Chapter 1

Neutrino Phenomenology

1.1 Standard Model Neutrinos

1.1.1 Discovery of the Neutrino

In 1930, Pauli hypothesised an additional particle named the neutron, as "a desperate remedy" to conserve energy, momentum, and spin [1] to solve the continuous electron energy spectrum of beta decay which was firstly observed in 1914. The particle was renamed as the neutrino by Fermi [2] in 1933 to avoid confusion with a recently discovered particle by Chadwick. Bethe and Peierls expanded Pauli's theory and showed that this new particle interacts very weakly [3]. Due to this property, the existence of the neutrino has not been experimentally confirmed for over a quarter-century until 1956 when Reines and Cowan [4] first observed the inverse beta decay at the Savannah River nuclear reactor, using two large tanks of water doped with cadmium chloride sandwiched between scintillator tanks, which gave a unique signature of an antineutrino interaction.

$$\overline{\nu} + p \rightarrow n + e^{+} \tag{1.1}$$

Later in 1959, Davis and Harmer [5] looked for a similar reaction using anti-neutrinos and found that it does not occur, which indicated that the neutrino and anti-neutrino are different particles.

$$\overline{\nu} + n \nrightarrow p + e^- \tag{1.2}$$

According to a rule introduced by Konopinski and Mahmoud in 1953 [6], the lepton number for the electron, the muon, the tau, and the neutrino is $L = +1$, and the lepton number for the corresponding antiparticle is $L = -1$. In 1957, Goldhaber's experiment established neutrino helicity [7], resulting in the discovery that neutrinos are left-handed (LH) while anti-neutrinos are right handed (RH). Therefore, two properties can be used to distinguish neutrinos and anti-neutrinos: lepton number and the helicity.

1.1.2 The Standard Model

The Standard Model(SM) of particle physics is a theory describing interactions of all three types of elementary particles, including the leptons, quarks, and their mediators, as shown in Figure 1.1. These elementary particles are classified as either fermions or bosons by spin.

Fermions are elementary particles with spin 1/2. In the SM, there are 12 kinds of fermions, each with an associated antiparticle (see Figure 1.1). According to their interaction, these fermions are composed into two sectors: leptons and quarks, which both can be classified into three generations. All the six leptons interact through the weak force and only the three charged leptons, electron, muon and tau, interact through the electromagnetic force as well.

Particles with integer spin are bosons. There are three forces in the SM, and each of them has an associated elementary boson that mediates the interactions: the strong force is mediated by the gluon, the electromagnetic force is mediated by the photon, and the weak force is mediated by W^+, W^- and Z. The Higgs Boson is a massive boson with no intrinsic spin, which can explain the mass generation

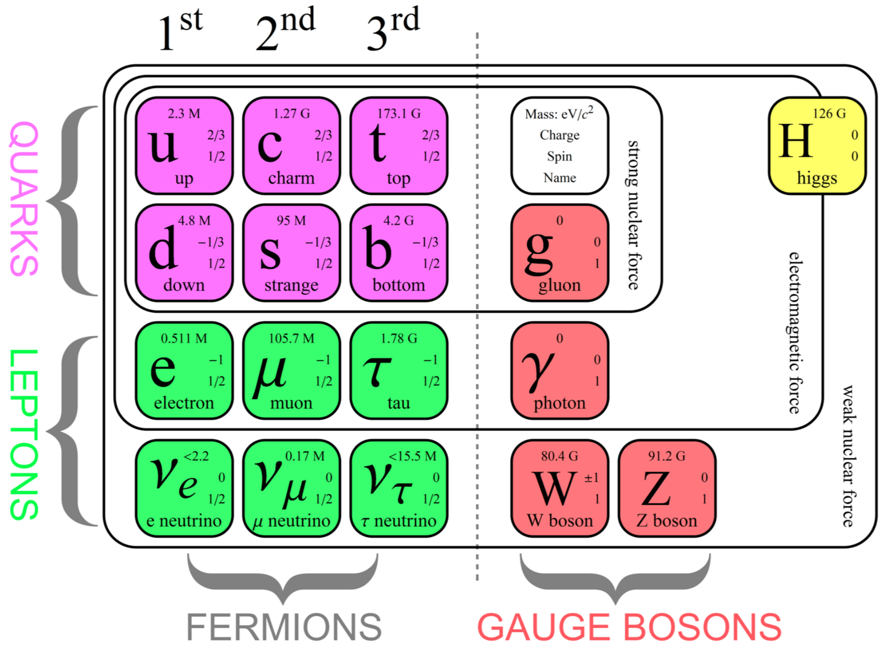


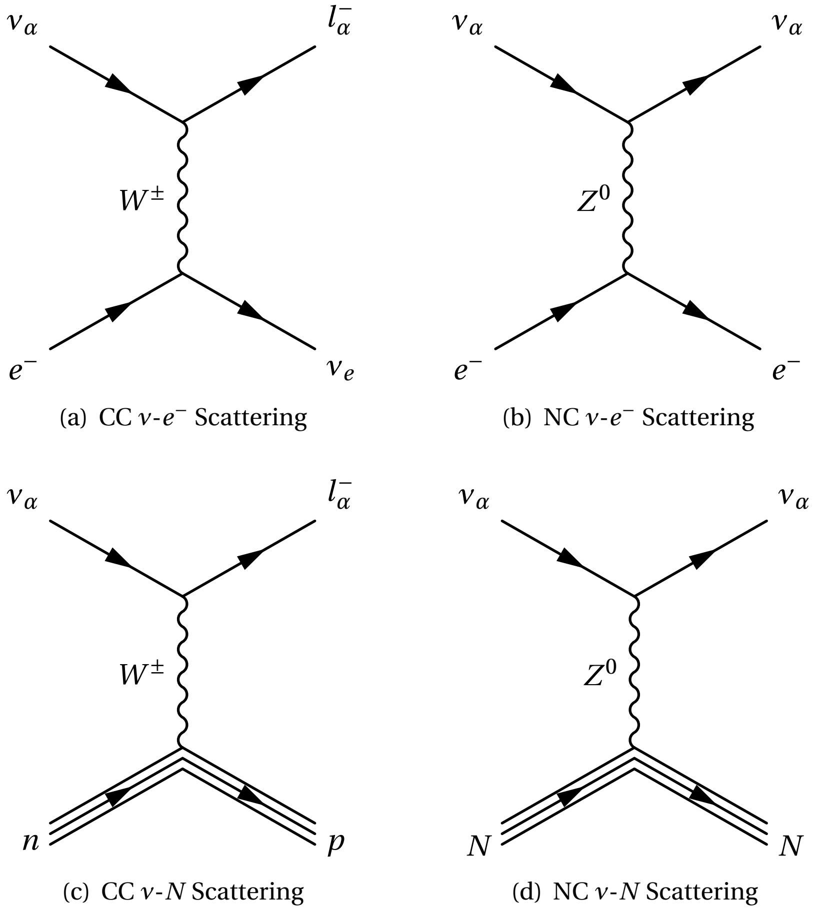
Figure 1.1: The Standard Model of elementary particles [8].

mechanism for bosons and fermions. The confirmation of the existence of Higgs Boson has completed the SM [9].

1.1.3 Neutrino Interactions

In the SM, neutrinos can interact only via the weak force, either through Charged Current (CC) interactions with the exchange of a $W^{\pm}$ boson, or Neutral Current (NC) interactions involving the exchange of Z^0 boson, as shown in Figure 1.2.

To observe these interactions, considerably large-scale detectors are required as the cross-sections are extremely small. In neutrino detectors, neutrinos scatter off nucleons or atomic electrons, and can therefore be detected through the recoil (or disintegration) of the target. For CC interactions, the outgoing lepton may also be observed, such that the flavour of the incoming neutrino can be determined.



(a) CC ν-e^- Scattering (b) NC ν-e^- Scattering (c) CC ν-N Scattering (d) NC ν-N Scattering

Figure 1.2: Two charged current and two neutral current neutrino interactions via which it is possible to detect neutrinos.

1.1.4 Neutrino Flavours

It was first experimentally confirmed that there was more than one flavour of the neutrino in 1962. A research group from Brookhaven National Laboratory observed the unambiguous signature of a CC interaction with an outgoing muon [10]. However, it was not until 2000 when the DOUNT collaboration reported their direct detection of the third flavour of neutrino, ν_τ, except from ν_e and ν_μ.

To date, three generations of neutrino been directly detected, ν_e, ν_μ and ν_τ, corresponding to three generations of charged leptons e, μ and τ. The number of active light neutrinos has been studied by four experiments at the LEP collider [11]. By measuring the width of Z^0 decay, the combined results from the four experiments show there are only three generations of neutrinos, as shown in Figure 1.3. However, the study is not sensitive to the sterile neutrinos, which are

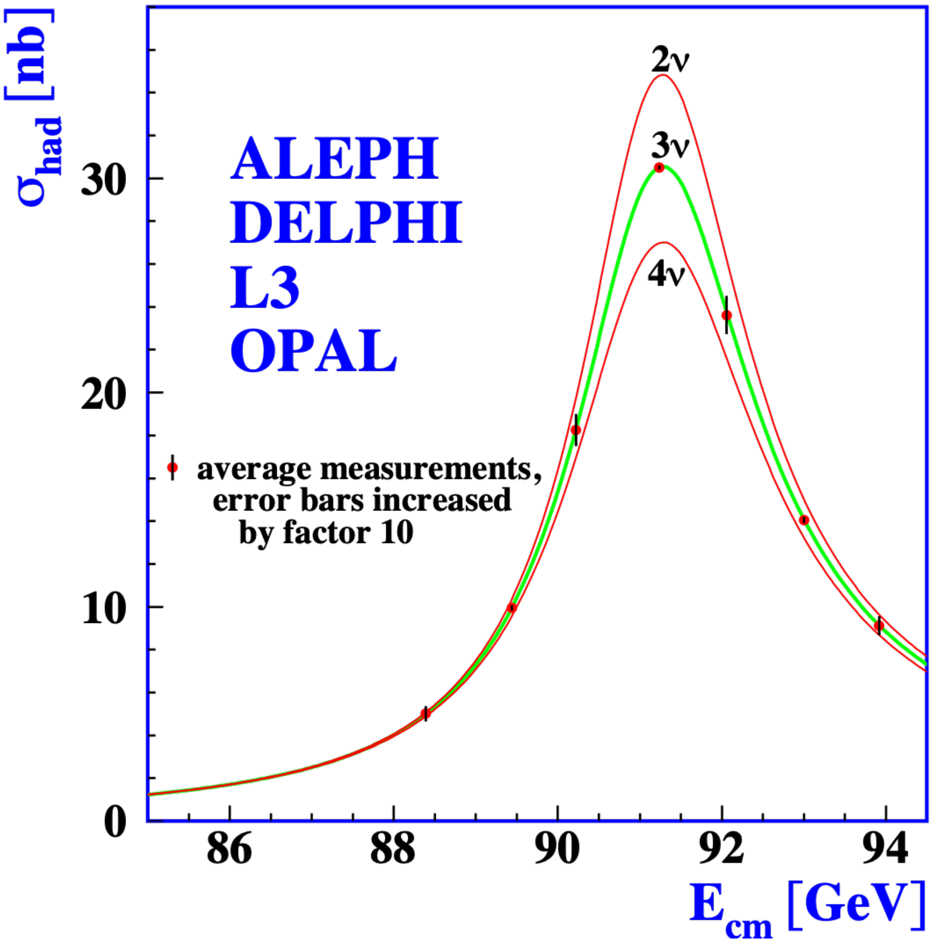


Figure 1.3: Combined LEP cross-section measurements for $e^+e^- \rightarrow hadrons$ around the Z^0 resonance. $N_\nu = 3$ is clearly favoured [11].

hypothetical particles not included in the Standard Model.

1.2 Neutrino Oscillations

Bahcall proposed a complete solar model to quantify the energy radiated from various fusion reactions occurring in the sun, including the energy carried by neutrinos. To validate the solar model proposed by J. Bahcall, the Homestake experiment led by American chemist Ray Davis measured neutrinos from the sun, using 100,000 gallons of cleaning perchloroethylene fluid. The detector is only sensitive to the CC interaction of electron neutrinos (Figure 1.2(c)). Their first results published in 1968 [12] were surprising in that the number of detected neutrinos were only one-third of the theoretical prediction, which became known as the "solar neutrino problem". In the following 30 years, about 2,000 solar neutrinos have been detected, but the conclusion remained unchanged.

There were many explanations for this, and one of the explanations is that the solar neutrinos oscillate. In this scenario, the ν_e from the sun becomes ν_μ and ν_τ, which would not be seen in the Homestake experiment since the energy is too low to produce a muon or tau.

The theory was first postulated by Pontecorvo, where neutrinos are not massless and as such could oscillate between different flavours [13, 14]. It was then further extended by Maki, Nakagawa, Sakata in 1962 [15].

It was not until 2001, when the Sudbury Neutrino Observatory (SNO) measured the solar neutrino flux in both the CC and NC channels [16] to obtain evidence of the disappearance of electron neutrinos, that the "solar neutrino problem" was finally solved. Their results of the number of detected neutrinos were consistent with Bahcall's solar models. The Kamiokande experiment performed the measurement of the solar neutrino flux performed independently. It also measured a significantly lower incoming solar neutrino flux, comparing with the predicted value [17].

1.2.1 Oscillations Phenomenology

The reason why neutrinos oscillate is that the neutrinos produced by weak interaction are not mass eigenstates. For neutrinos, the flavours are ν_e, ν_μ, ν_τ, and the mass eigenstates are ν_1, ν_2, ν_3. The neutrino flavour eigenstate $|\nu_\alpha\rangle$ under weak interaction can be expressed as a linear superposition of the neutrino mass eigenstates $|\nu_i\rangle$. The relationship between them can be described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, U, which is a unitary matrix similar to the Cabibo-Kobayashi-Masukawa (CKM) matrix for quarks.

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \tag{1.3}$$

$$U = \overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}^{\text{Atmospheric}} \overbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}^{\text{Cross-mixing}} \overbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{\text{Solar}} D_M \tag{1.4}$$

Where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, θ_{ij} is a mixing angle which defines the degree of mixing between mass states i and j, and δ is the CP-violating phase. All mixing angles θ_{ij} have been determined experimentally in the reactor and accelerator experiments. The last ~~one~~ measured, θ_{13}, has a non-zero value, thus gives access to the neutrino CP-violating phase.

Furthermore, if the neutrino is a Majorana particle, the additional Majorana CP-violating phases ϕ_1 and ϕ_2, are included in the diagonal matrix D_M:

$$D_M = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.5}$$

The evolution of a mass eigenstate $|\nu_i\rangle$ over space and time obeys the Schrödinger equation, such that, in natural units,

$$|\nu_i(t)\rangle = e^{-i(E_i t - p_i L)}|\nu_i(0)\rangle \tag{1.6}$$

where t is time, and E_i and p_i refer to the energy and momentum of neutrinos in the lab frame. Assuming that the mass of neutrinos is negligible compared to the energy, and $t \approx L$,

$$E_i = \sqrt{{p_i}^2 + m_i^2} = p_i + \frac{m_i^2}{2p_i} \tag{1.7}$$

$$|\nu_i(t)\rangle \approx e^{-i(m_i^2/2p_i)L}|\nu_i(0)\rangle \tag{1.8}$$

Considering $E \approx p$ for a relativistic neutrino of flavour , the equation of propagation becomes

$$|\nu_\alpha(L)\rangle \approx \sum_i U_{\alpha i} e^{-i(m_i^2/2E)L}|\nu_i\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-i(m_i^2/2E)L}\left|\nu_\beta\right\rangle \tag{1.9}$$

Consider a neutrino flavour transition from α to β, the transition amplitude can be written in term of the distance travelled

$$A(\alpha \rightarrow \beta)(L) = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-i(m_i^2/2E)L} \tag{1.10}$$

Thus the probability of transition can is:

$$P(\alpha \rightarrow \beta)(L) = \left|A(\alpha \rightarrow \beta)(L)\right|^2 = \left|\sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-i(m_i^2/2E)L}\right|^2 \tag{1.11}$$

Considering the simplified two neutrino case, in a vacuum, the probability of a neutrino changing from flavour α to β is given by:

$$P(\alpha \rightarrow \beta)(L) = sin^2(2\theta_{ij}) sin^2(\Delta m^2_{ij} \frac{1.27L}{E}) \tag{1.12}$$

where Δm^2_{ij} is the mass mixing splitting between the two neutrino mass eigenstates $|\nu_i\rangle and \left|\nu_j\right\rangle andisdefinedas : \Delta m^2_{ij} \equiv m_i^2 - m_j^2 (1.13)$

Therefore, the probability of oscillation is determined by the parameters in the PMNS matrix and mass splittings. Since the probability depends only on Δm^2_{ij}, measuring the oscillations parameters does not give an answer to the absolute neutrino mass.

1.2.2 Oscillations in Matter

All neutrino flavours can interact with matter (protons, neutrons and electrons) through NC, this is identical for all flavours and so would not affect the oscillation probabilities between them. However, ν_e interact with electrons in matter via CC interaction, exchanging $W^{\pm}$. This effect, known as the Mikheyev-Smirnov-Wolfenstein (MSW) effect, is proportional to the electron density and the energy of the neutrino and modifies the expressin for the neutrino oscillation probability.

1.2.3 Oscillations Parameters

In the past few decades, flavour oscillation of solar, atmospheric, reactor, and accelerator neutrinos have been studied to determine the value of the PMNS matrices parameters. The latest best-fit limits of the three mixing angles and two mass-squared differences are summarised in Table 1.2.

Parameter	Ordering	Best-fit	3σ range
$\sin^2\theta_{12}/10^{-1}$	Normal/Inverse	3.10	2.75 - 3.50
$\sin^2\theta_{23}/10^{-1}$	Normal	5.58	4.27 - 6.09
$\sin^2\theta_{23}/10^{-1}$	Inverse	5.63	4.30 - 6.12
$\sin^2\theta_{13}/10^{-2}$	Normal	2.241	2.046 - 2.440
$\sin^2\theta_{13}/10^{-2}$	Inverse	2.261	2.066 - 2.461
$\Delta m^2_{21}/10^{-5}eV^2$	Normal/Inverse	7.39	6.79 - 8.01
$\Delta m^2_{32}/10^{-3}eV^2$	Normal	2.449	2.358 - 2.544
$\Delta m^2_{32}/10^{-3}eV^2$	Inverse	2.509	2.603 2.416

Table 1.1: Best current estimates for neutrino mixing parameters from a global fit [18], Δm^2 defined as $m_3^2 - (m_1^2 + m_2^2)/2$ [19].

The neutrino oscillation experiments are only sensitive to the squared mass differences ($\Delta m^2_{ji} = \Delta m^2_j - \Delta m^2_i$). Current oscillation experiments have measured two of the squared mass differences, Δm^2_{21} and Δm^2_{32}. As a consequence, there are two possible mass ordering status: the normal ordering(NO, $\Delta m^2_{31} > 0$) and the inverse ordering(IO, $\Delta m^2_{31} < 0$), see Section 1.5.3. The PMNS mixing matrix also contains at least one CP-violating phase, see Section 1.5.4, and there will be two more if neutrinos are Majorana fermions. The Dirac CP-violating phase can be measured by the accelerator experiments or atmospheric experiments.

Experiment	Dominant	Important
Solar Experiments	θ_{12}	$\Delta m^2_{21}, \theta_{13}$
Reactor LBL	Δm^2_{21}	θ_{12}, θ_{13}
Reactor MBL	$\Delta m^2_{31}, \Delta m^2_{32}$	
Atmospheric Experiments		$\Delta m^2_{31}, \Delta m^2_{32}, \theta_{13}, \delta_{CP}$
Accel LBL $\nu_\nu, \bar{\nu}_\nu$, Disapp	$\Delta m^2_{31}, \Delta m^2_{32}, \theta_{23}$	
Accel LBL $\nu_e, \bar{\nu}_e$ App	δ_{CP}	θ_{13}, θ_{23}

Table 1.2: Experiments contributing to the present determination ofthe oscillation parameters [19].

1.3 Neutrino Mass

The observation of the neutrino oscillation has provided clear evidence that neutrinos have non-zero mass. Several attempts on extending the SM to add the neutrino mass have been studied.

Two distinct mass terms were considered for addition to the SM, depending on the nature of neutrinos. The first one is the Dirac mass term, where the neutrino is treated as a Dirac particle, the same as other SM fermions. The second is the Majorana mass term, where the neutrino is regarded as its own antiparticle. A combination of both Dirac and Majorana mass is also considered.

1.3.1 Dirac Mass

In the SM, neutrinos are all LH particles, which interact weakly. To allow a massive neutrino, the RH neutrino can be added to the SM as an extension, which is sterile since experiments show it does not participate in the weak interaction.

Similar to the way that other charged leptons and quarks acquire mass in the SM, via the introduction of the RH field, the Dirac neutrino mass term can be added through the coupling of LH and RH fields with the Higgs field.

Although the minimum extension to the SM is introducing only one RH field, it is more natural to include three RH field singlets $\nu_{e,R}$, $\nu_{\mu,R}$ and $\nu_{\tau,R}$ corresponding the three generations.

The Lagrangian for a massive Dirac neutrino in the simplified, single-flavour case is,

$$\mathscr{L}_D = -\frac{1}{2} m_D \left(\overline{\nu_L} \nu_R + \overline{\nu_R} \nu_L \right) + \text{h.c.} \tag{1.14}$$

where m_D is a constant mass term that represents the Yukawa coupling between the neutrino and Higgs fields, and h.c. is the Hermitian conjugate of the first two terms. The neutrino and anti-neutrino fermionic field are both composed of two chiral fields:

$$\nu = \nu_L + \nu_R, \overline{\nu} = \overline{\nu_L} + \overline{\nu_R} \tag{1.15}$$

Here, neutrinos and anti-neutrinos are fundamentally different particles. This method seems like a small and natural extension to the SM; however, several drawbacks arise. Firstly, it introduces a sterile neutrino which cannot be directly experimentally detected. Secondly, it does not answer why the Higgs-neutrino Yukawa coupling needs to be extremely small. The coupling constant will be $< 0^{-12}$, corresponding to the neutrino mass $< 1eV$, which is $> 10^9$ times lower than the coupling constant for τ, and $> 10^6$ times smaller than the one for e.

1.3.2 Majorana Mass

It is possible to form a non-zero mass term by charge-conjugating the RH field and contracting it with itself, as the neutrino is neutral and it does not need to conserve charge. In 1936, Ettore Majorana proposed a new type of mass term, known as Majorana mass [20], in which the two chiral fields ν_L and ν_R are considered to be non-independent. The charge conjugation is the same as the arbitrary phase as:

$$\nu_R^c = C\nu_R \equiv i\gamma^2 \nu_R \tag{1.16}$$

where C is the charge conjugation operator. In the Majorana case, only two of the four components in the Dirac case are needed, and Majorana neutrino is its own antineutrino:

$$\nu = \nu_L + \nu_L^C, \nu^C = \nu \tag{1.17}$$

The Lagrangian for the Majorana neutrino is:

$$\mathscr{L}_M = -\frac{1}{2} m_R \overline{\nu_R^c} \nu_R + \text{h.c.} \tag{1.18}$$

where m_R is a constant Majorana mass term and h.c. is the Hermitian conjugate. It should be noted that in the Equatio 1.18, lepton number is not conserved as the incoming neutrino is destroyed and an outgoing anti-neutrino is created. Thus, the existence of Majorana mass term implies lepton number violation, including neutrinoless double beta decay.

1.3.3 See-Saw Mechanism

It is possible to bring Dirac and Majorana descriptions together to explain the small mass of neutrinos, where the Lagrangian is:

$$\mathscr{L}_{M+D} = \mathscr{L}_D + \mathscr{L}_M \tag{1.19}$$

$$\mathscr{L}_{M+D} == -\frac{1}{2} m_D \left(\overline{\nu_L} \nu_R + \overline{\nu_R} \nu_L \right) - \frac{1}{2} m_R \overline{\nu_R^c} \nu_R + \text{h.c.} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \mathscr{M} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{h.c.}$$

where $\mathscr{M}$ is given by:

$$\mathscr{M} = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \tag{1.20}$$

It should be noted that the neutrino states in Equation 1.20 are weak eigenstates rather than mass eigenstates which can be found by be diagonalising the $\mathscr{M}$. The eigenvalues of $\mathscr{M}$ can be derived from Equation 1.20:

$$m_{1,2} = \frac{1}{2} m_R \pm \frac{1}{2} \sqrt{m_R^2 + 4 m_D^2} \tag{1.21}$$

Since the standard module does not have any requirement on the value of the right-hand Majorana term, m_R can be very large, where $m_R \gg m_D$. In this scenario, the two reduced eigenstates are:

$$m_1 \approx m_R, m_2 \approx \frac{m_D^2}{m_R} \tag{1.22}$$

Notice that if the neutrino of mass m_1 is heavy, then the mass of the other neutrino m_2 is very light due to the suppression factor $\frac{1}{m_R}$. This is well known as the See-Saw Mechanism. It can answer the question of why the neutrino is much lighter than other charged leptons. Assuming that the Dirac mass is $\sim 1Gev$, in the same range with the other charged leptons, and the right-hand Majorana mass term is at the scale of Grand Unification Theory ($\sim 10^{15}GeV$), then the mass of the light neutrino is in the meV scale, as it is observed. The See-Saw Mechanism introduced a very heavy neutrino, and CP violations in the decay of this heavy neutrino in the early universe could explain the matter-antimatter asymmetry.

1.4 Neutrino Mass Constraints From Experiments

There are mainly four types of experiments that offer information about neutrino mass. Tritium Decay, $0\nu\beta\beta$, and cosmological models can measure the absolute neutrino mass, while neutrino oscillation experiments can only measure the squared mass differences. Through the precise measurement of $|\Delta m_{23}^2|$, which is the largest mass splitting, a lower limit of the heaviest neutrino can be set as $> 0.06\,(\sqrt{|\Delta m_{23}^2|})$, since the lightest neutrino cannot be less than zero.

1.4.1 Tritium Decay Experiment

Tritium Decay is one of the standard methods to measure the mass of the electron neutrino by measuring the endpoint of the beta decay spectrum with high energy resolution. The underlying mechanism for this data decay is:

$$^3\mathrm{H} \rightarrow {}^3\mathrm{He} + \mathrm{e}^- + \overline{\nu_e} \tag{1.23}$$

The maximum allowable energy for the electron from the decay kinematics, called the end point, is Q_β, and it equals the difference of masses between the ^{3}H and ^{3}He $+ e^-$ at rest. However, if the neutrino mass has a non-zero value, the curve of the energy spectrum will be affected slightly. In addition, the Q_β will be reduced by the neutrino mass, as shown in Figure 1.4.

chapters/neutrino-theory/tritiumSpectrum.png

Figure 1.4: Sample spectrum for tritium decay, showing exaggerated distortion in the high energy tail due to a neutrino of mass 30 [21].

This kind of direct measurement is challenging due to the very small neutrino mass. The number of electrons near the end point of the spectrum is small, so the statistical error will be large. As well as this, the energy resolution of the detector should be excellent. However, the advantage of a direct measurement is obvious in that it only relies on energy and momentum conservation to extract $\langle m_\beta \rangle$, and thus the result is model independent. The KATRIN experiment, which started data taking in 2018, aims to reach a sensitivity of 200 meV at 90% CL. It has produced the first result of < 1.1 eV, which improved the previous ^{3}H constraint by a factor of 2 [22].

1.4.2 Cosmology

The analysis of the cosmic background (CMB) and its anisotropies has also made it possible to constrain the mass of neutrinos. A large number of photons generated in the Big Bang were left behind after the end of the Great Thermal Explosion, which red-shifted and cooled as the universe expanded, forming the Cosmic Microwave Background (CMB) radiation we observe today. Similarly, a large number of neutrinos produced during the Big Bang were also left behind, creating a Cosmic Neutrino Background (CNB). This background has not been detected so far, but it is possible to measure it indirectly using cosmological observation data. The combined analysis of the anisotropy of the cosmic background (CMB), baryon acoustic oscillations, and large scale structure formation have also made it possible to constrain the mass of neutrinos, although it is heavily dependent on cosmological models. The current best limits are $\sum m_i$ <0.11 eV at 95% CL [23].

1.4.3 Double Beta Decay

Double beta decay is a rare process of weak interaction, involving two beta decays in one nucleus at the same time. It occurs only when the single beta decay is forbidden from the energetic point of view. The two-neutrino double beta decay ($2\nu\beta\beta$) is allowed by the SM and has already been observed for several nuclei. However, neutrinoless double beta decay ($0\nu\beta\beta$) is prohibited by the SM because it violates leptonic number conservation. If neutrinos are indeed Majorana particles, $0\nu\beta\beta$ is sensitive to the effective Majorana neutrino mass, $\langle m_{\beta\beta}\rangle$:

$$\langle m_{\beta\beta}\rangle = \left|\sum_i U_{ei}^2 m_i\right| \tag{1.24}$$

The current best limit of $\langle m_{\beta\beta}\rangle$ is $< 61-165$ meV, given by the combined analysis of Kamland-Zen and the EXO experiment in ^{136}Xe []. More details about $0\nu\beta\beta$ will be discussed in Chapter **??**.

1.5 Outstanding Questions

Since the discovery of neutrino oscillation, significant progress has been made in neutrino physics. However, there are still many outstanding questions left for current and next generation oscillation experiments, double beta decay experiments, cosmology, and beta decay experiments to solve. The unresolved problem of neutrino and the corresponding experimental techniques summarised in Table 1.3.

1.5.1 Number of Neutrinos

Three generations of light neutrinos (ν_e, ν_μ and ν_τ) have been detected, as discussed in Section 1.1.4. However, there is still the possibility of one or more

Property	Oscillation	Cosmology	β-decay	$0\nu\beta\beta$
Number of Neutrinos	✓	✓		
Absolute Mass		✓	✓	✓
Mass Ordering	✓			✓
Dirac or Majorana				✓
Dirac CP-violation	✓			
Majorana CP-violation				✓

Table 1.3: Unknown properties of the neutrino that can be observed using the four main experimental techniques.

sterile neutrinos. The results from the short baseline (SBL) experiment such as LSND and MiniBoone show some hints of the existence of an eV-range sterile neutrino and some cosmological results favour the keV-range sterile neutrino. So far, there is no definitive evidence confirming or refuting the existence of sterile neutrino with sufficient confidence, thus is left to the next generation of short baseline neutrino oscillation experiments to answer.

1.5.2 Absolute Neutrino Mass

The current and next generation of direct measurements, such as KATRIN, and cosmology experiments are working on determining the absolute neutrino mass. In the next few years, if the neutrino is Majorana particle, $0\nu\beta\beta$ experiment will also be able to provide information about neutrino mass down to 30meV. In the meantime, cosmology, and direct measurement such as KATRIN will keep working on determining the absolute mass. But an unrealistic large-scale spectrometer will be required to reach a better sensitivity for KATRIN experiment; thus it will be a real technical challenge to surpass the current limit of 0.2 eV in the near future. There are other ideas on determining the electron energy precisely, such as Project-8 which measure the frequency of cyclotron radiation emission from single electrons [24].

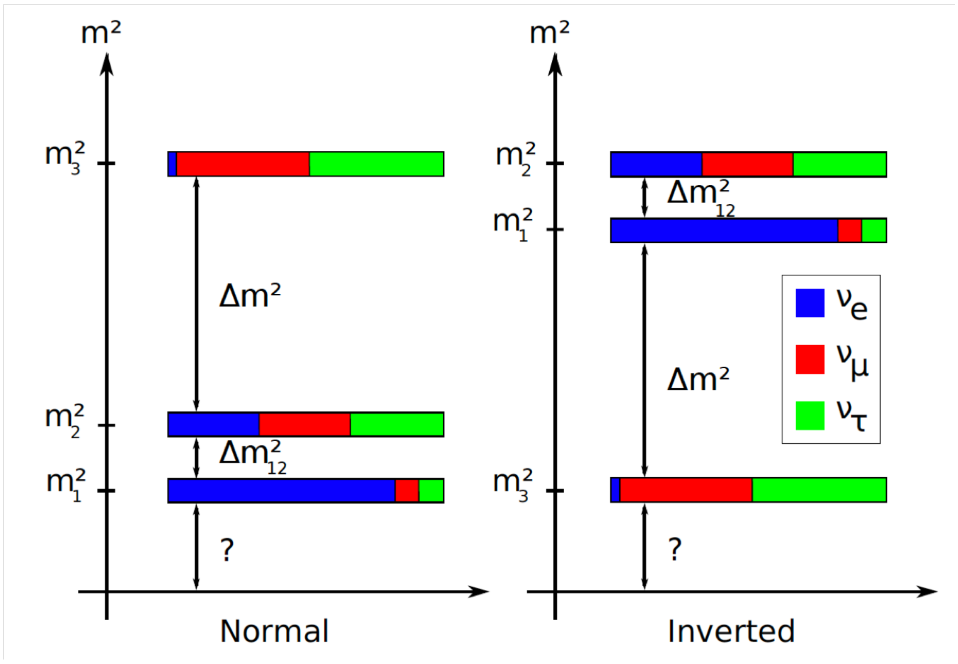


Figure 1.5: A representation showing the "normal" and "inverted" mass hierarchies of absolute neutrino masses.

1.5.3 Neutrino Mass Ordering

Oscillation experiments have confirmed that $m_1 < m_2$ through precise measurements of the Δm_{12}^2. But there are still two possibilities whether $m_1 < m_2 < m_3$ known as normal ordering (NO) or $m_3 < m_1 < m_2$ known as the inverted ordering(IO), shown in Figure 1.5. Thus the neutrino mass ordering remains an open question to be solved. The current and next generation of neutrino oscillation experiments will measure Δm_{32}^2 to give the information of neutrino mass ordering.

The $0\nu\beta\beta$ search is dependent on the neutrino mass ordering. The effective majorana mass $\langle m_{\beta\beta}\rangle$ which is related to the decay rate, as a function of the lighest neutrino mass m_light is shown in Figure 1.6

1.5.4 CP violation

In the SM, CP violation is caused by a complex phase in the CKM matrix describing quark mixing. Similar to the quark sector, the CP violating phase in

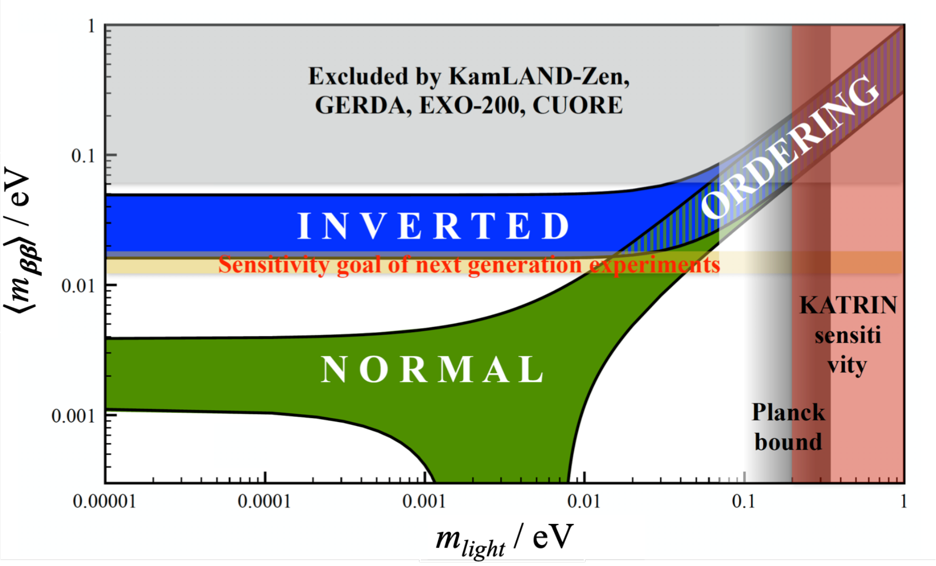


Figure 1.6: The effective Majorana mass $\langle m_{\beta\beta} \rangle$ as a function of the lightest neutrino mass m_light, with best fit values with 2σ errors of the oscillation parameters, for normal hierarchy (green) and inverted ordering (blue) [25].

the PMNS matrix can cause a difference in the oscillation probability between neutrinos and anti-neutrinos. Future long baseline (LBL) accelerator neutrino oscillation experiments can detect this difference and determine the Dirac CP violating phase. If the neutrinos are Majorana particles, there are two additional CP violating phase in the PMNS matrix, which do not affect the neutrino oscillation probability and therefore, unable to be measured with the oscillation experiments. The $0\nu\beta\beta$ experiment is the only practical way to measure the Majorana CP violating phase.

1.5.5 Dirac or Majorana

The nature of the neutrino, whether it is a Dirac particle or a Majorana particle, is one of the most fundamental questions. And it can only be determined by the $0\nu\beta\beta$ experiments. The current and near future experiments will be able to reach the sensitivity of $\langle m_{\beta\beta} \rangle$ if the neutrino mass ordering is inverted; however, it will be more challenging if the mass ordering is normal.

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