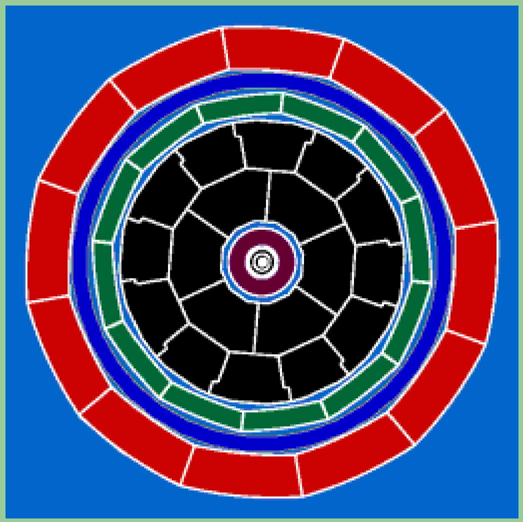
Particle Physics



UCL

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Practicalities (I)

- Contact details
  - My office: D16, 1st floor Physics, UCL
  - My e-mail: n.konstantinidis@ucl.ac.uk
  - Web page: www.hep.ucl.ac.uk/~nk/teaching/PH4442
- Office hours for the course (this term)
  - Tuesday 13h00 – 14h00
  - Friday 12h00 – 13h00
- Problem sheets
  - To you at week 1, 3, 5, 7, 9
  - Back to me one week later
  - Back to you (marked) one week later

Practicalities (II)

- Textbooks
  - I recommend (available for £27 – ask me or Dr. Moores)
    - Griffiths "Introduction to Elementary Particles"
  - Alternatives
    - Halzen & Martin "Quarks & Leptons"
    - Martin & Shaw "Particle Physics"
    - Perkins "Introduction to High Energy Physics"
- General reading
  - Greene "The Elegant Universe"

Course Outline

1. Introduction (w1)
2. Symmetries and conservation laws (w2)
3. The Dirac equation (w3)
4. Electromagnetic interactions (w4,5)
5. Strong interactions (w6,7)
6. Weak interactions (w8,9)
7. The electroweak theory and beyond (w10,11)

Week 1 – Outline

- Introduction: elementary particles and forces
- Natural units, four-vector notation
- Study of decays and reactions
- Feynman diagrams/rules & first calculations

The matter particles

		Family			EM Charge	Interactions		
		1st	2nd	3rd	(units of *e*)	Strong	EM	Weak
Quarks	U-type	u	c	t	+2/3	Y	Y	Y
	D-type	d	s	b	–1/3	Y	Y	Y
Leptons	Charged	e	μ	τ	–1	N	Y	Y
	Neutral	ν_e	ν_μ	ν_τ	0	N	N	Y

- All matter particles (a) have spin ½; (b) are described by the same equation (Dirac's); (c) have antiparticles
- Particles of same type but different families are identical, except for their mass:

  m_e = 0.511MeV m_μ=105.7MeV m_τ=1777MeV

- Why three families? Why they differ in mass? Origin of mass?
- Elementary ⇨ point-like (...but have mass/charge/spin!!!)

The force particles

Force	Name	Symbol	Number
Strong	gluons	g	8
Weak	W and Z	W^+, W^-, Z^0	3
EM	photon	γ	1

- All force particles have spin 1 (except for the graviton, still undiscovered, expected with spin 2)
- Many similarities but also major differences:
  - $m_\gamma = 0$ vs. $m_{W,Z} \sim 100$GeV
  - Unlike photon, strong/weak “mediators” carry their “own charge”
- The SM provides a unified treatment of EM and Weak forces (and implies the unification of Strong/EM/Weak forces); but requires the Higgs mechanism (→ the Higgs particle, still undiscovered!).

Natural Units

- SI units not "intuitive" in Particle Physics; e.g.
  - Mass of the proton $\sim 1.7\times10^{-27}$kg
  - Max. momentum of electrons @ LEP $\sim 5.5\times10^{-17}$kg·m/sec
  - Speed of muon in pion decay (at rest) $\sim 8.1\times10^{7}$m/sec
- More practical/intuitive: $\hbar = c = 1$; this means energy, momentum, mass have same units
  - $E^2 = p^2c^2 + m^2c^4$ ➡ $E^2 = p^2 + m^2$
  - E.g. m_p=0.938GeV, max. p_{LEP}=104.5GeV, v_μ=0.27
  - Also:

$$E = h\nu \rightarrow 2\pi\nu \qquad E = \frac{hc}{\lambda} \rightarrow \frac{2\pi}{\lambda}$$

- Time and length have units of inverse energy!
  - $1\text{GeV}^{-1} = 1.973\times10^{-16}$m $\qquad 1\text{GeV}^{-1} = 6.582\times10^{-25}$sec

4-vector notation (I)

Lorentz Transformations

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

$$\mathrm{x}' = \Lambda \mathrm{x}$$

or

$$(\mathrm{x}')^{\mu} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} \mathrm{x}^{\nu}$$

or

$$(\mathrm{x}')^{\mu} = \Lambda^{\mu}_{\nu} \mathrm{x}^{\nu}$$

4-vector: "An object that transforms like x^{μ} between inertial frames"

E.g. **momentum :** $p \equiv (p^0, p^1, p^2, p^3) \equiv (E, p_x, p_y, p_z)$

Invariant: "A quantity that stays unchanged in all inertial frames"

E.g. 4-vector scalar product: $a \cdot b \equiv a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

4-vector length: $p^2 = p^0 p^0 - p^1 p^1 - p^2 p^2 - p^3 p^3 = E^2 - |\vec{\mathbf{p}}|^2 = \mathrm{m}^2$

Length can be:

	>0	timelike
	<0	spacelike
	$=0$	lightlike 4-vector

4-vector notation (II)

- Define matrix g: $g_{00}=1$, $g_{11}=g_{22}=g_{33}=-1$ (all others 0)
- Also, in addition to the standard 4-v notation (*contravariant* form: indices up), define *covariant* form of 4-v (indices down):

$$a_\nu = g_{\mu\nu} a^\mu$$

- Then the 4-v scalar product takes the tidy form:

$$a \cdot b = g_{\mu\nu} a^\mu b^\nu = a_\nu b^\nu$$

- An unusual 4-v is the four-derivative:

$$\partial^\mu \equiv \frac{\partial}{\partial x_\mu} \equiv \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \equiv \left(\frac{\partial}{\partial t}, \quad \vec{\nabla}\right)$$

- So, $\partial_\mu a^\mu$ is invariant; e.g. the EM continuity equation becomes:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{\mathbf{J}} = 0 \quad \rightarrow \quad \partial_\mu J^\mu = 0$$

What do we study?

- Particle Decays ($A \to B + C + \ldots$)
  - Lifetimes, branching ratios etc...
- Reactions ($A + B \to C + D + \ldots$)
  - Cross sections, scattering angles etc...
- Bound States
  - Mass spectra etc...

Study of Decays ($A \to B+C+\ldots$)

- Decay rate Γ: **"The probability per unit time that a particle decays"**

$$dN = -\Gamma N dt \quad \Rightarrow \quad N(t) = N(0)e^{-\Gamma t}$$

- Lifetime τ: "The average time it takes to decay" (at particle's rest frame!)

$$\tau = 1/\Gamma$$

- Usually several decay modes

$$\Gamma_{tot} = \sum_i \Gamma_i \quad \text{and} \quad \tau = 1/\Gamma_{tot}$$

- Branching ratio BR

$$BR(\text{decay mode } i) = \Gamma_i / \Gamma_{tot}$$

- We measure Γ_{tot} (or τ) and BRs; we calculate Γ_i

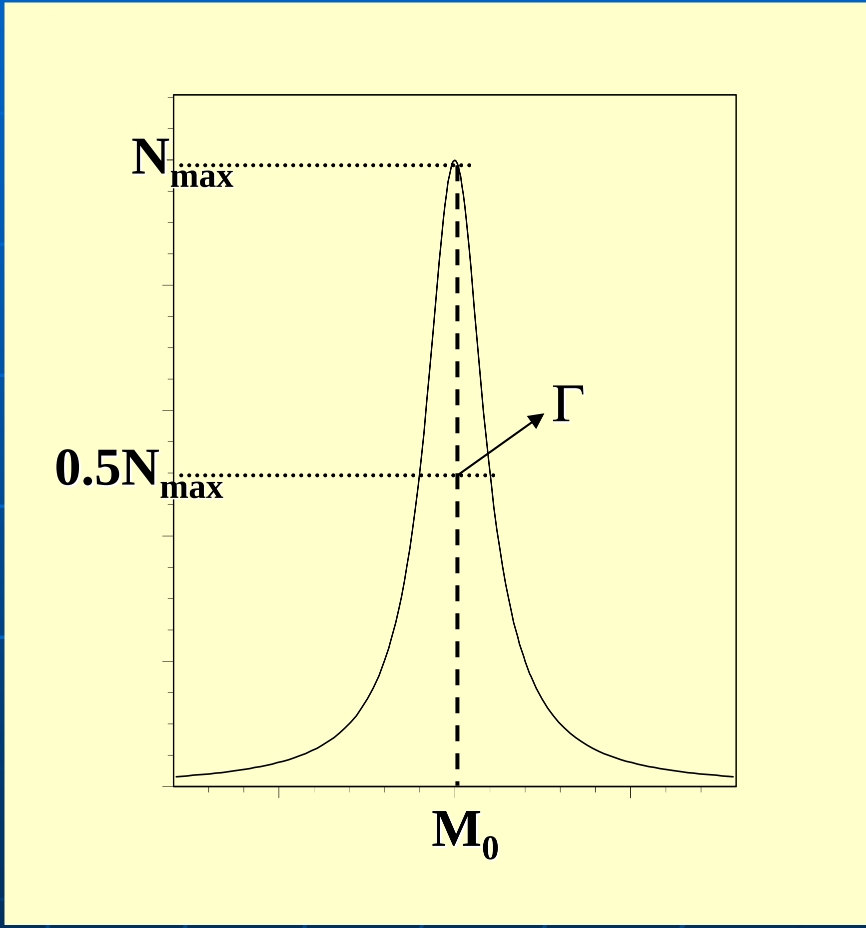
Γ as decay width

- Unstable particles have no fixed mass due to the uncertainty principle:

$$\Delta m \times \Delta t \approx \hbar$$

- The Breit-Wigner shape:

$$N(m) = N_{max} \frac{(\Gamma/2)^2}{(m - M_0)^2 + (\Gamma/2)^2}$$



- We are able to measure only one of Γ, τ of a particle

($1 GeV^{-1} = 6.582 \times 10^{-25}$ sec)

Study of reactions (A+B→C+D+…)

- Cross section σ
  - The "effective" cross-sectional area that A sees of B (or B of A)
  - Has dimensions $[L]^2$ and is measured in (subdivisions of) barns

    $1b = 10^{-28}\ m^2$ $\quad$ $1\mu b = 10^{-34}\ m^2$ $\quad$ $1pb = 10^{-40}\ m^2$

- Often measure "differential" cross sections
  - $d\sigma/d\Omega$ or $d\sigma/d(\cos\theta)$

- Luminosity $\mathcal{L}$
  - Number of particles crossing per unit area and per unit time

$$\mathcal{L} = \frac{\text{(particles in beam 1)} \times \text{(particles in beam 2)}}{\text{(transverse area of the bunches)}} \times \text{(rate of beam crossings)}$$

  - Has dimensions $[L]^{-2}[T]^{-1}$; measured in $cm^{-2}s^{-1}$ $(10^{31} – 10^{34})$

Study of reactions (cont'd)

- Event rate (reactions per unit time)

$$\textbf{Event rate} = \sigma \times \mathcal{L}$$

- Ordinarily use "integrated" Luminosity (in pb^{-1}) to get the total number of reactions over a running period

$$\textbf{Total number of Events} = \sigma \times \int \mathcal{L}dt$$

- In practice, $\mathcal{L}$ measured by the event rate of a reaction whose σ is well known (e.g. Bhabha scattering at LEP: $e^+e^- \rightarrow e^+e^-$). Then cross sections of new reactions are extracted by measuring their event rates

Feynman diagrams

- Feynman diagrams: schematic representations of particle interactions

- They are purely symbolic! Horizontal dimension is (…can be) time (*except in Griffiths!*) but the other dimension DOES NOT represent particle trajectories!

- Particle going backwards in time => antiparticle forward in time

- A process A+B→C+D is described by all the diagrams that have A,B as input and C,D as output. The overall cross section is the sum of all the individual contributions

- Energy/momentum/charge etc are conserved in each vertex

- Intermediate particles are "*virtual*" and are called *propagators*; The more virtual the propagator, the less likely a reaction to occur
  - Virtual: $p^2 = p_\mu p^\mu = E^2 - |\vec{\mathrm{p}}|^2 \neq \mathrm{m}^2$

Fermi's "Golden Rule"

- Calculation of Γ or σ has two components:
  - Dynamical info: (Lorentz Invariant) Amplitude (or Matrix Element) $\mathcal{M}$
  - Kinematical info: (L.I.) Phase Space (or Density of Final States)

- FGR for decay rates (1→2+3+...+n)

$$d\Gamma = |\mathcal{M}|^2 \frac{S}{2m_1}\left[\left(\frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\right)\left(\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\right)\cdots\left(\frac{d^3\vec{p}_n}{(2\pi)^3 2E_n}\right)\right] \times (2\pi)^4 \delta^4(p_1 - p_2 - p_3 \ldots - p_n)$$

- FGR for cross sections (1+2→3+4+...+n)

$$d\sigma = |\mathcal{M}|^2 \frac{S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}}\left[\left(\frac{d^3\vec{p}_3}{(2\pi)^3 2E_3}\right)\left(\frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}\right)\cdots\left(\frac{d^3\vec{p}_n}{(2\pi)^3 2E_n}\right)\right] \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n)$$

Feynman rules to extract $\mathcal{M}$

Toy theory: A, B, C spin-less and only ABC vertex

1. Label all incoming/outgoing 4-momenta $p_1, p_2, \ldots, p_n$;
   Label internal 4-momenta $q_1, q_2 \ldots, q_n$.
2. *Coupling constant*: for each vertex, write a factor $-ig$
3. *Propagator*: for each internal line write a factor $i/(q^2 - m^2)$
4. *E/p conservation*: for each vertex write $(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$;
   k's are the 4-momenta at the vertex (+/– if incoming/outgoing)
5. *Integration over internal momenta*: add $1/(2\pi)^4 d^4q$ for each internal line and integrate over all internal momenta
6. *Cancel the overall Delta function* that is left: $(2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n)$

What remains is: $-i\mathcal{M}$

Summary

- The SM particles & forces [1.1->1.11, 2.1->2.3]
- Natural Units
- Four-vector notation [3.2]
- Width, lifetime, cross section, luminosity [6.1]
- Fermi's G.R. and phase space for 1+2–>3+4 [6.2]
- Mandelstam variables [Exercises 3.22, 3.23]
- δ-functions [Appendix A]
- Feynman Diagrams [2.2]
- Feynman rules for the ABC theory [6.3]
- $d\sigma/d\Omega$ for A+B–>A+B [6.5]
- Renormalisation, running coupling consts [6.6, 2.3]