PH4442 - Problem Sheet 2

(answers should be returned on 31/01/2006)

1. Particle A decays at rest to particles B and C ($A \to B + C$). Show that the energy of, say, B is given by

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A}\,.$$

2. How does the u_1 free Dirac spinor change under the Parity transformation, $\hat{P}$? Show that this can be written as

$$\hat{P}u_1 = \gamma^0 u_1\,.$$

(This is a general result, independent of the representation for the Dirac spinors and the γ matrices.)

Show that the quantity $\bar{u}_1\gamma^\mu\partial_\mu u_1$, where $\bar{u}_1 = u_1^\dagger\gamma^0$, is invariant under the Parity transformation, while replacing γ^μ with $\gamma^\mu(1-\gamma^5)$ breaks the $\hat{P}$ invariance. (When we discuss Lagragians, we will see that this is the key difference between QED and the weak interactions.)

3. The charge conjugation operator (C) takes a Dirac spinor ψ into its charge-conjugate ψ_C, given by

$$\psi_C = \imath\gamma^2\psi^*$$

Find the charge-conjugates of u_1 and u_2 and compare them with v_1 and v_2.

4. Using only the properties of the Pauli matrices (i.e. WITHOUT picking a specific representation), show that for any two vectors $\vec{\mathrm{a}}$ and $\vec{\mathrm{b}}$

$$(\vec{\sigma}\cdot\vec{\mathrm{a}})(\vec{\sigma}\cdot\vec{\mathrm{b}}) = \vec{\mathrm{a}}\cdot\vec{\mathrm{b}} + \imath\vec{\sigma}\cdot\vec{\mathrm{a}}\times\vec{\mathrm{b}}$$

and hence that $(\vec{\sigma}\cdot\vec{\mathrm{a}})^2 = |\vec{\mathrm{a}}|^2$.

(You can find all the main properties of the Pauli matrices in problems 4.19 and 4.20 of Griffiths.)

5. Derive the completeness relation

$$\sum_{s=1,2} u^{(s)}(p)\bar{u}^{(s)}(p) = \not{p} + m\,,$$

where $\not{p} = p_\mu\gamma^\mu$.

(Hint: DO NOT expand in 4 dimensions. 2×2 for the γ matrices and 2×1 for the u's is sufficient. You will also need to use the result from problem 3 above.)