi}{2}$ is $E_1 = \frac{E}{2}$? **[2]**

4. Factorise the following as far as possible given that you should only try factors equal to ± 1 and ± 2 , in the first three problems.

$$x^3 + 2x^2 - x - 2\,,$$

$$x^3 - x^2 - x - 2\,,$$

$$x^4 - 1 \text{ and}$$

$$x^{\frac{3}{2}} - a^{\frac{3}{2}}\,.$$

[8]

5. In a certain collision problem the kinematics are determined by the following three equations:

$$u = v_1 \cos\theta + \sqrt{2} v_2 \, ,$$

$$v_1 \sin\theta = \sqrt{2} v_2 \quad \text{and}$$

$$\frac{1}{2} m u^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} (2m) v_2^2 \, .$$

Use the first two equations to express v_1 and v_2 in terms of u, $\sin\theta$ and $\cos\theta$. Then substitute in the third equation and obtain solutions for $\sin\theta$. **[6]**