Chapter 1

Double Beta Decay

1.1 Beta Decay

Beta decay (β decay) is mediated by the weak force. It transmutes a nucleus to a different element accompanied by the emission of a neutrino or antineutrino. There are three forms of this process: β^- decay, β^+ decay, and electron capture (EC).

In β^- decay, a neutron converts to a proton, emitting one electron and one antineutrino:

$$n \to p + e^- + \overline{\nu_e} \tag{1.1}$$

In β^+ decay, a proton converts to a neutron, emitting one positron and one neutrino:

$$p \to n + e^+ + \nu_e \tag{1.2}$$

EC converts a proton and an electron to a neutron, accompanied by the emission of a neutrino. In EC, an atomic electron exchanges a W boson with a quark in the nucleus.

$$p + e^- \to n + \nu_e \tag{1.3}$$

All three types of β decay process are allowed only when the mass of the nucleus in the initial state is higher than that of the final state:

$$M(A, Z_i) > M(A, Z_f) \tag{1.4}$$

where $M(A,Z)$ is the mass of the atom with A nucleons and Z protons, and the subscripts i and f refer to the initial and final states respectively. The mass of a nucleus, $M(A,Z)$, can be estimated using the semi-empirical mass formula (SEMF) [1], therefore it is possible to predict whether a β decay is allowed or forbidden. The SEMF gives the mass of a nucleus, m, as:

$$m = Zm_p + (A-Z)m_n - a_V A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A} + \delta(A,Z) \tag{1.5}$$

where

$$\delta(A,Z) = \begin{cases} \frac{a_p}{A^{1/2}} & Z, N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ \frac{-a_p}{A^{1/2}} & Z, N \text{ odd } (A \text{ even}) \end{cases} \tag{1.6}$$

The first two terms calculate the masses of individual protons and neutrons respectively. The rest are the volume term, the surface, the Coulomb term, the asymmetry term, and the pairing term, which provide the correction to the approximation. For fixed A, parabolic curves are generated as a function of Z, which dictate which β decays are energetically allowed. If A is odd, only one curve will exist, see Figure 1.1.

1.2 Two Neutrino Double Beta Decay

In Figure 1.1, it can be seen that the beta decay process of isotope (c) → (d) is energetically forbidden. However, it can undergo the process (c) → (e) where two β decays simultaneously where two neutrons decay to two protons, emiting two electrons and two antineutrinos. This rare process is called the two neutrino double beta decay ($2\nu 2\beta$) and it was first postulated by Goeppert-Mayer in 1935

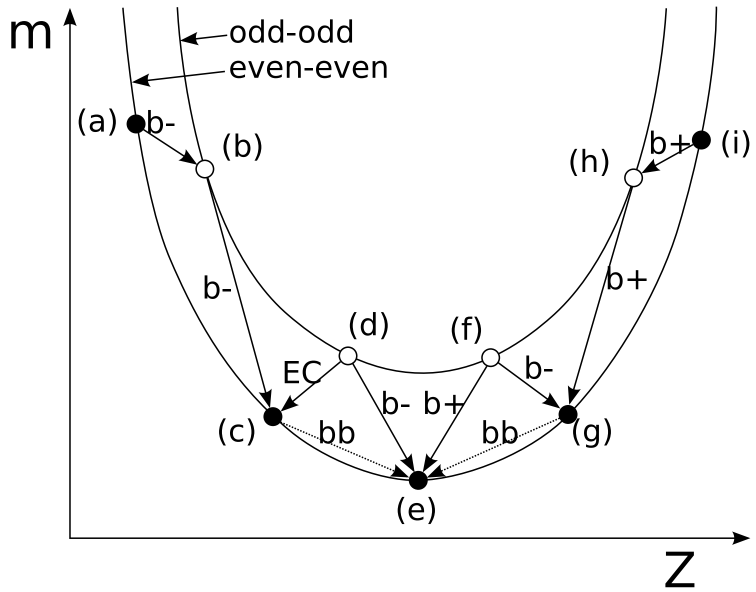

Figure 1.1: Predictions of the SEMF for an even value of A. The arrows between the two parabolae show the energetically allowed β decays [2].

[3].

$$(A, Z) \rightarrow (A, Z+2) + 2e^{-} + 2\overline{\nu_e} \tag{1.7}$$

It is clear that this process only occurs in even-even nuclei where β decay is energetically impossible, or strongly suppressed by conservation principles. The Feynman diagram of the $2\nu2\beta$ process is shown in Figure 1.2.

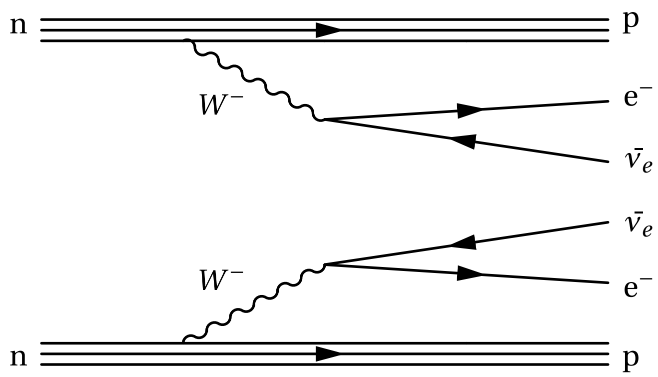

Figure 1.2: Feynman diagram for $2\nu2\beta$, a second order SM process.

Similar to the spectrum of the β decay, the spectrum of the total energy of the emitted electrons in the $2\nu2\beta$ decay is also continuous and the end-point is at the nuclear transition energy $Q_{\beta\beta}$ (see Figure 1.3).

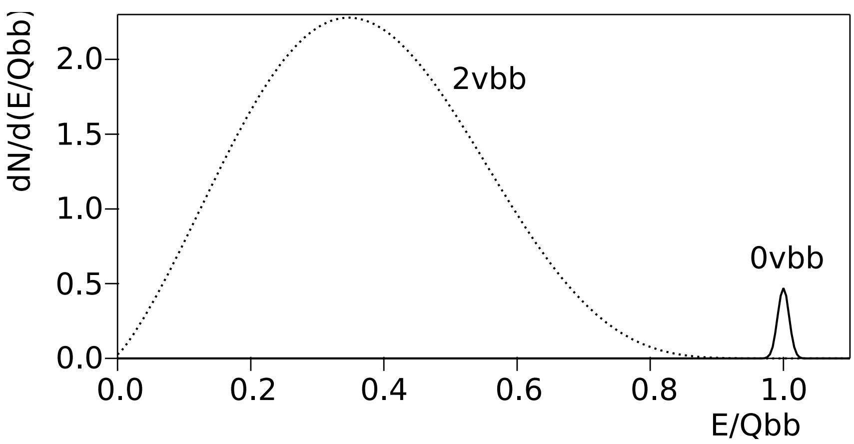

Figure 1.3: Distribution of the sum of electron energies for $2\nu 2\beta$ and $0\nu 2\beta$. The curves assume that $T^{0\nu}_{1/2}$ is 1% of $T^{2\nu}_{1/2}$, with an energy resolution of 2% [4].

$Q_{\beta\beta}$ is the total energy released in the process and it can be calculated using:

$$Q_{\beta\beta} = M(A,Z) - M(A,Z+2) \tag{1.8}$$

The half-life of the decay is parameterised as

$$\left(T^{2\nu}_{1/2}(A,Z)\right)^{-1} = G^{2\nu}(Q_{\beta\beta},Z)\left|M^{2\nu}(A,Z)\right|^2 \tag{1.9}$$

where $G^{2\nu}$ is a four-body phase space factor that can be calculated exactly, and $M^{2\nu}$ is the $2\nu 2\beta$ nuclear matrix element (NME) for the decay, which is effectively a nuclear structure calculation of the transition probability from the initial to final states. It should be noted that the NME calculation is heavily model-dependent, as mentioned in 1.4, and as such experimental information is vital to tune models appropriately. Although the study of this process does not allow one to discriminate between the Dirac or Majorana nature of the neutrino, it remains nonetheless crucial because it constitutes the ultimate background to searching for the neutrinoless double beta decay.

1.3 Neutrinoless Double Beta Decay

Neutrinoless double beta decay($0\nu 2\beta$) is a hypothesised decay which was first proposed by W.H. Furry in 1939 [5]. In the $0\nu 2\beta$, two β decays occur simultaneously,resulting in the emission of two electrons without anti-neutrinos. It is clear that this process violates lepton number and is thus forbidden in the SM.

$$(A,Z) \rightarrow (A,Z+2) + 2e^- \tag{1.10}$$

All isotopes that can result in $2\nu 2\beta$ are also candidates for $0\nu 2\beta$. The half-life of this process is:

$$(T^{0\nu}_{1/2}(A,Z))^{-1} = m^2_{\beta\beta} \cdot |M^{0\nu}(A,Z)|^2 \cdot G^{0\nu}(Q_{\beta\beta},Z) \tag{1.11}$$

where $G^{0\nu}$ is now a two-body phase space factor which can be calculated exactly, $M^{0\nu}$ is the $0\nu 2\beta$ NME, and $m_{\beta\beta}$ is the effective Majorana mass of the neutrino defined as:

$$m_{\beta\beta} = \left| \sum_i U^2_{ei} m_i \right| \tag{1.12}$$

where U_{ei} is the elements of the neutrino mixing matrix and m_i are the neutrino mass eigenstates. The effective Majorana mass depends on the mechanism of $0\nu 2\beta$, which is still not yet clear. It is still not yet clear whether light Majorana mass term is the leading mechanism of $0\nu 2\beta$. There are many alternatives mechanisms of the $0\nu 2\beta$, such as the neutrino mass mechanism, right-handed currents, Majorona emission, R-parity violating SUSY, etc. The most common mechanism among them is the neutrino mass mechanism, or light neutrino exchange mechanism, which requires the least modification to the SM.

In the neutrino mass mechanism, a light RH (R) Majorana neutrino undergoes a helicity flip, being absorbed as a light LH Majorana neutrino (see Figure 1.4).

There is an inherent dependence of the decay rate on the effective mass due to the requirement of a helicity flip.

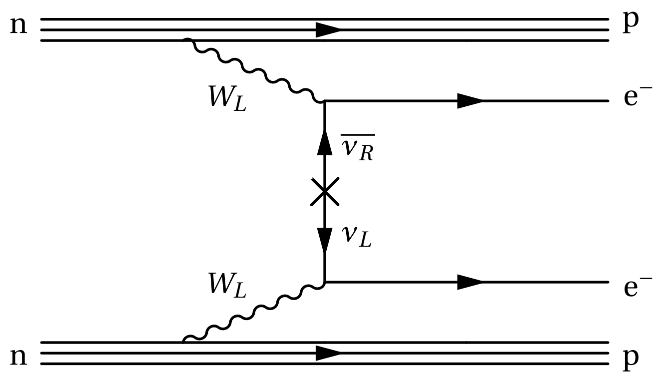

Figure 1.4: Feynman diagram of $0\nu2\beta$ for the neutrino mass mechanism. The decay is facilitated by the exchange of a Majorana neutrino.

In the SM, the weak interaction is only propagated by a LH W boson, W_L. A Left-Right Symmetric models have been proposed to resolve this apparent asymmetry, where a new RH gauge boson is introduced. The new boson may be completely new, such as a W' boson, or be an addition to the SM W boson such that W is an admixture of W_L and W_R. These new models can lead to $0\nu2\beta$ without a helicity flip, as can be seen from Figure 1.5 [6].

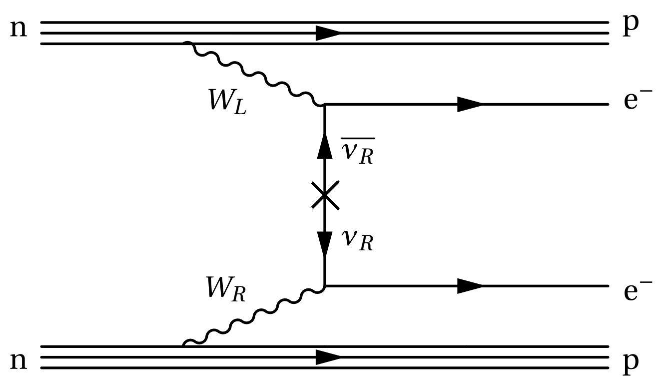

Figure 1.5: Feynman diagram from $0\nu2\beta$ using a right handed weak current, described by the $\langle\lambda\rangle$ decay mode.

RH currents will produce a different distribution of individual electron energies and the opening angle between them which can only be distinguished in an experiment which exploits a SuperNEMO-like technology [7], see Figure 1.6.

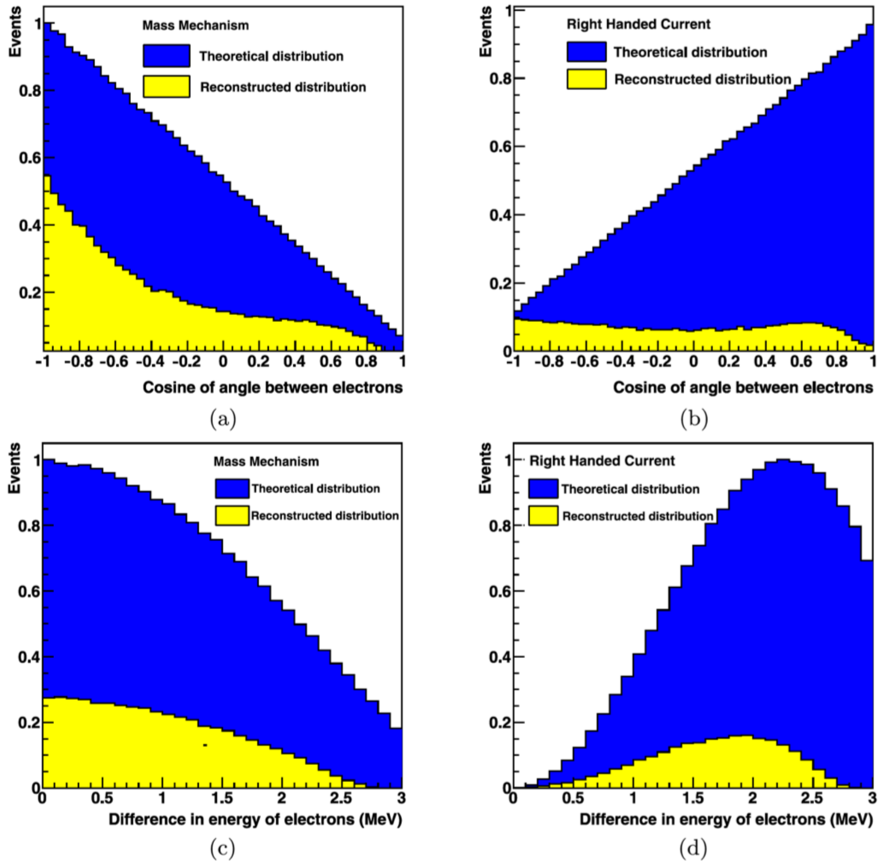

Figure 1.6: Theoretical and experimental electron angular distributions for (a) mass mechanism and (b) right-handed current. Theoretical and experimental electron energydifference distributions for (c) mass mechanism and (d) right-handed currents. All distributions are shown for the isotope ^{82}Se and the reconstructed distributions are normalised to the theoretical distribution to show signal efficiency [7].

1.4 Nuclear Matrix Elements

Searching for the $0\nu 2\beta$ process can produce a result of the half-life of the decay, $T^{0\nu}_{1/2}$, which can be converted to the physics parameter of interest, usually the effective neutrino mass $\langle m_{\beta\beta}\rangle$ that can not be directly measured. The appropriate NME is required for this conversion (Equation 1.11). To calculate the NME, the many-body Schrodinger equation need to be solved, accounting for each nucleon-nucleon interaction, given the total initial and final state nuclear wave functions. Due to the complexity of the calculations, various approximations and simplifications have been applied [8].

To date, five main approaches based on different approximations have been applied with regards to this issue, all of which involve two stages. The first stage is to create a many-body Hamiltonian that describes the nucleon-nucleon interactions at short distances. The second stage is to introduce a mean field which incorporates information about the nuclear structure and residual interactions.

1.4.1 Interacting Shell Model

The interacting shell model (ISM) [9], considers only a limited number of nuclear orbitals close to the Fermi level, but all possible correlations for these orbitals are included. Compared to other correction methods, this approach tends to reduce the value of the NMEs due to the limited number of orbits considered. The ISM is useful for calculating single particle states that are close to Fermi level, and is usually reliable for small nuclei such as ^{48}Ca, ^{76}Ge, and ^{82}Se. In particular, a doubly magic nucleus, such as ^{48}Ca, is considered to be a test bench for ISM calculations. However, it has difficulties in calculating deformed nuclei, such as ^{150}Nd and heavy isotopes.

1.4.2 Quasiparticle Random Phase Approximation

The quasiparticle random phase approximation (QRPA) [10] considers more nuclear orbitals but simpler interactions between nucleons compared to the ISM. In the QRPA, the nucleus is described using nucleon-nucleon pairs, and these quasiparticles are then treated as bosons. The coupling constant g_{pp}, which quantify the proton-proton interaction, is a free parameter of the model but can be constrained by $2\nu2\beta$ results. Experimental inputs help reduce uncertainties on the model, but might not describe the $0\nu2\beta$ accurately. The QRPA is more reliable for calculation of the large nuclei in comparison to ISM.

1.4.3 Interacting Boson Model

The interacting boson model (IBM) [11] is similar to the ISM but considers only bosons made of pairs of nucleons with the angular momentum states being restricted to $L = 0$ or $L = 2$. IBM shares similar advantages and disadvantages as the ISM.

1.4.4 Projected Hartree-Fock-Bogoluibov Method

In the projected Hartree-Fock-Bogoluibov (PHFB) [12] model, nuclear wave functions with good particle number and angular momentum are constructed by projection on the HFB wavefunctions. The nuclear Hamiltonian includes only quadrupole interactions. It describes neutron pairs with even angular momenta and positive parity only (non-0^+ pairs are heavily suppressed in comparison to others).

1.4.5 Energy Density Functional Method

The energy density functional (EDF) method [13] is an improvement based on the PHFB method. It includes modified inter-nucleon interaction to reproduce the Gogny interaction.

1.4.6 Comparison of different NME calculations

All five of these models have been used to calculate several isotopes for similar processes, including $2\nu2\beta$ decay and β decay. However, the computation result for the $0\nu2\beta$ decay differs to each other, typically by a factor 2-3 ~~times,~~ which is problematic as the variation goes directly into an uncertainty in the obtained $0\nu2\beta$ decay limits from the experiments.

Comparing the NME results evaluated by each method can help to understand the effect of each of these assumptions and the associated systematic error with the resulting NMEs. In the conversion from an experimental half-life to $\langle m_{\beta\beta}\rangle$, the phase space factor, $G^{0\nu}$, enters into the calculation alongside the NME. This factor exhibits a proportionality given by

$$G^{0\nu} \sim \frac{g_A^4}{R_A^2} \tag{1.13}$$

where g_A is the ratio of the vector and axial-vector couplings and R_A is the atomic radius, commonly parameterised as $r_0 A^{1/3}$ [14]. In the past, different NME calculations are performed using different values of g_A and r_0 so that the phase space factors, and therefore NME results, are not directly comparable. Common values for g_A are either 1.0 or 1.25 and for r_0 either 1.1 to 1.2 fm.

Figure 1.7 shows the difference between the NME of the 11 $0\nu2\beta$ candidate isotopes in the mass mechanism, calculated using the five methods described previously. Also, for a meaningful comparison, g_A and r_0 are adjusted to be same values, 1.25 and 1.2 respectively.

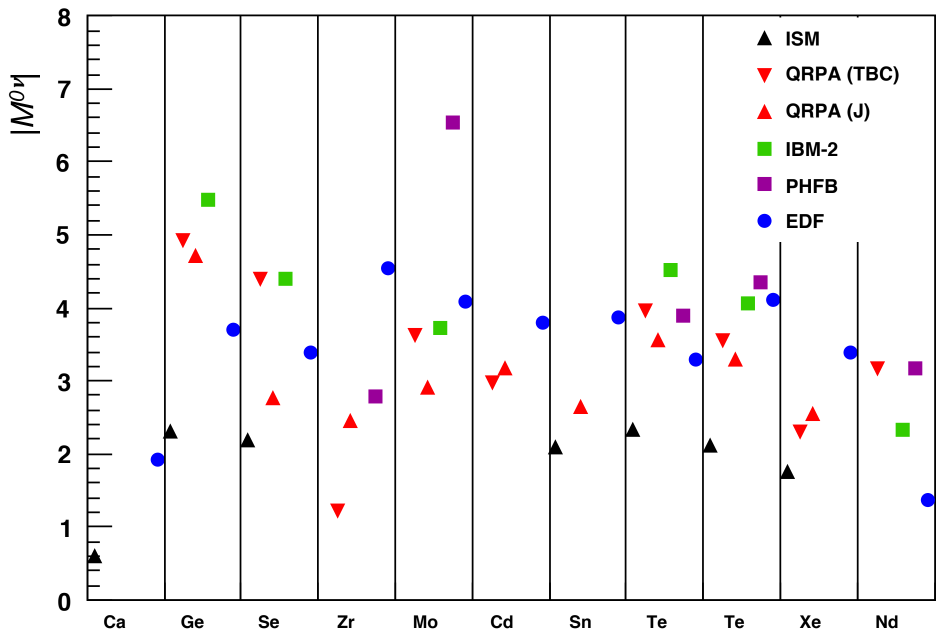

Figure 1.7: $0\nu2\beta$ NME for the neutrino mass mechanism, calculated with five different approaches. QRPA(T) and QRPA(J) show the results of the Tübingen-Bratislava-Caltech and Jyväskylä groups. $|M^{0\nu}|$ values taken from [8]. Conversions for $g_A = 1.25$ and $r_0 = 1.2$ fm have been made where necessary.

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