Invariant Set Theory:
Geometrising the Quantum Potential?

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Foundations of Physics

What do you want to give up?

- Determinism (realism)
- Local causality (events determined by data in past light cone)
- Measurement independence (measurement settings independent of the ontic variables of particles being measured)

Who Cares?

- The grossly incorrect quantum field theoretic estimates of dark energy may be indicating that quantum theory is going badly wrong in situations where gravity is important.

- We must sort out these foundational issues if we are ever to replace quantum theory with something more compatible with general relativity, and which can readily account for the dark universe.

Cauchy Sequences

$\{1,\ 1.4,\ 1.41,\ 1.414,\ 1.4142,\ 1.41421\ldots\}$

is a Cauchy sequence relative to the Euclidean metric

$d(a,b) = |a-b|,\ a,b \in \mathbb{Q}$

$\{1,\ 1+2,\ 1+2+2^2,\ 1+2+2^2+2^3,\ 1+2+2^2+2^3+2^4\ldots\}$

is a Cauchy sequence relative to the metric $d_2(a,b)$ where for $a-b \neq 0$

$d_p(a,b) = p^{-\mathrm{ord}_p(a-b)}$

and

$\mathrm{ord}_p x =$ the highest power of p that divides x, if $x \in \mathbb{Z}$ (generalises for $x \in \mathbb{Q}$)

e.g. $d_2(1+2+2^2,\ 1+2) = 2^{-2} = 1/4,\ d_2(1+2+2^2+2^3,\ 1+2+2^2) = 2^{-3} = 1/8$

These metrics provide two (and only two) inequivalent ways of completing the field of rational numbers $\mathbb{Q}$:

$d(a,b)\ :\mathbb{Q}\Rightarrow\mathbb{R}$ consistent with and applicable to Euclidean Geometry

$d_p(a,b):\mathbb{Q}\Rightarrow\mathbb{Q}_p$ consistent with and applicable to Fractal Geometry

"We [number theorists] tend to work almost as much p-adically as with the reals or complexes nowadays, and in fact it is usually best to consider all at once." Andrew Wiles personal communication.

I believe physicists should also use reals, p-adics and complexes equally:

$d(a,b)$ for describing distances in space-time;

$d_p(a,b)$ for describing distances in state space (phase space).

Cantor Sets and the p-adic Metric

$$C_2 = \bigcap_{k\in\mathbb{N}} C_2(k)$$

Now

$$F \ : \sum_{k=0}^{\infty} \frac{2a_k}{3^{k+1}} \to \sum_{k=0}^{\infty} a_k 2^k \quad a_k \in \{0,1\}$$

is a bijection between C_2 and $\mathbb{Z}_2 \subset \mathbb{Q}_2$.

More generally, $F \ : \mathrm{C}_p \to \mathbb{Z}_p$. For C_p, divide the unit interval into $2p-1$ equal subintervals and remove every second open subinterval …

Can do algebra, calculus and Fourier analysis on fractals using p-adic numbers.

The Key Physical Point

- If $a, b \in C_p$ and $d(a,b) \ll 1$, then $d_p(a,b) \ll 1$

- However, if $a \in C_p$ and $b \notin C_p$ then $d_p(a,b) \geq p$ even if $d(a,b) \ll 1$.

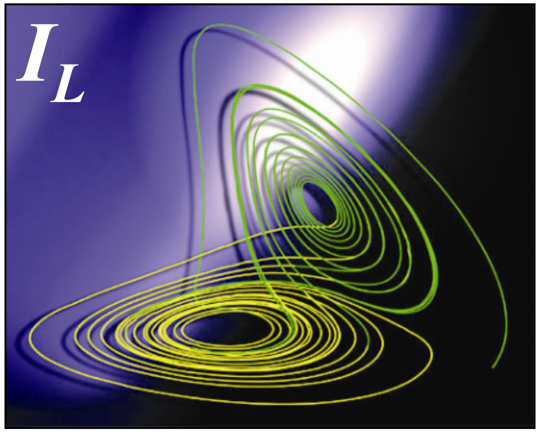
- If $p \gg 1$, then a perturbation which is very small relative to the Euclidean metric, but takes a point $a \in C_p$ off C_p, is a very large-amplitude perturbation relative to the p-adic metric.

Fractal Invariant Sets are Generic in Nonlinear Dynamical Systems Theory (Classical Physics)

E.g.

$$\dot{X} = -\sigma X + \sigma Y$$
$$\dot{Y} = -XZ + rX - Y$$
$$\dot{Z} = XY - bZ$$

Non-computable



Locally

$$I_L = \mathbb{R} \times (\text{Cantor Set})$$

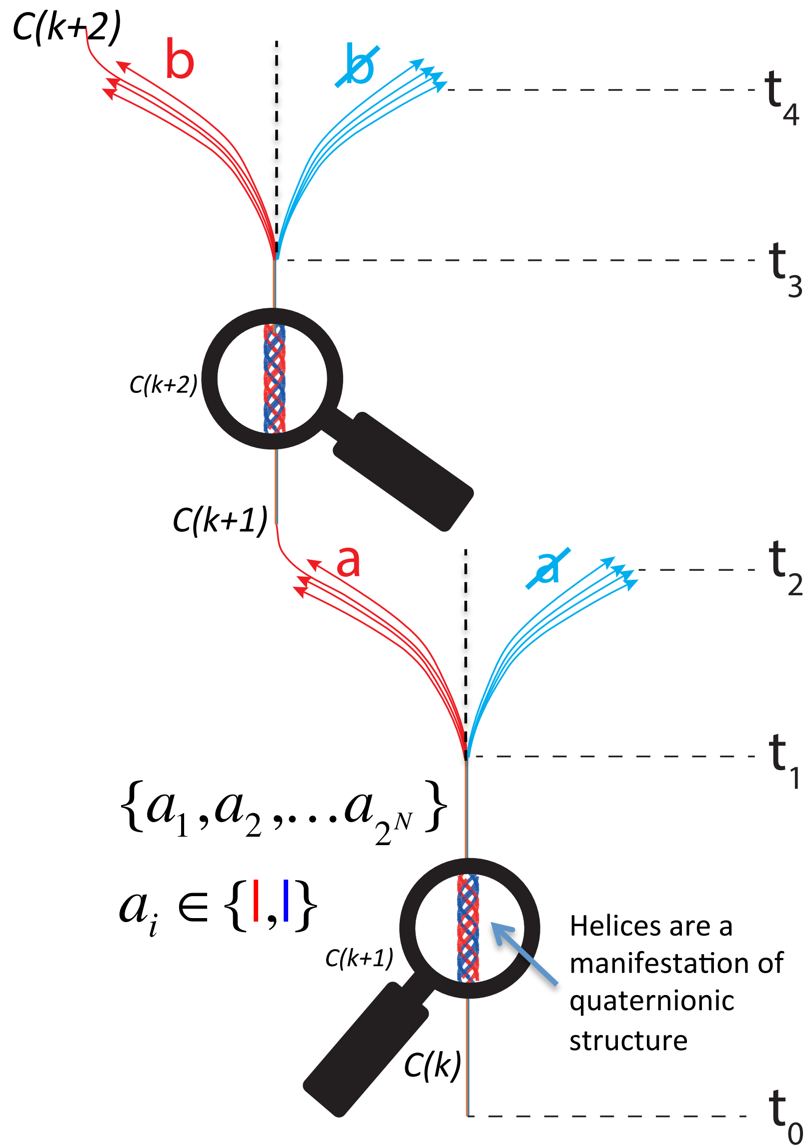
Undivided Wholeness and The Cosmological Invariant Set Postulate

Proc Roy Soc, 2009: Contemporary Physics, 2014

Suppose the most primitive expression of the laws of physics is not a set of differential evolution equations (in space-time, configuration or state space) but rather a mathematical representation of (fractal) geometry in the state space of the universe as a whole.

Realistic, Locally Causal, but **Nonclassical**

Measurement and the Complex Hilbert Space



Based on a geometric model

of $\mathbb{Z}_p$ where $p = 2^N + 1 \gg 0$

There is a correspondence between I_U and complex Hilbert Space vectors

$$\{a_1,a_2\ldots a_{2^N}\}\mapsto \cos\frac{\theta}{2}|\ |\ \rangle+\sin\frac{\theta}{2}e^{i\phi}|\ |\ \rangle$$

$a_i \in \{|,|\}$

but only when

$$\cos\theta \in \mathbb{Q}(N), \quad \frac{\phi}{\pi} \in \mathbb{Q}(N)$$

$\mathbb{Q}(N)$ denotes the set of rationals describable by N bits.

Complex numbers as bit-string permutation operators

$$S = \{a_1 a_2 a_3 a_4\}$$

$$a_i \in \{|,|\} \qquad \neg | = | \qquad \neg | = |$$

Define $e^{i\pi/4}(S) \equiv \{\neg a_4 a_3 a_1 a_2\}$

$$\Rightarrow \ \mathrm{e}^{i\pi/4} \circ e^{i\pi/4}(S) \equiv e^{i\pi/2}(S) = \{\neg a_2 a_1 \neg a_4 a_3\}$$

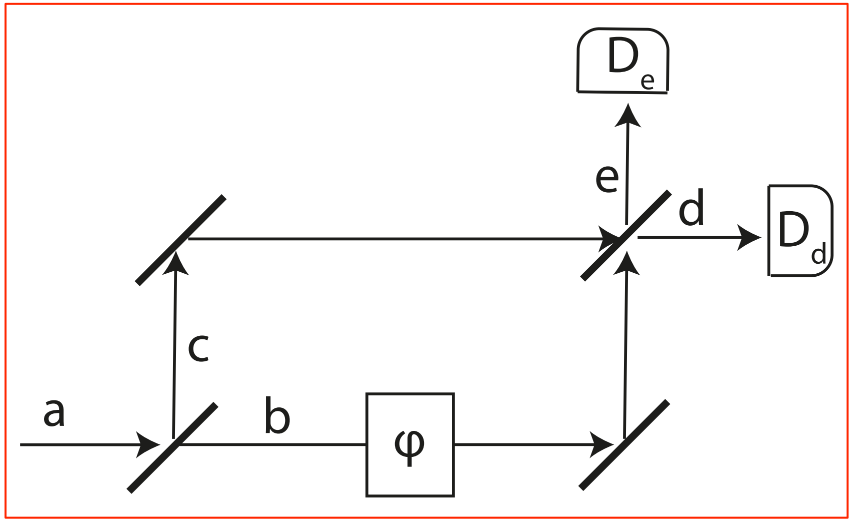
$$\Rightarrow e^{i\pi/2} \circ e^{i\pi/2}(S) \equiv e^{i\pi}(S) = \{\neg a_1 \neg a_2 \neg a_3 \neg a_4\} \equiv -S$$

Generalises to $e^{i\phi}(S)$ with $S = \left\{a_1 a_2 \ldots a_{2^N}\right\}$

for all ϕ / π describable by N bits.

Quantum Interference

$p =$



$\in I_U$

$$U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad V = \begin{pmatrix} 1 & \\ & e^{i\phi} \end{pmatrix}$$

$$|a\rangle \xrightarrow{UVU} \cos\frac{\phi}{2}|d\rangle + \sin\frac{\phi}{2}|e\rangle$$

With $\cos\phi \in \mathbb{Q}(N)$ then $\exists$ the correspondence

$$\cos\frac{\phi}{2}|d\rangle + \sin\frac{\phi}{2}|e\rangle \mapsto \{a_1, a_2, \ldots a_{2^N}\}$$

✔

Welcher Weg?

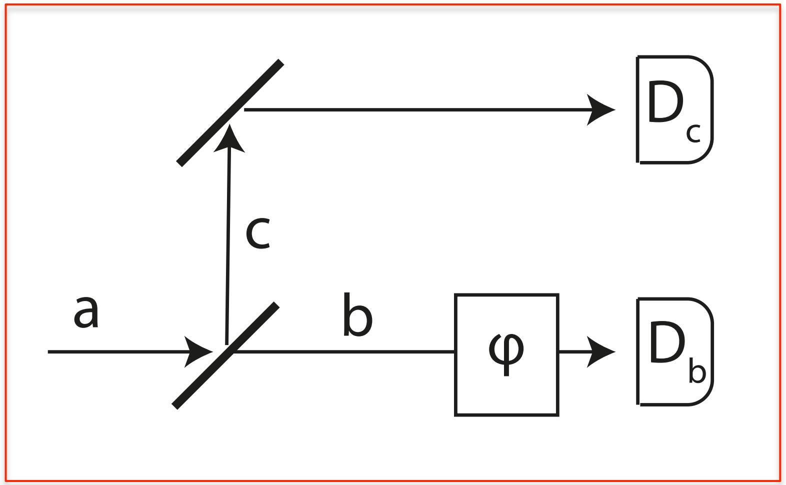
$$|a\rangle \xrightarrow{VU} \frac{1}{\sqrt{2}}\{|c\rangle + e^{i\phi}|b\rangle\}$$

For $0 < \phi < \pi/2$, $\cos\phi \in \mathbb{Q}(N) \Rightarrow \phi/\pi \notin \mathbb{Q}(N)$

$\Rightarrow$ no $\{a'_1, a'_2 \ldots a'_{2^N}\}$ corresponding to $|c\rangle + e^{i\phi}|b\rangle$

$\Rightarrow$ ✗

$p' =$



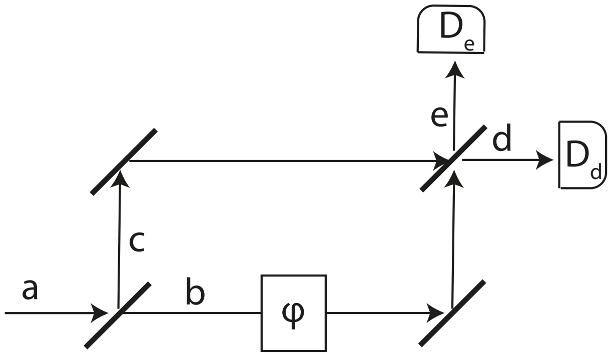
$\notin I_U$

Is this fine tuning gone mad?
No!

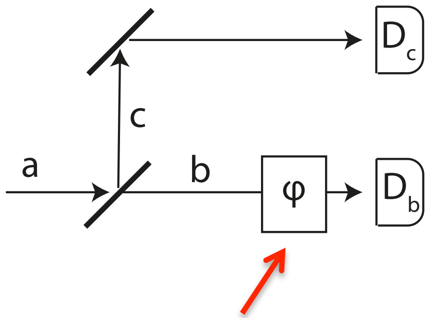
- Let U_1 denote the universe where a particle (e.g. with ontic variable λ) passes through MZ(ϕ). $U_1 \in I_U$.
- Let U_2 denote the universe where a Which Way experiment is performed in MZ(ϕ) on the same particle. $U_2 \notin I_U$.
- Let U_3 denote the universe where a Which Way experiment is performed in MZ(ϕ') on the same particle. $U_3 \in I_U$.
- $d_p(U_3, U_2) \gg 1$ even though $d(\phi', \phi)$ may be smaller than the accuracy with which the phase shifter can be set experimentally.
- If I_U is considered primitive, then the p-adic metric is a more physically relevant measure of distance than the Euclidean metric.

Heisenberg Uncertainty Principle from Number Theory!!

a)



b)



If a) lies on I_U, then b) does not lie on I_U, and vice versa – if you measure momentum, could cannot measure position and vice versa.

I.e. Heisenberg Uncertainty Principle implied by the number-theoretic non-commensurateness of ϕ and cos ϕ

Violates measurement independence assumption.

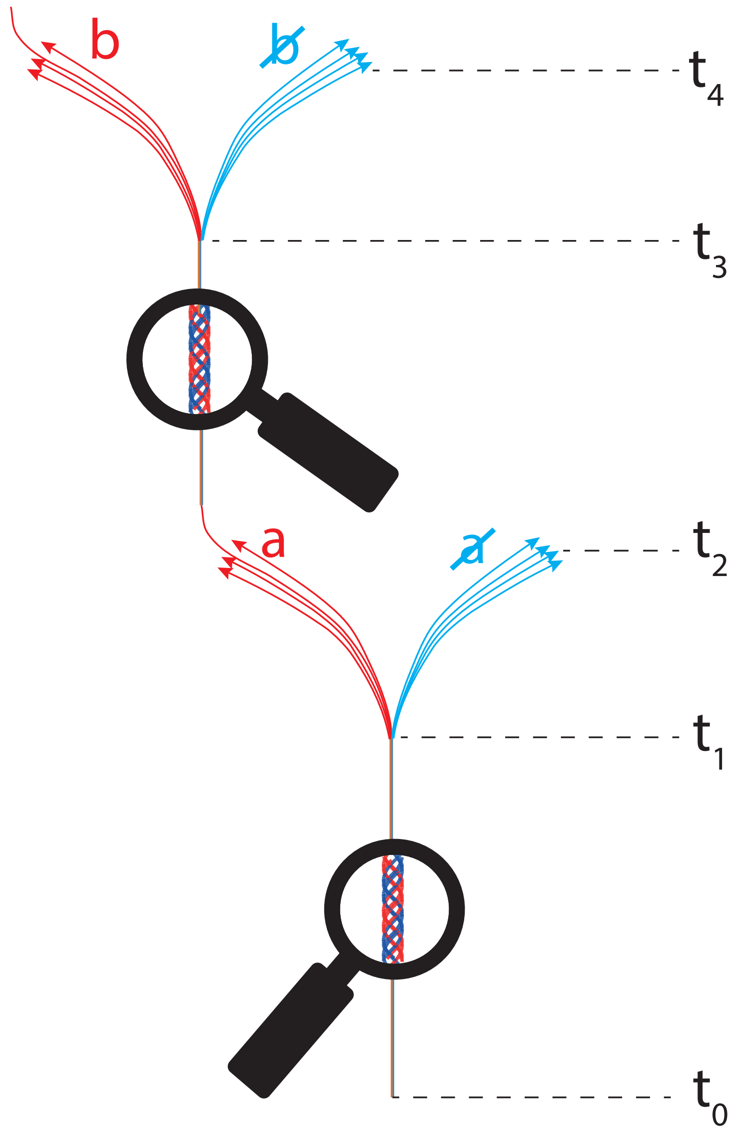
However, no violation of experimenter free will because of non-computability of I_U (cf delayed choice expt).

De Broglie Relationships

$$E = \hbar\omega \quad p = \hbar k$$

- In GR, a test particle's inertial properties are inherited from the neighbouring geometry of space-time (whose structure is determined by the locally euclidean pseudo-riemannian metric).

- In Invariant Set Theory, a particle's energy-momentum in space-time are inherited from the neighbouring helical periodic state-space geometry of I_U (whose structure is determined by the p-adic metric).



The Dark Universe:

Where the Euclidean metric meets the p-adic metric?

If I_U is to be considered primitive, we must also generalise GR e.g. from

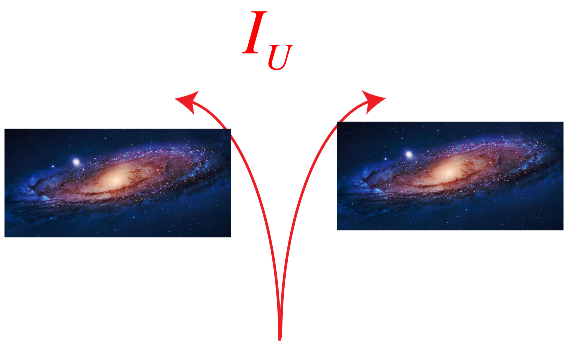
$$G_{ab}(\mathcal{M}) = \frac{8\pi G}{c^4} T_{ab}(\mathcal{M})$$

to

$$G_{ab}(\mathcal{M}) \approx \frac{8\pi G}{c^4} \int_{\mathcal{N}(\mathcal{M})} F(\mathcal{M},\mathcal{M}') T_{ab}(\mathcal{M}') d\mu'$$

where μ is a Haar measure and F a propagator on the fractal Invariant Set.

Dark Matter?



Dark Energy?

Phase-space convergence associated with space-time singularities

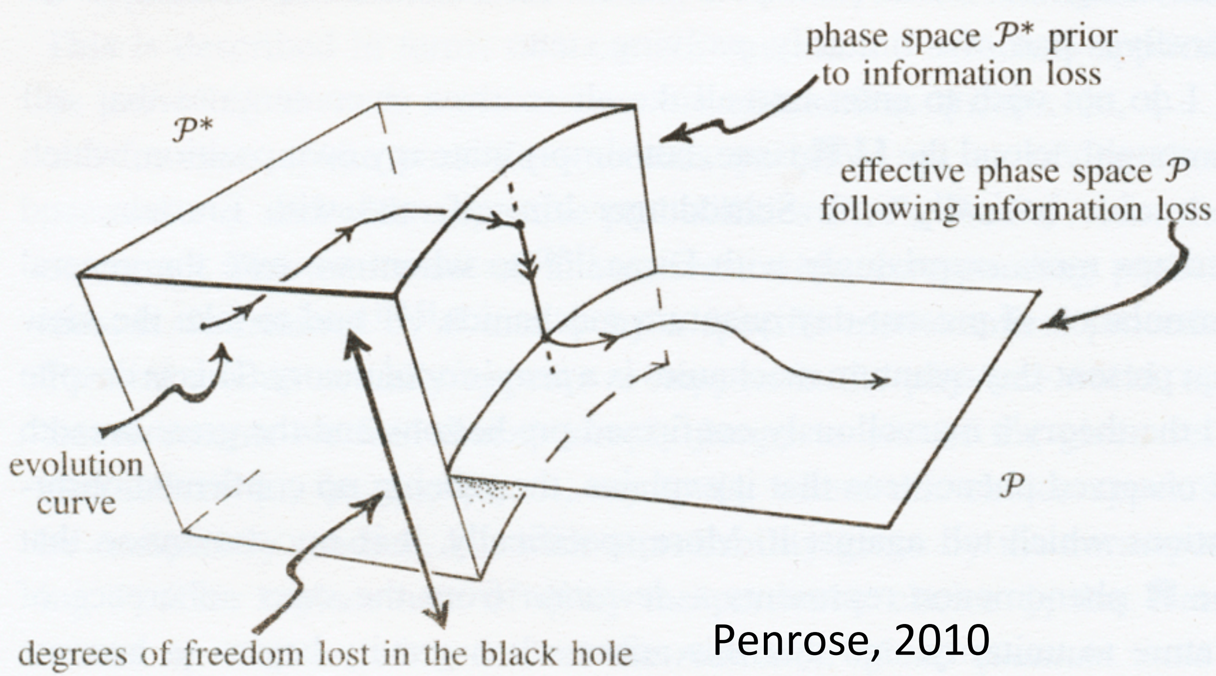


Fig. 3.14 Evolution in phase space following black-hole information loss.

Need space-time singularities to understand quantum physics, as much as we need quantum physics to understand space-time singularities.

Invariant Set Theory

- Non-classical but still realistic/ deterministic and locally causal.
- Violates measurement independence, but no constraint on human free will because of noncomputability.

Geometrising the Quantum Potential?

Is the quantum potential a coarse-grain (L^2) representation of the fractal invariant set geometry of the universe?

Extra Slides

Where Does Quantum Theory Fit ?

- Fractal dimension of C_p is log p/ log (2p-1) ➔1 as p ➔∞. But [0,1] is the **singular limit** of C_p at p=∞ because the measure of C_p is strictly zero for all finite p.

- Similarly, the complex Hilbert Space arises as the **singular limit** of IS state space at p=∞.

- Old theories are generically the **singular limit** of new theories (Michael Berry).

Conspiracy?

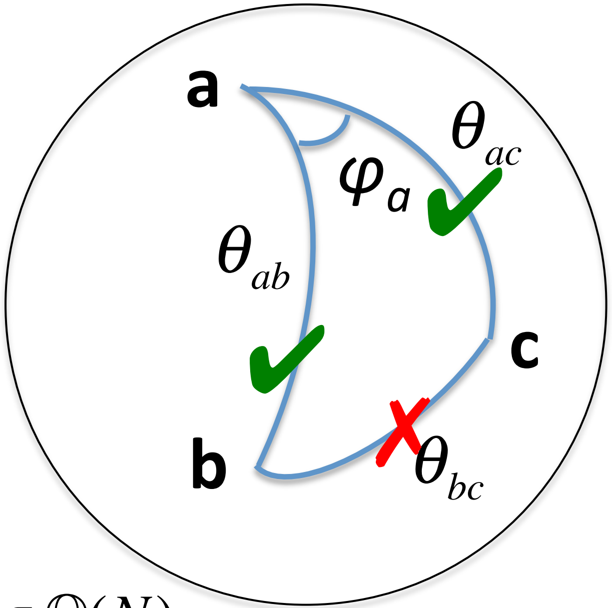
"Suppose the [CHSH] instruments are set… [by] random number generators… Suppose that the choice between two possible outputs depends on the oddness or evenness of the digit in the millionth decimal place of some input variable. … But this peculiar piece of information is unlikely to be the vital piece for any distinctively different purpose, i.e. is otherwise rather useless. In this sense the output of such a device is indeed a sufficiently free variable for the purpose at hand."

John Bell: Free variables and local causality (1977).

Reasoning incorrect if $d_p(U_{even}, U_{odd}) >> 0$

"Of course it might be that these reasonable [i.e. intuitive] ideas about physical randomizers are just wrong – for the purposes at hand."

EPR according to IS theory



Consider an entangled particle pair.

Suppose Alice measures her particle wrt **a**,

and **b** and **c** are two possible choices for Bob.

IS theory demands that $\cos\theta_{ab}$, $\cos\theta_{ac}$, $\phi_a/\pi \in \mathbb{Q}(N)$.

Cosine rule for spherical triangles

$$\cos\theta_{bc} = \cos\theta_{ab}\cos\theta_{ac} + \sin\theta_{ab}\sin\theta_{ac}\cos\phi_a$$

$\Rightarrow$ $\cos\theta_{bc} \notin \mathbb{Q}(N)$

$\Rightarrow$ The counterfactual universe where Alice measures wrt **b** and Bob wrt **c** is not on I_U

Bell's Theorem

$$|\mathrm{Corr}_\rho(\mathbf{a},\mathbf{b}) - \mathrm{Corr}_\rho(\mathbf{a},\mathbf{c})| \le 1 + \mathrm{Corr}_\rho(\mathbf{b},\mathbf{c})$$

where $\mathrm{Corr}_\rho(\mathbf{a},\mathbf{b}) = \cos\,\theta_{ab}$ etc

Suppose $\cos\theta_{ab},\ \cos\theta_{ac},\ \phi_a/\pi \in \mathbb{Q}(N)$.

$\Rightarrow$ By triangle rule, $\mathrm{Corr}_\rho(\mathbf{b},\mathbf{c}) = \cos\theta_{bc} \notin \mathbb{Q}(N)$

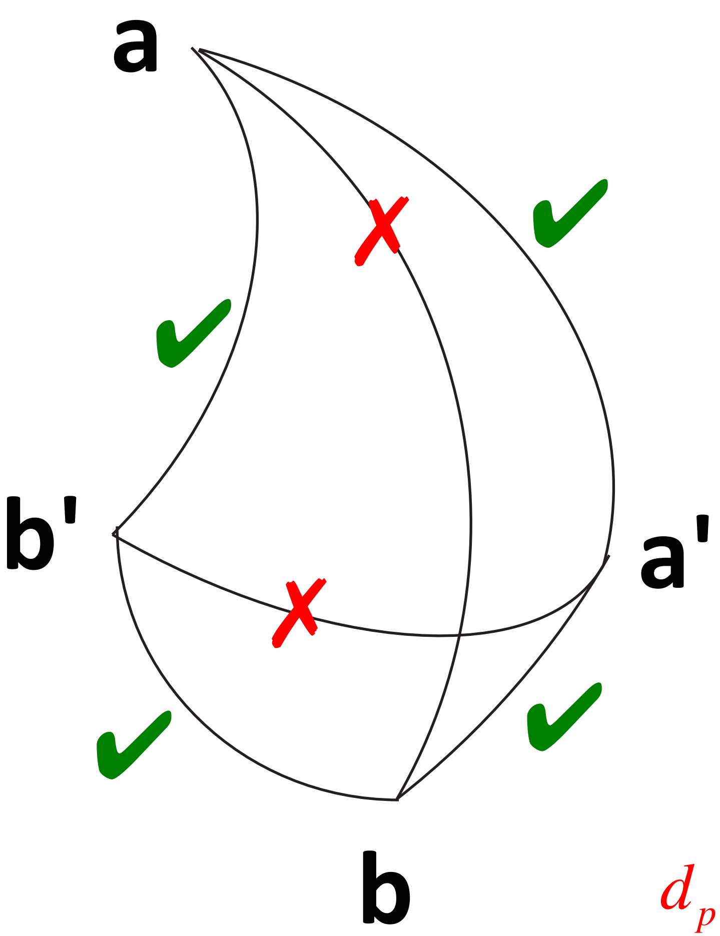
$\Rightarrow$ If $(\mathbf{a},\mathbf{b})$ and $(\mathbf{a},\mathbf{c})$ describe universes on I_U, then $(\mathbf{b},\mathbf{c})$ doesn't.

According to IS theory, what is measured experimentally is

$\mathrm{Corr}_\rho(\mathbf{a},\mathbf{b})$, $\mathrm{Corr}_\rho(\mathbf{a},\mathbf{c})$, $\mathrm{Corr}_\rho(\mathbf{b},\mathbf{c}')$

where $\mathrm{Corr}_\rho(\mathbf{b}\ ,\mathbf{c}') \in \mathbb{Q}(N)$ and $\theta_{bc} \approx \theta_{bc'}$

$|\theta_{bc'} - \theta_{bc}| <$ the finite precision of any experimental apparatus



CHSH

Alice can choose either **a** or **a’**.
Bob can choose either **b** or **b’**.

If Alice chooses **a** and Bob **b’** in universe *U* on I_U then the counterfactual universe *U’* where Alice chooses **a** and Bob **b** (or Alice **a’** and Bob **b’**) does not lie on I_U .

$d_p(U,U') \gg 1$ even though $d(U,U') \ll 1$

$$-2 \le Corr(\mathbf{a},\mathbf{b}) - Corr(\mathbf{a},\mathbf{b}') + Corr(\mathbf{a}',\mathbf{b}) + Corr(\mathbf{a}',\mathbf{b}') | \le 2$$

CHSH
What is measured experimentally?

$$-2 \le Corr(\mathbf{a},\mathbf{b}) - Corr(\mathbf{a},\tilde{\mathbf{b}}') + Corr(\tilde{\mathbf{a}}',\mathbf{b}) + Corr(\mathbf{a}',\mathbf{b}')| \le 2$$

where

$|\theta_{\tilde{\mathbf{a}}'\mathbf{a}'}|$, $|\theta_{\tilde{\mathbf{b}}'\mathbf{b}'}|$ < finite precision of measuring apparatuses

IS theory violates measurement independence, i.e.

$\rho(\lambda \mid \mathbf{b}, \mathbf{c}') \neq \rho(\lambda \mid \mathbf{b}, \mathbf{c})$

e.g. LHS $\neq 0$, RHS=0.

Fine Tuned? Conspiratorial? No!

Bell violation robust to uncountably many (p-adic) small-amplitude perturbations.

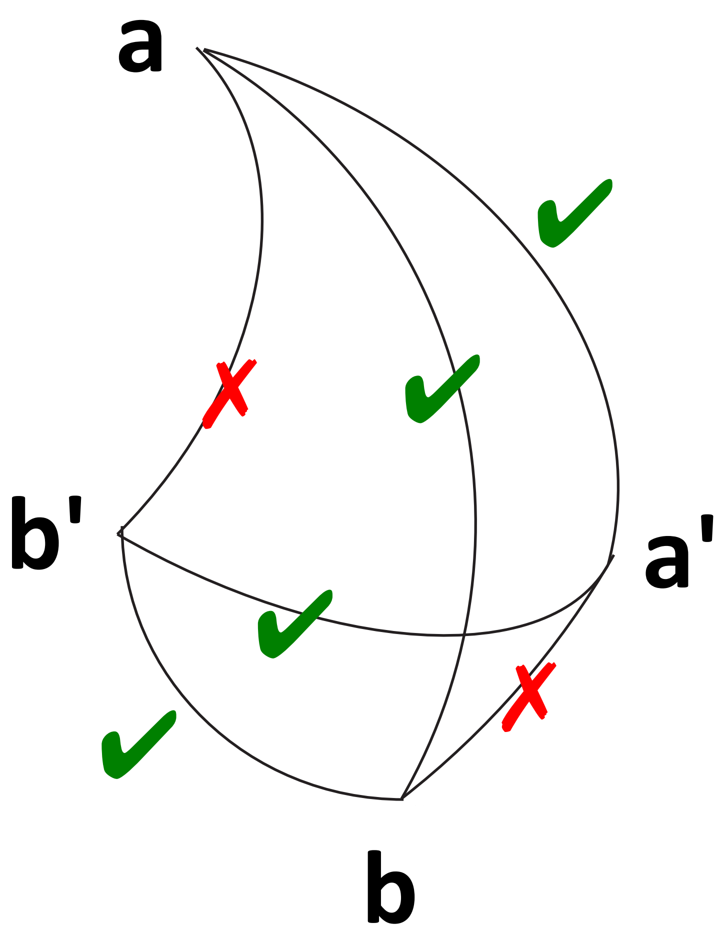
even though it is not robust - nor should it be - to (p-adic) large-amplitude perturbations.

Violates experimenter free will? No!

We are free when there is an absence of constraints preventing us from doing what we want to do (Hobbes, Hume, Mill...)

There are no computably knowable constraints that might conflict with Alice choosing either $\mathbf{a}$ or $\mathbf{a}'$, or Bob $\mathbf{b}$ or $\mathbf{b}'$.

Alice and Bob's choice of orientation setting is determined the action of their neurons, which are operating on I_U and therefore necessarily consistent with the structure of I_U. In particular, I_U cannot constrain these neurons from doing what they "want to do".



$$-2 \le Corr(\mathbf{a},\mathbf{b}) - Corr(\mathbf{a},\mathbf{b}') + Corr(\mathbf{a}',\mathbf{b}) + Corr(\mathbf{a}',\mathbf{b}') | \le 2$$