Overview: Diffraction in *ep* and *pp* collisions

Graeme Watt

University College London

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In collaboration with H. Kowalski, A.D. Martin, L. Motyka and M.G. Ryskin

Outline

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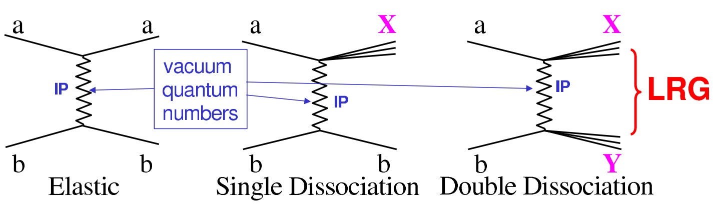
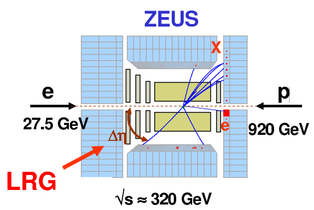
What is diffraction?

Two equivalent definitions of what is meant by 'diffraction' in high-energy physics:

1. "A reaction in which no quantum numbers are exchanged between the colliding particles is, at high energies, a diffractive reaction." [Good–Walker,'60]
2. "A diffractive reaction is characterized by a Large, non-exponentially suppressed, Rapidity Gap (LRG) in the final state." [Bjorken,'92]

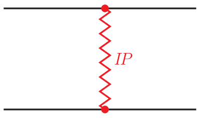


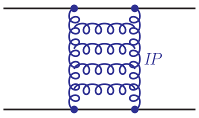


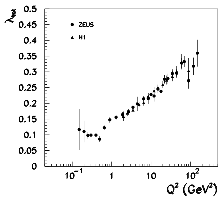
Diffraction is 'Pomeron' ($\mathbb{P}$) exchange!

What is the 'Pomeron'?

- In Regge theory, asymptotic behavior ($s \gg |t|$) of cross sections given by
$$\sigma_{\rm tot} \sim s^{\alpha(0)-1}, \quad {\rm d}\sigma_{\rm el}/{\rm d}t \sim s^{2[\alpha(t)-1]}, \quad \alpha(t) = \alpha(0) + \alpha'\, t.$$
- The 'Pomeron' was a Regge trajectory with intercept $\alpha_{\mathbb{P}}(0) = 1$ introduced to explain the asymptotically constant total cross sections expected in the '60s.
- Fit to soft hadron–hadron data gives 'soft' (or non-perturbative) Pomeron [Donnachie–Landshoff, hep-ph/9209205]:
$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t = 1.08 + (0.25\ {\rm GeV}^{-2})\, t.$$
- In pQCD, the 'hard' (or perturbative) Pomeron is a parton ladder, usually satisfying either BFKL evolution (strongly-ordered longitudinal momenta) or DGLAP evolution (strongly-ordered transverse momenta).
- Regge phenomenology useful in quantifying whether a process is 'soft' or 'hard'. 'Hard' processes have a higher effective $\alpha_{\mathbb{P}}(0)$, e.g., fit small-$x_{\rm Bj}$ HERA data to
$$\sigma_{\rm tot}^{\gamma^* p}(x_{\rm Bj}, Q^2) = c(Q^2)\, x_{\rm Bj}^{-\lambda_{\rm tot}(Q^2)}, \quad \text{where} \quad \lambda_{\rm tot} = \alpha_{\mathbb{P}}(0) - 1.$$
- Open question: how to unify 'soft' and 'hard' Pomerons? Insight from gauge/string duality [e.g. Brower–Polchinski–Strassler–Tan, hep-th/0603115]?







Some introductory references

- Proceedings of "HERA and the LHC: A Workshop on the implications of HERA for LHC physics" [hep-ph/0601013].
  - See in particular:
    M. Arneodo and M. Diehl, "Diffraction for non-believers" [hep-ph/0511047].
  - Ongoing meetings: see http://www.desy.de/~heralhc/.
- L. Frankfurt, M. Strikman and C. Weiss, "Small-x physics: From HERA to LHC and beyond", Ann. Rev. Nucl. Part. Sci. **55**, 403 (2005) [hep-ph/0507286].
- B. Z. Kopeliovich, I. K. Potashnikova and I. Schmidt, "Diffraction in QCD", hep-ph/0604097.
- Textbooks:
  - V. Barone and E. Predazzi, "High-energy particle diffraction", Heidelberg, Germany: Springer-Verlag (2002).
  - J. R. Forshaw and D. A. Ross, "Quantum chromodynamics and the Pomeron", Cambridge, UK: Univ. Pr. (1997).

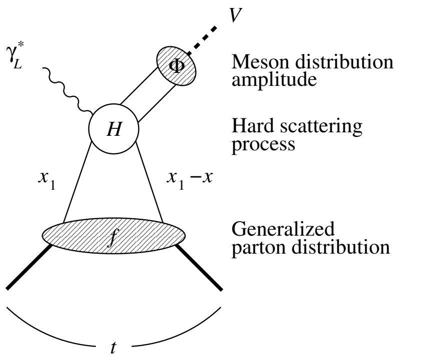
Outline

1. Introduction
2. Exclusive diffraction in *ep* collisions
   - Exclusive processes within collinear factorization
   - Exclusive processes within the dipole picture
     - Dipole picture in the non-forward direction
     - Impact parameter dependent dipole cross sections
     - Description of HERA vector meson data
     - Description of HERA DVCS data
3. Inclusive diffraction in *ep* collisions
   - Diffractive DIS kinematics and structure functions
   - Collinear factorization in DDIS
   - Perturbative Pomeron contribution to DDIS
   - Analysis of HERA DDIS data
   - Non-linear evolution of inclusive PDFs
4. Diffraction in pp and $p\bar{p}$ collisions
   - Factorization breaking in diffractive hadron–hadron collisions
   - Exclusive diffractive Higgs production at LHC
5. Summary and outlook

Collinear factorization for exclusive hard processes

Collins, Frankfurt, Strikman [hep-ph/9611433]:

$$\mathcal{A}(\gamma_L^* p \to Vp) \;=\; \sum_{i,j} \int_0^1 \mathrm{d}z \int \mathrm{d}x_1\, \overbrace{f_{i/p}(x_1, x - x_1, t; \mu)}^{\text{GPD}}\; \overbrace{H_{ij}(x_1, x, z, Q^2; \mu)}^{\text{Hard scattering}}\; \overbrace{\phi_j^V(z, \mu)}^{\text{Meson dist. amp.}}$$
$$+ \quad \text{power-suppressed corrections.}$$



Kinematic variables:

- Label 4-momenta of photon, incoming proton, and outgoing proton by q, p, and p'.
- Photon virtuality, $q^2 = -Q^2$.
- $\gamma^* p$ center-of-mass energy squared, $W^2 = (p+q)^2$.
- $x_{\rm Bj} = Q^2/(2p\cdot q) \simeq Q^2/(Q^2+W^2)$.
- $t = (p - p')^2$.

Alternative approach valid at high-energy (small-$x_{\rm Bj}$) is the dipole picture:

$\mathcal{A}(\gamma^* p \to Vp) \sim$ (photon wave function)·(dipole cross section)·(meson wave function).

Exclusive processes at HERA within the dipole picture

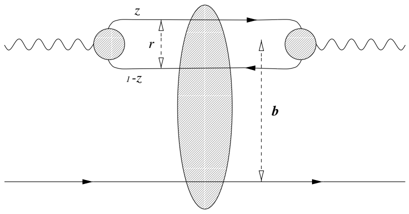
Kowalski, Motyka, G.W. [hep-ph/0606272]

Munier, Staśto, Mueller [hep-ph/0102291]

Kowalski, Teaney [hep-ph/0304189]

$$\boxed{\mathcal{A}(\gamma^* p \to E p) = \int \mathrm{d}^2\mathbf{r} \int_0^1 \frac{\mathrm{d}z}{4\pi}\, \Psi^*(\gamma^* \to q\bar{q})\, \mathcal{A}(q\bar{q}+p \to q\bar{q}+p)\, \Psi(q\bar{q} \to E),}$$

where $E = \gamma^*$ (inclusive DIS), $E = \gamma$ (DVCS) or $E = V$ (vector meson production).



- z = photon's light-cone momentum fraction.
- $\mathbf{r}$ = transverse dipole size.
- $t = (p - p')^2 = -\Delta^2$.
- $\mathbf{b}$ = impact parameter: Fourier conjugate variable to $\mathbf{\Delta}$.

- Elastic amplitude for $q\bar{q}$ dipole scattering on the proton:

$$\mathcal{A}(q\bar{q}+p \to q\bar{q}+p) = \mathrm{i}\int \mathrm{d}^2\mathbf{b}\, \mathrm{e}^{-\mathrm{i}\mathbf{b}\cdot\mathbf{\Delta}}\, 2\,[1 - S(x,r,b)]\,.$$

- Dipole cross section from optical theorem:

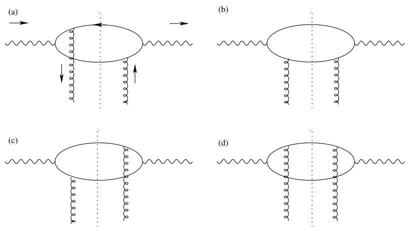
$$\sigma_{q\bar{q}}(x,r) = \mathrm{Im}\ \mathcal{A}(q\bar{q}+p \to q\bar{q}+p)|_{\Delta=0}$$

$$\Rightarrow \quad \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2[1 - \mathrm{Re}\, S(x,r,b)].$$

(Will take S-matrix to be predominantly real.)

Dipole picture in the non-forward direction

Bartels, Golec-Biernat, Peters (BGBP) [hep-ph/0301192]

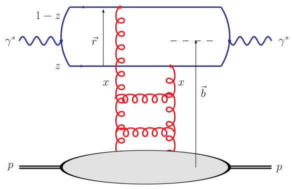


- Non-forward photon impact factor calculated in the high-energy limit.
- Transform from momentum space to coordinate space ($\mathbf{k} \to \mathbf{r}$), then to impact parameter space ($\mathbf{\Delta} \to \mathbf{b}$).
- Results obtained in color dipole picture: amplitude factorizes into (wave function)·(dipole cross section)·(wave function).
- Non-forward wave functions can be written as forward wave functions multiplied by $\exp[\pm\mathrm{i}(1-z)\mathbf{r}\cdot\mathbf{\Delta}/2]$.
- Effectively, the momentum transfer $\mathbf{\Delta}$ should conjugate to $\mathbf{b} + (1-z)\mathbf{r}$, the transverse distance from the centre of the proton to one of the two quarks of the dipole, rather than to $\mathbf{b}$.

$$\boxed{\mathcal{A}_{T,L}^{\gamma^* p \to E p}(x,Q,\Delta) = \mathrm{i}\int \mathrm{d}^2\mathbf{r} \int_0^1 \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2\mathbf{b}\, (\Psi_E^*\Psi)_{T,L}\, \mathrm{e}^{-\mathrm{i}[\mathbf{b}-(1-z)\mathbf{r}]\cdot\mathbf{\Delta}}\, \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}}}$$

Inclusive DIS in the dipole picture

Total cross section for inclusive DIS is



$$\sigma_{T,L}^{\gamma^* p}(x,Q) = \mathrm{Im}\,\mathcal{A}_{T,L}^{\gamma^* p\to\gamma^* p}(x,Q,\Delta=0)$$
$$= \sum_f \int \mathrm{d}^2\mathbf{r} \int_0^1 \frac{\mathrm{d}z}{4\pi} (\Psi^*\Psi)_{T,L}^f \, \sigma_{q\bar{q}}(x,r),$$

i.e. depends on dipole cross section integrated over impact parameter **b**.
The squared (forward) photon wave functions are calculable in pQCD:

$$(\Psi^*\Psi)_T^f \equiv \frac{1}{2} \sum_{\substack{h,\bar{h}=\pm\frac{1}{2}\\ \lambda=\pm 1}} \Psi^*_{h\bar{h},\lambda}\Psi_{h\bar{h},\lambda} = \frac{2N_c}{\pi}\alpha_{\mathrm{em}} e_f^2 \left\{\left[z^2+(1-z)^2\right]\epsilon^2 K_1^2(\epsilon r) + m_f^2 K_0^2(\epsilon r)\right\},$$

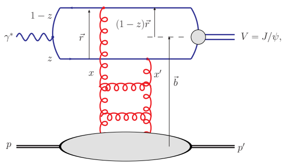
$$(\Psi^*\Psi)_L^f \equiv \sum_{h,\bar{h}=\pm\frac{1}{2}} \Psi^*_{h\bar{h},\lambda=0}\Psi_{h\bar{h},\lambda=0} = \frac{8N_c}{\pi}\alpha_{\mathrm{em}} e_f^2 Q^2 z^2 (1-z)^2 K_0^2(\epsilon r),$$

where $\epsilon^2 \equiv z(1-z)Q^2 + m_f^2$ and $K_{0,1}$ are modified Bessel functions.

DVCS and exclusive diffractive vector meson production

$$\boxed{\mathcal{A}_{T,L}^{\gamma^* p\to Ep}(x,Q,\Delta) = \mathrm{i}\int \mathrm{d}^2\mathbf{r}\int_0^1 \frac{\mathrm{d}z}{4\pi}\int \mathrm{d}^2\mathbf{b}\,(\Psi_E^*\Psi)_{T,L}\,\mathrm{e}^{-\mathrm{i}[\mathbf{b}-(1-z)\mathbf{r}]\cdot\mathbf{\Delta}}\,\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}},}$$

where $E=\gamma$ (DVCS) or $E=V$ (vector meson production). The differential cross sections are then obtained from



$$\frac{\mathrm{d}\sigma_{T,L}^{\gamma^* p\to Ep}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\mathcal{A}_{T,L}^{\gamma^* p\to Ep}\right|^2 [1+\tan^2(\pi\lambda/2)], \quad \text{with} \quad \lambda \equiv \frac{\partial \ln\left(\mathcal{A}_{T,L}^{\gamma^* p\to Ep}\right)}{\partial \ln(1/x)}.$$

For DVCS,

$$(\Psi_\gamma^*\Psi)_T^f = \frac{2N_c}{\pi}\alpha_{\mathrm{em}} e_f^2 \left\{\left[z^2+(1-z)^2\right]\epsilon K_1(\epsilon r) m_f K_1(m_f r) + m_f^2 K_0(\epsilon r) K_0(m_f r)\right\}.$$

For vector meson production, however, some modeling of the wave functions, Ψ_V, is required. Constraints from normalization and experimental decay width. Will consider two alternatives denoted "Gaus-LC" and "boosted Gaussian" (see hep-ph/0606272 for details).

Review of (selected) dipole cross sections

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2[1 - \mathrm{Re}\, S(x,r,b)] \equiv 2\mathcal{N}(x,r,b),$$

where $\mathcal{N} \in [0,1]$ and $\mathcal{N} = 1$ is the unitarity limit. First dipole models were integrated over $\mathbf{b}$ assuming proton is a disc in transverse plane:

$$\mathcal{N}(x,r,b) = \mathcal{N}(x,r)\,\Theta(b_S - b) \quad \Rightarrow \quad \sigma_{q\bar{q}}(x,r) = \sigma_0 \mathcal{N}(x,r) \quad \text{with} \quad \sigma_0 = 2\pi b_S^2.$$

Golec-Biernat–Wüsthoff (GBW) [hep-ph/9807513]:

$$\mathcal{N}(x,r) = 1 - \mathrm{e}^{-r^2 Q_s^2(x)/4}, \quad \text{where} \quad Q_s^2(x) = (x_0/x)^{\lambda}\ \mathrm{GeV}^2.$$

Decrease of λ_{tot} with decreasing Q^2 entirely due to saturation effects.

Bartels–Golec-Biernat–Kowalski (BGBK) [hep-ph/0203258]:

$$\mathcal{N}(x,r) = 1 - \exp\left[-\pi^2 r^2 \alpha_s(\mu^2) x g(x,\mu^2)/(3\sigma_0)\right]$$

DGLAP evolution of gluon distribution. Charm neglected.

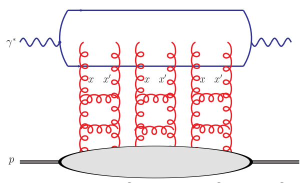
Iancu–Itakura–Munier (CGC) [hep-ph/0310338]:

$$\mathcal{N}(x,r) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda Y}\ln\frac{2}{rQ_s}\right)} & : \quad rQ_s \le 2 \\ 1 - \mathrm{e}^{-A\ln^2(BrQ_s)} & : \quad rQ_s > 2 \end{cases},$$

where $Y = \ln(1/x)$, $\gamma_s = 0.63$, $\kappa = 9.9$. Approximate solution of Balitsky–Kovchegov equation (sums ‘fan’ diagrams). Charm neglected.

Impact parameter dependent dipole cross sections

Kowalski–Teaney [hep-ph/0304189] used the Glauber–Mueller dipole cross section:



$$\boxed{\mathcal{N}(x,r,b) = 1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)\right),}$$

where $\mu^2 = 4/r^2 + \mu_0^2$. Gluon density, $xg(x,\mu^2)$, is evolved from a scale μ_0^2 up to μ^2 using LO DGLAP evolution without quarks:

$$\frac{\partial xg(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \mathrm{d}z\ P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z},\mu^2\right).$$

- Initial gluon density taken in the form $xg(x,\mu_0^2) = A_g\, x^{-\lambda_g}\,(1-x)^{5.6}$.
- Take $x = x_{\mathrm{Bj}}$ for light quarks, $x = x_{\mathrm{Bj}}(1 + 4m_c^2/Q^2)$ for charm quarks, and $x = x_{\mathrm{Bj}}(1 + M_V^2/Q^2)$ for vector meson production. Will use quark masses $m_{u,d,s} = 0.14$ GeV and $m_c = 1.4$ GeV.
- **Exclusive processes:** $xg(x,\mu^2) \to R_g xg(x,\mu^2)$ accounts for skewness of gluon distribution in the limit that $x' \ll x \ll 1$, where [Shuvaev *et al.*, hep-ph/9902410]

$$R_g(\lambda) = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda+4)}, \quad \text{with} \quad \lambda \equiv \frac{\partial \ln\left[xg(x,\mu^2)\right]}{\partial \ln(1/x)}.$$

Impact parameter dependent dipole cross sections

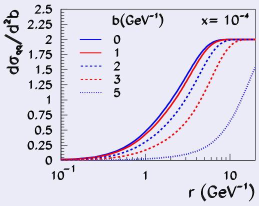
"b-Sat" model

$$\boxed{\mathcal{N}(x,r,b) = 1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) xg(x,\mu^2) T(b)\right)}$$

Assume $T(b)$ to have a Gaussian form:

$$T(b) = \frac{1}{2\pi B_G} \exp\left(-\frac{b^2}{2B_G}\right),$$

where $B_G = 4$ GeV^{-2} from description of vector meson t-distributions.



"b-CGC" model

$$\mathcal{N}(x,r,b) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda Y}\ln\frac{2}{rQ_s}\right)} & : \quad rQ_s \le 2 \\ 1 - \mathrm{e}^{-A\ln^2(BrQ_s)} & : \quad rQ_s > 2 \end{cases},$$

where

$$Q_s \equiv Q_s(x,b) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{b^2}{2B_{\mathrm{CGC}}}\right)\right]^{\frac{1}{2\gamma_s}},$$

and $B_{\mathrm{CGC}} = 5.5$ GeV^{-2} from description of vector meson t-distributions.

Fits to the total DIS cross section

Determine parameters in dipole cross section from fits to ZEUS $\sigma_{\mathrm{tot}}^{\gamma^* p}(x_{\mathrm{Bj}}, Q^2)$ data with $x_{\mathrm{Bj}} \le 0.01$:

Model	Q^2/GeV^2	μ_0^2/GeV^2	A_g	λ_g	$\chi^2/\mathrm{d.o.f.}$
b-Sat	[0.25,650]	1.17	2.55	0.020	$193.0/160 = 1.21$
Model	**Q^2/GeV^2**	**$\mathcal{N}_0$**	**$x_0/10^{-4}$**	**λ**	**$\chi^2/\mathrm{d.o.f.}$**
b-CGC	[0.25,45]	0.417	5.95	0.159	$211.2/130 = 1.62$

b-Sat model better than b-CGC model.

Model	Q^2/GeV^2	Charm?	σ_0/mb	$x_0/10^{-4}$	λ	$\chi^2/\mathrm{d.o.f.}$
GBW	[0.25,45]	No	20.1	5.16	0.289	$216.5/130 = 1.67$
GBW	[0.25,45]	Yes	23.9	1.11	0.287	$204.9/130 = 1.58$
GBW	[0.25,650]	Yes	22.5	1.69	0.317	$414.4/160 = 2.59$
CGC	[0.25,45]	No	25.8	0.263	0.252	$117.2/130 = 0.90$
CGC	[0.25,45]	Yes	35.7	0.00270	0.177	$116.8/130 = 0.90$
CGC	[0.25,650]	Yes	34.5	0.00485	0.188	$173.7/160 = 1.09$

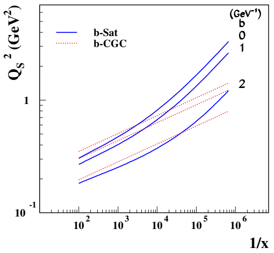
GBW model gives a relatively poor description; CGC better. Inclusion of data at larger Q^2 worsens fit: need DGLAP evolution. Presence of charm important.

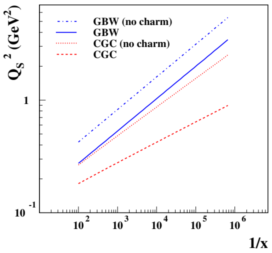
The saturation scale Q_S^2

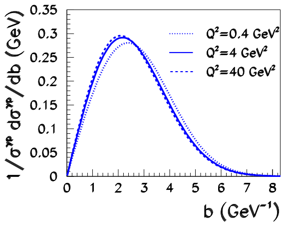
Define saturation scale $Q_S^2 \equiv 2/r_S^2$, where r_S^2 is the dipole size where

$$\mathcal{N} = 1 - \mathrm{e}^{-1/2} \simeq 0.4.$$

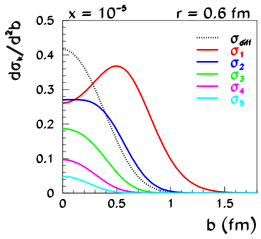
(This definition of the saturation scale coincides with $Q_s^2 = (x_0/x)^\lambda$ GeV^2 in the GBW model, but is **not** equal to Q_s^2 in the CGC and b-CGC models.)





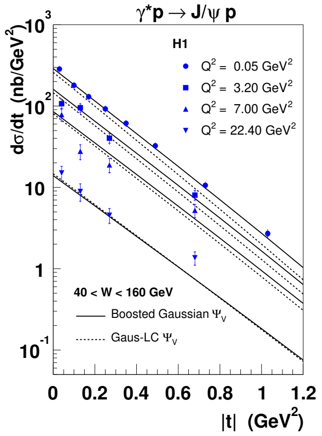


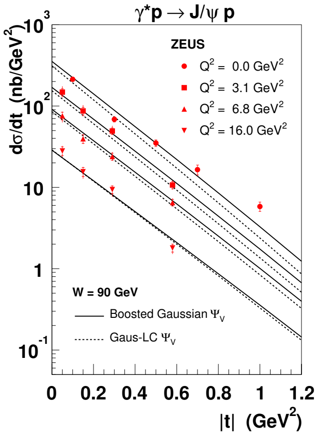
- DGLAP evolution, presence of charm, impact parameter dependence: all lower saturation scale.
- Median $b \simeq 2.6$ GeV^{-1} $\Rightarrow$ saturation effects not important for inclusive DIS in HERA kinematic regime.
- Detailed study of multiple interactions in b-Sat model by H. Kowalski [HERA–LHC proceedings, hep-ph/0601013]. Multiple scattering enhanced for large dipole sizes with $b \approx 0$.

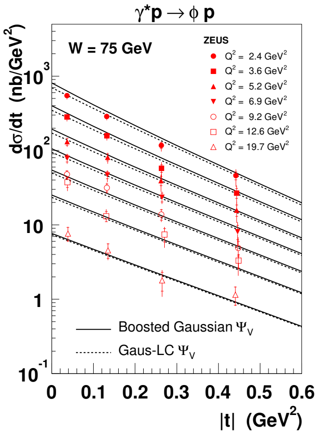


Description of HERA vector meson data in the b-Sat model

- Input gluon density fitted to $\sigma_{\mathrm{tot}}^{\gamma^* p}$ data.
- Parameter $B_G = 4$ GeV^{-2} giving width of Gaussian $T(b)$ chosen to give best overall description of vector meson t-distributions.





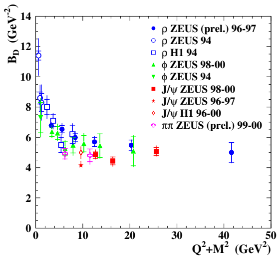


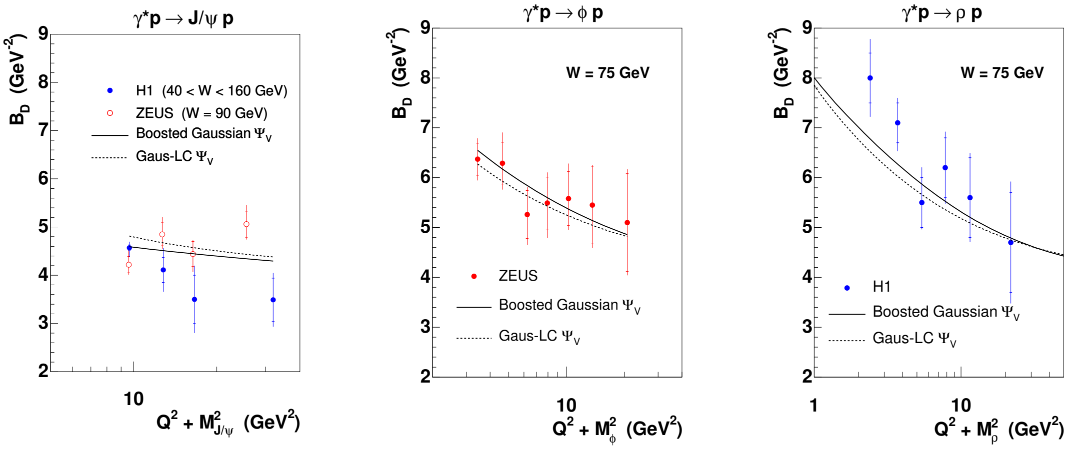
t-slope parameter B_D

t-dependence of exclusive processes at HERA are well described by an exponential form:

$$\frac{\mathrm{d}\sigma^{\gamma^* p \to Ep}}{\mathrm{d}t} \propto \mathrm{e}^{-B_D|t|},$$

where B_D is process dependent, but approaches a universal value at large $(Q^2 + M_V^2)$.





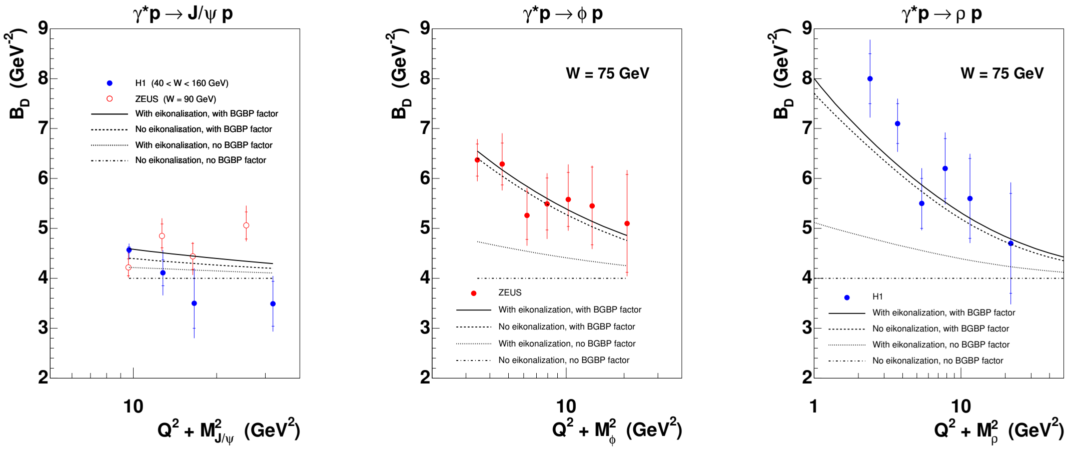
Transverse proton shape $T(b)$

- In limit of small dipole sizes:

$$T(b) = \frac{1}{2\pi B_G}\exp\left(-\frac{b^2}{2B_G}\right) \quad \Rightarrow \quad \frac{\mathrm{d}\sigma^{\gamma^* p \to Ep}}{\mathrm{d}t} \propto \left| \int \mathrm{d}^2\mathbf{b}\, \mathrm{e}^{-\mathrm{i}\mathbf{b}\cdot\mathbf{\Delta}}\; T(b) \right|^2 \propto \mathrm{e}^{-B_G|t|},$$

so $B_D = B_G = 4\ \mathrm{GeV}^{-2}$ and $\sqrt{\langle b^2 \rangle} = \sqrt{2B_G} = 0.56$ fm.

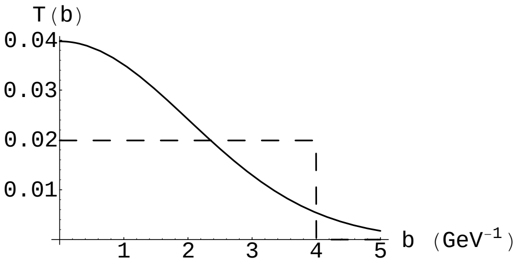
- cf. proton charge radius 0.870 ± 0.008 fm [PDG].
- However, important modifications to B_D for finite dipole sizes due to eikonalization and BGBP factor, $\exp[\mathrm{i}(1-z)\mathbf{r}\cdot\mathbf{\Delta}]$, in amplitude:

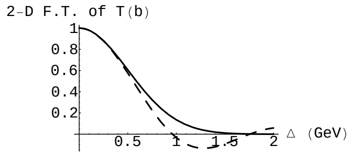


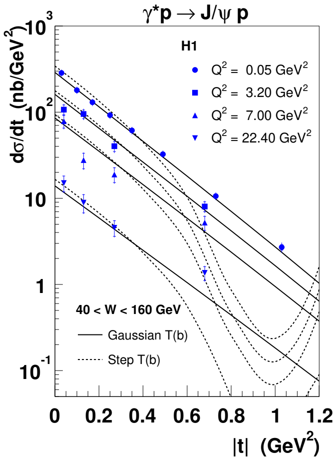
Alternative proton shape: step $T(b)$

$$\boxed{T(b) = \frac{1}{\pi b_S^2}\Theta(b_S - b)\,,}$$

with $b_S = 4\ \mathrm{GeV}^{-1} \Rightarrow \langle b^2 \rangle = b_S^2/2 = 8\ \mathrm{GeV}^{-2}$.

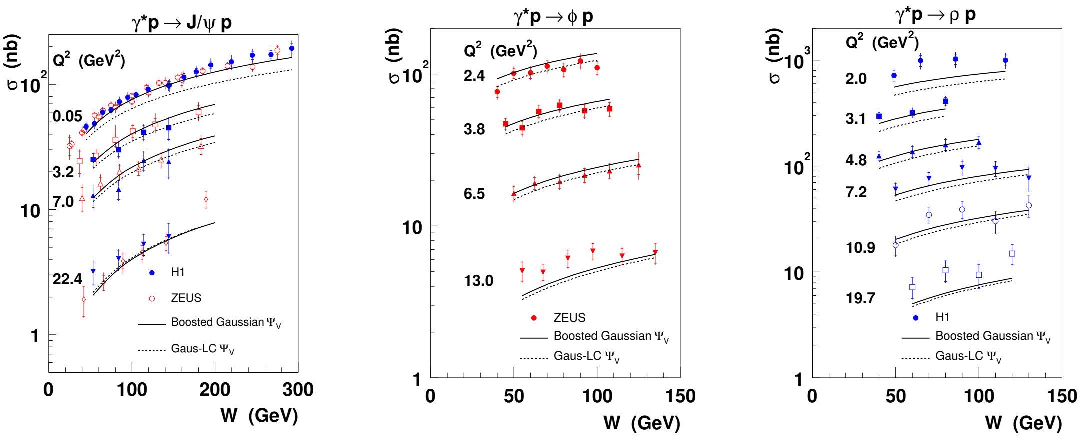






- Assume a step $T(b)$ instead of a Gaussian. Recall that this is implicitly assumed in b-independent dipole models (e.g. GBW, CGC).
- t-distribution from step $T(b)$ not supported by J/ψ data.

W dependence of exclusive diffractive vector meson data

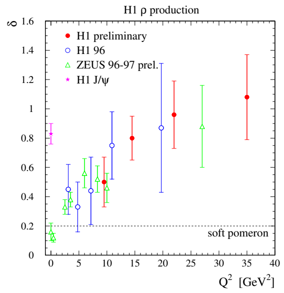


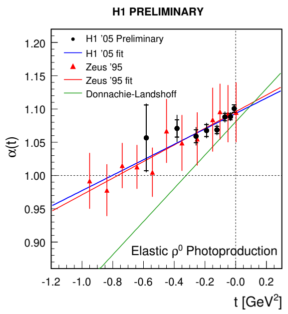
- Good agreement of b-Sat model with HERA data in both shape and normalization.

‘Soft’ vs. ‘hard’ Pomeron from W dependence

- Fit $\sigma \propto W^{\delta}$. In language of Regge theory, $\delta = 4[\alpha_{\mathbb{P}}(\langle t \rangle) - 1]$, where $\alpha_{\mathbb{P}}(\langle t \rangle) = \alpha_{\mathbb{P}}(0) - \alpha'_{\mathbb{P}}/B_D$, and $\delta \simeq 0.2$ corresponds to the ‘soft’ Pomeron.
- Fit $\mathrm{d}\sigma/\mathrm{d}t \propto W^{4[\alpha_{\mathbb{P}}(t)-1]}$, where $\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t$.

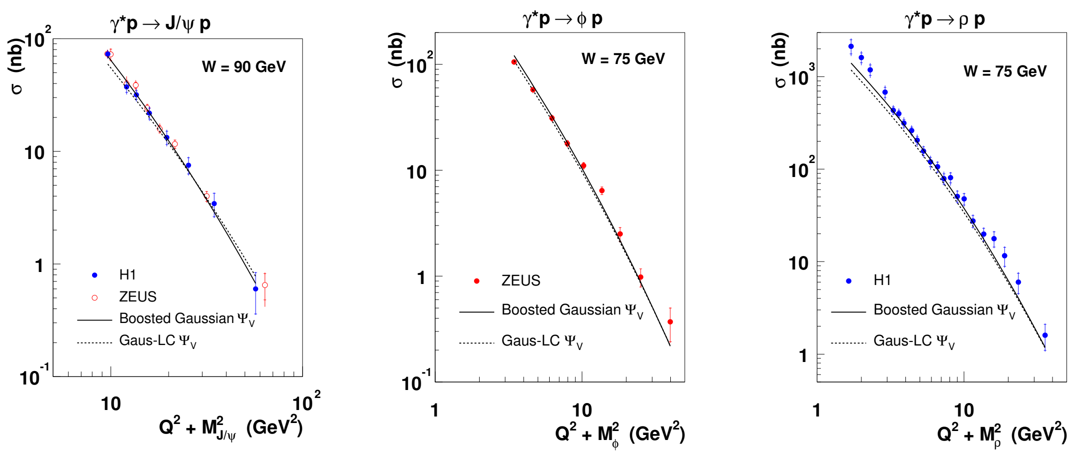
	$\alpha_{\mathbb{P}}(0)$	$\alpha'_{\mathbb{P}}$ (GeV^{-2})
H1 $\gamma p \to J/\psi\ p$	1.22 ± 0.02	0.16 ± 0.04
ZEUS $\gamma p \to J/\psi\ p$	1.20 ± 0.01	0.12 ± 0.02
H1 $\gamma p \to \rho\ p$	1.09 ± 0.01	0.12 ± 0.05
ZEUS $\gamma p \to \rho\ p$	1.10 ± 0.02	0.13 ± 0.04
Donnachie–Landshoff	1.08	0.25





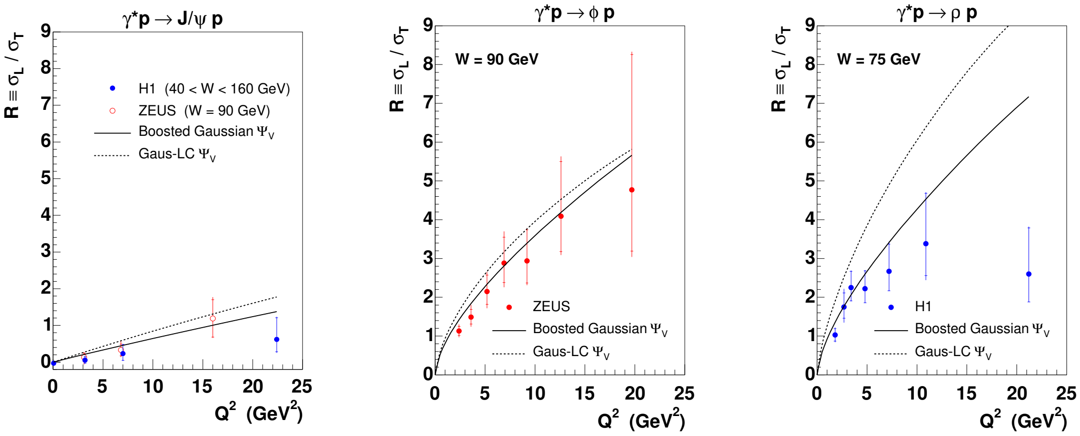
- ρ photoproduction is a ‘soft’ process: $\alpha_{\mathbb{P}}(0)$ compatible with ‘soft’ Pomeron value of 1.08.
- $\alpha'_{\mathbb{P}}$ in γp only half value in pp, due to stronger absorptive effects in pp.
- J/ψ photoproduction is a ‘hard’ process: larger $\alpha_{\mathbb{P}}(0)$ but similar $\alpha'_{\mathbb{P}}$ compared to ρ photoproduction.

Q^2 dependence of exclusive diffractive vector meson data



- Again, good agreement of b-Sat model with HERA data in both shape and normalization.

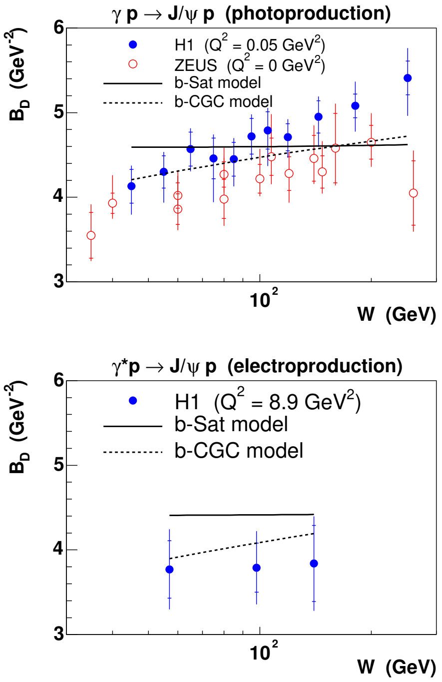
σ_L/σ_T of exclusive diffractive vector meson data



- σ_L/σ_T sensitive to details of the vector meson wave functions.

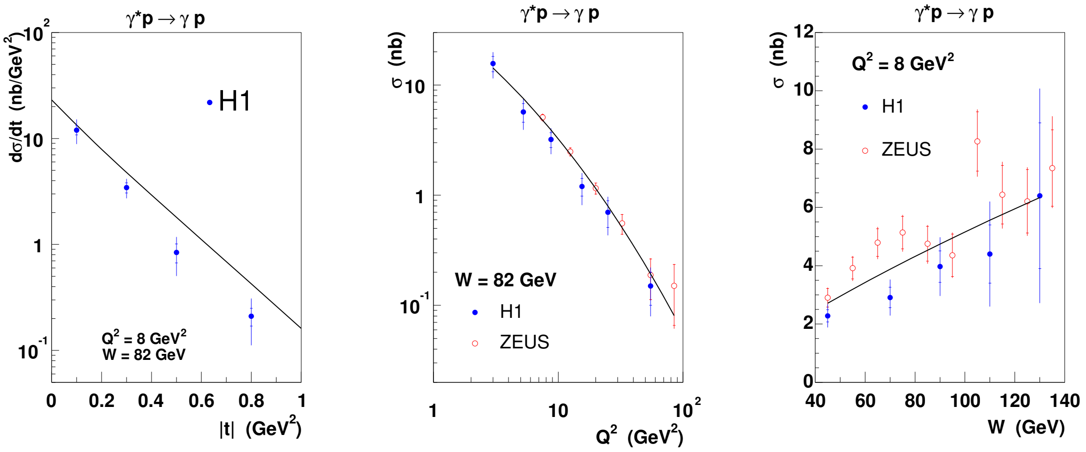
$\alpha'_{\mathbb{P}}$ from b-Sat and b-CGC models

$$B_D = B_0 + 4\alpha'_{\mathbb{P}} \ln(W/W_0)$$



- The b-Sat model gives a much better overall description of both the total DIS cross section and exclusive processes than the b-CGC model.
- However, the b-Sat model predicts $\alpha'_{\mathbb{P}} \approx 0$ due to the assumed factorisation of $T(b)$ from $xg(x,\mu^2)$, and the small saturation effects.
- In the b-CGC model, the W (or x) dependence is not factorized from the b dependence, therefore an appreciable $\alpha'_{\mathbb{P}}$ is achievable.

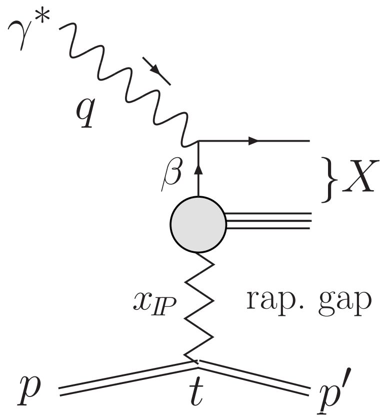
Deeply virtual Compton scattering at HERA



- DVCS ($\gamma^* p \longrightarrow \gamma p$) is a much cleaner process to describe theoretically than exclusive vector meson production.
- Predicted t-slope $B_D = 5.29$ GeV^{-2} from b-Sat model slightly underestimates experimental value of $B_D = 6.02 \pm 0.52$ GeV^{-2}.
- Good agreement of b-Sat model with data for Q^2 and W distributions in both shape and normalisation.

Outline

Diffractive DIS kinematics



- Inclusive diffractive DIS (DDIS): $\gamma^* p \to X + p$.
- $q^2 \equiv -Q^2$
- $W^2 \equiv (q+p)^2 = -Q^2 + 2\, p \cdot q$
  $\Rightarrow \quad x_{\rm Bj} \equiv \frac{Q^2}{2\, p \cdot q} = \frac{Q^2}{Q^2+W^2}$ (fraction of proton's momentum carried by struck quark)
- $t \equiv (p-p')^2 \approx 0$, $(p-p') \approx x_{\mathbb{P}}\, p$

- $M_X^2 \equiv (q+p-p')^2 = -Q^2 + x_{\mathbb{P}}(Q^2+W^2)$
  $\Rightarrow \quad x_{\mathbb{P}} = \frac{Q^2+M_X^2}{Q^2+W^2}$
  (fraction of proton's momentum carried by Pomeron)
- $\beta \equiv \frac{x_{\rm Bj}}{x_{\mathbb{P}}} = \frac{Q^2}{Q^2+M_X^2}$ (fraction of Pomeron's momentum carried by struck quark)

Diffractive reduced cross section $\sigma_r^{\rm D(3)}$

- Diffractive cross section (integrated over t):

$$\frac{{\rm d}^3\sigma^{\rm D}}{{\rm d}x_{\mathbb{P}}\,{\rm d}\beta\,{\rm d}Q^2} = \frac{2\pi\alpha_{\rm em}^2}{\beta\, Q^4}\left[1+(1-y)^2\right]\,\sigma_r^{\rm D(3)}(x_{\mathbb{P}},\beta,Q^2),$$

  where $y = Q^2/(x_{\rm Bj}s)$, $s = 4E_eE_p$, and

$$\sigma_r^{\rm D(3)} = F_2^{\rm D(3)} - \frac{y^2}{1+(1-y)^2}\,F_L^{\rm D(3)} \approx F_2^{\rm D(3)}(x_{\mathbb{P}},\beta,Q^2),$$

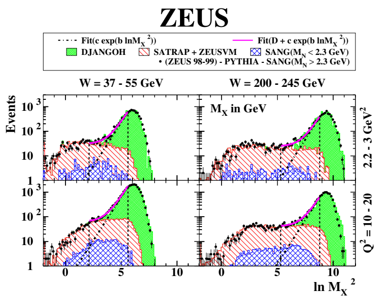
  for small y or assuming that $F_L^{\rm D(3)} \ll F_2^{\rm D(3)}$
- Measurements of $\sigma_r^{\rm D(3)} \Rightarrow$ *diffractive* parton density functions (DPDFs)
  $a^{\rm D}(x_{\mathbb{P}},z,Q^2) = zq^{\rm D}(x_{\mathbb{P}},z,Q^2)$ or $zg^{\rm D}(x_{\mathbb{P}},z,Q^2)$,
  where $\beta \le z \le 1$, cf. $x_{\rm Bj} \le x \le 1$ in DIS.

Recent measurements of DDIS using three methods

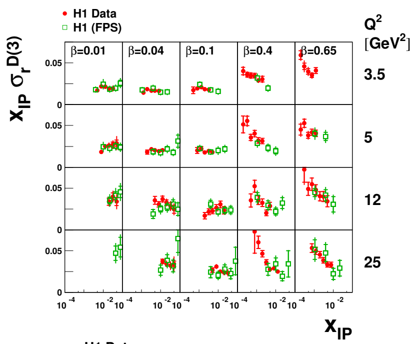
1. Detect leading proton. No proton dissociation background. Can measure t-dependence. Higher $x_{\mathbb{P}}$ accessible. But low statistics due to poor acceptance. Both Pomeron ($\mathbb{P}$) and secondary Reggeon ($\mathbb{R}$) contributions. [ZEUS LPS: `hep-ex/0408009`, H1 FPS: `hep-ex/0606003`]
2. Look for Large Rapidity Gap (LRG). (Non-diffractive contribution is exponentially suppressed as a function of the gap size.) Proton dissociation background (H1: $M_Y < 1.6$ GeV). Used in all final-state DDIS measurements (e.g. dijet and D^* meson production). Both $\mathbb{P}$ and $\mathbb{R}$ contributions. [H1 LRG: `hep-ex/0606004`]
3. Use "M_X method". Subtract non-diffractive (including $\mathbb{R}$) contribution in each (W, Q^2) bin by fitting (in a limited range of $\ln M_X^2$):

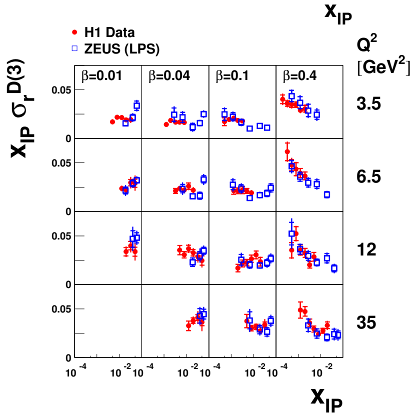
$$\frac{\mathrm{d}N}{\mathrm{d}\ln M_X^2} = D + \underbrace{c\exp(b\ln M_X^2)}_{\text{non-diffractive}}$$

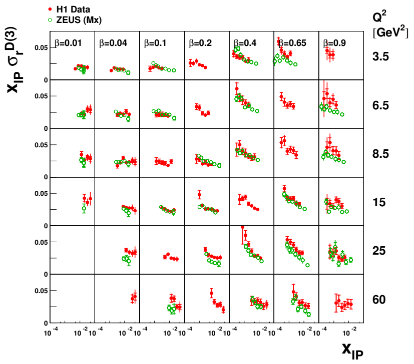
Proton dissociation background (ZEUS: $M_Y < 2.3$ GeV). Motivated by Regge theory assuming $M_X^2 \gg Q^2$, $\alpha_{\mathbb{P}}(0) \approx \alpha_{\mathbb{P}}(\langle t\rangle) \approx 1$, and neglecting $\mathbb{P}$–$\mathbb{R}$ interference. Validity in general? [ZEUS M_X: `hep-ex/0501060`]



Comparison of H1 LRG with other data sets



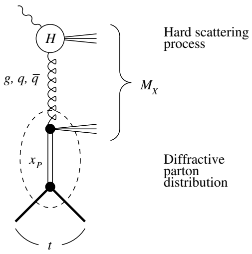




- H1 FPS and ZEUS LPS scaled to $M_Y < 1.6$ GeV by an overall factor 1.23: good agreement with H1 LRG.
- ZEUS M_X scaled to $M_Y < 1.23$ by an overall factor 0.86. Difference expected at large $x_{\mathbb{P}}$ due to lack of $\mathbb{R}$ contribution. However, some difference in Q^2 dependence even at low $x_{\mathbb{P}}$ $\Rightarrow$ diffractive gluon density extracted from ZEUS M_X data roughly half gluon density from H1 LRG data.

Leading-twist collinear factorization in DDIS

$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} \quad + \quad \text{power-suppressed corrections}, \ (1)$$

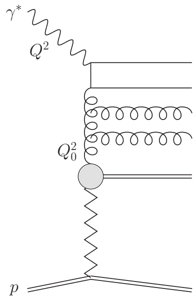


where $C_{2,a}$ are the **same** coefficient functions as in inclusive DIS. The DPDFs $a^{\mathrm{D}} = zq^{\mathrm{D}}$ or zg^{D} satisfy DGLAP evolution:

$$\frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}} \qquad (2)$$

"The factorization theorem applies when Q is made large while x_{Bj}, $x_{\mathbb{P}}$, and t are held fixed." [Collins, hep-ph/9709499]

- Says little about the mechanism for diffraction: information about the diffractive exchange ('Pomeron') needs to be parameterized at an input scale Q_0 and fit to data. Will show later that assuming a perturbative Pomeron contribution, we need to modify both (1) and (2).
- Factorization should also hold for final states (jets etc.) in DDIS, but is broken in hadron–hadron collisions, although hope that same formalism can be applied with extra suppression factor calculable from eikonal models (more later).



LO diffractive dijet photoproduction: resolved photon contribution should be suppressed, but direct photon contribution unsuppressed. Complications at NLO [Klasen–Kramer, hep-ph/0506121].

H1 2006 extraction of DPDFs [hep-ex/0606004]

- Assume Regge factorization [Ingelman–Schlein,'85]:

$$\boxed{a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, a^{\mathbb{P}}(z, Q^2)} \qquad (3)$$

- Pomeron flux factor from Regge phenomenology:

$$\boxed{f_{\mathbb{P}}(x_{\mathbb{P}}) = \int_{t_{\mathrm{cut}}}^{t_{\mathrm{min}}} \mathrm{d}t \ \mathrm{e}^{B_{\mathbb{P}}\, t}\ x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)}} \qquad (\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t)$$

"Regge factorization relates the power of $x_{\mathbb{P}}$ measured in DDIS to the power of s measured in hadron–hadron elastic scattering." [Collins, hep-ph/9709499]

- Fit to H1 FPS data gives $\alpha_{\mathbb{P}}(t) = 1.11 + 0.06\, t$. Fit to H1 LRG data gives $\alpha_{\mathbb{P}}(0) = 1.12$ if $\alpha'_{\mathbb{P}} = 0.06$, or $\alpha_{\mathbb{P}}(0) = 1.15$ if $\alpha'_{\mathbb{P}} = 0.25$.
- So the Pomeron in DDIS is **not** the 'soft' Pomeron with $\alpha_{\mathbb{P}}(t) = 1.08 + 0.25\, t$. By Collins' definition, Regge factorization is broken. H1 assume that the $x_{\mathbb{P}}$ dependence factorizes as eq.(3) regardless, with the fitted $\alpha_{\mathbb{P}}(0)$ independent of β and Q^2. However, this $x_{\mathbb{P}}$ factorization is also broken, see later.

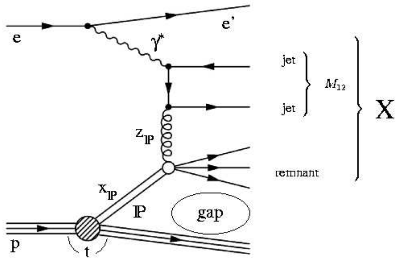
H1 2006 extraction of DPDFs

`hep-ex/0606004`

- Pomeron PDFs $a^{\mathbb{P}}(z,Q^2) = z\Sigma^{\mathbb{P}}(z,Q^2)$ or $zg^{\mathbb{P}}(z,Q^2)$ are DGLAP-evolved from arbitrary inputs at $Q_0^2 \simeq 2$ GeV^2:

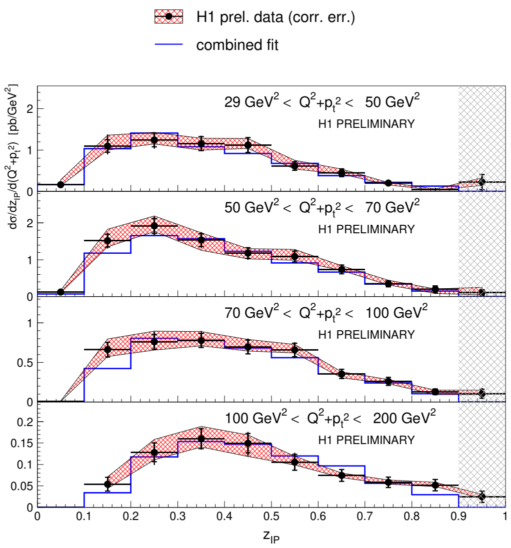
$$a^{\mathbb{P}}(z,Q_0^2) = A_a z^{B_a}(1-z)^{C_a}\, \mathrm{e}^{-\frac{0.01}{1-z}}.$$

- Secondary Reggeon contribution with $\alpha_{\mathbb{R}}(0) = 0.50$ included using pion PDFs. Normalisation fitted to data.
- Fit to H1 LRG data with $M_X \geq 2$ GeV and $Q^2 \geq 8.5$ GeV^2 (190 data points).
- H1 2006 Fit A: fix $B_g = 0$ $\Rightarrow \chi^2 = 158$.
- H1 2006 Fit B: fix $B_g = 0$ and $C_g = 0$ $\Rightarrow \chi^2 = 164$.

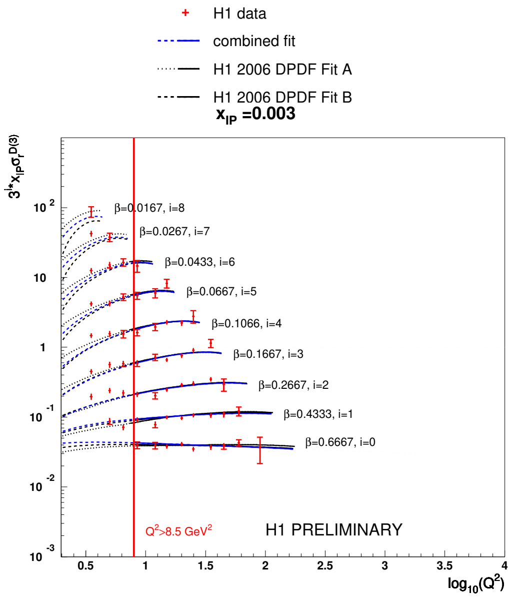


- Also, combined fit of inclusive DDIS data with diffractive dijet data [`H1prelim-06-011`].
- Constrains gluon density at $z = z_{\mathbb{P}}$ directly, rather than indirectly from scaling violations.
- Dijet cross sections calculated using NLOJET++ by Z. Nagy.

H1 2006 extraction of DPDFs

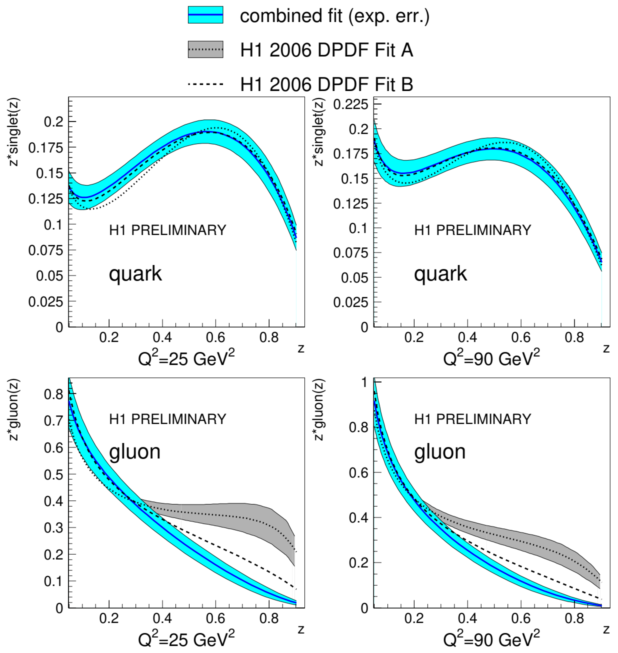


- Dijet $z_{\mathbb{P}}$ distribution mainly constains z dependence of g^{D}.



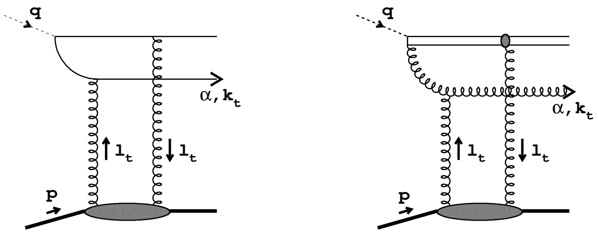
- Cut of $Q^2 \geq 8.5$ GeV^2 needed to achieve stable fit.

H1 2006 extraction of DPDFs

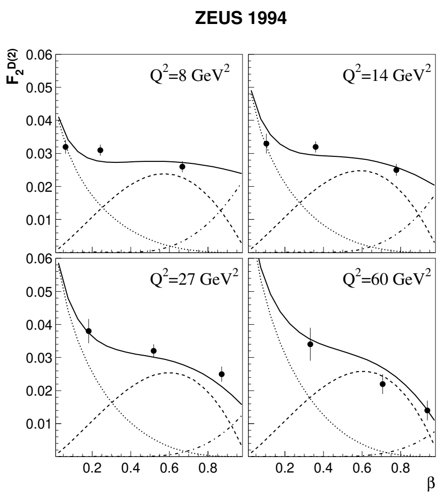


- Gluon density smaller at high z on inclusion of the dijet data.
- χ^2 for inclusive DDIS data increases from 158 (H1 Fit A) to 169 (H1 combined fit).
- Suggests some tension between inclusive DDIS and dijet data using this approach. Gluon determined **directly** from dijet data different from gluon determined **indirectly** from scaling violations of inclusive DDIS data.
- How reliable is the theory used in these fits?

Alternative approach: two-gluon exchange calculations



Two-gluon exchange calculations are the basis for the color dipole model description of DDIS.



- Right: $x_{\mathbb{P}} F_2^{\mathrm{D}(3)}$ for $x_{\mathbb{P}} = 0.0042$ as a function of β [Golec-Biernat–Wüsthoff, hep-ph/9903358].
  - dotted lines: $\gamma_T^* \to q\bar{q}g$,
  - dashed lines: $\gamma_T^* \to q\bar{q}$,
  - dot-dashed lines: $\gamma_L^* \to q\bar{q}$,

  important at low, medium, and high β respectively.
- $\gamma_T^* \to q\bar{q}g$ and $\gamma_T^* \to q\bar{q}$ are partly higher-twist, $\gamma_L^* \to q\bar{q}$ is **purely** higher-twist, but H1 DPDFs only include leading-twist contributions.

Comparison of two approaches

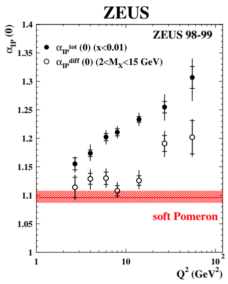
'Regge factorization' approach	Two-gluon exch. (e.g. dipole model)
• $\mathbb{P}$ is purely non-perturbative, i.e. a Regge pole.	• $\mathbb{P}$ is purely perturbative, i.e. a gluon ladder.
• Q^2 dependence given by DGLAP.	• Q^2 dependence predicted.
• Need to fit β dependence.	• β dependence predicted.
• $x_{\mathbb{P}}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.	• $x_{\mathbb{P}}$ dependence given by square of skewed gluon density (or dipole cross section).
• $x_{\mathbb{P}}$ dependence factorizes.	• $x_{\mathbb{P}}$ dependence doesn't factorize.
• Only leading-twist.	• Goes beyond leading-twist.
• Full DGLAP evolution in Pomeron structure function.	• Only $q\bar{q}$ and $q\bar{q}g$ final states as products of photon dissociation.
• Extract universal DPDFs.	• No concept of DPDFs.
• Only applies to inclusive DDIS.	• Also explains exclusive processes.

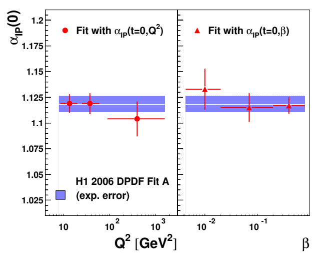
Is inclusive DDIS 'soft' or 'hard'?

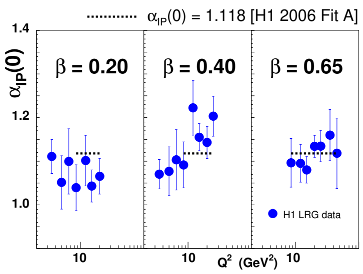
Is inclusive DDIS 'soft' or 'hard'?

	B_D (GeV^{-2})	$\alpha_{\mathbb{P}}(0)$	$\alpha'_{\mathbb{P}}$ (GeV^{-2})
ZEUS LPS	$7.9^{+1.0}_{-0.7}$	1.16 ± 0.3	0.25 (assumed)
H1 FPS	6	$1.11^{+0.05}_{-0.03}$	$0.06^{+0.19}_{-0.06}$
H1 LRG	—	$1.12^{+0.03}_{-0.01}$	0.06 (assumed)
H1 combined fit	—	1.15	0.06 (assumed)
cf. $\gamma p \to \rho\, p$	~ 10	~ 1.09	~ 0.12
cf. $\gamma p \to J/\psi\, p$	~ 4	~ 1.20	~ 0.12

- Inclusive DDIS is **harder** than exclusive ρ photoproduction, but **softer** than exclusive J/ψ photoproduction (and **softer** than inclusive DIS at the same Q^2).
- Above $\alpha_{\mathbb{P}}(0)$ values are averaged over β and Q^2. Perturbative Pomeron contribution should break $x_{\mathbb{P}}$ factorization to some degree.
- **Left plot**: ZEUS M_X data show rise of $\alpha_{\mathbb{P}}(0)$ with Q^2.
  Middle plot: H1 LRG data show constant $\alpha_{\mathbb{P}}(0)$ when averaged over β or Q^2.
  Right plot (G.W.): H1 LRG data show rise of $\alpha_{\mathbb{P}}(0)$ with Q^2 in some β bins.







Combination of two approaches

Martin, Ryskin, G.W. [hep-ph/0406224, hep-ph/0504132, hep-ph/0511333]

- In reality, both non-perturbative and perturbative Pomeron contributions to inclusive DDIS. Improve two-gluon exchange calculations by introducing DGLAP evolution in 'Pomeron structure function' allowing universal DPDFs to be extracted.

Non-perturbative $\mathbb{P}$ contribution

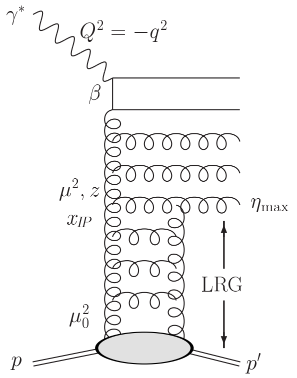
- $\mathbb{P}$ is ~~purely~~ partly non-perturbative, i.e. a Regge pole.
- Q^2 dependence given by DGLAP.
- Need to fit β dependence.
- $x_{\mathbb{P}}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.
- $x_{\mathbb{P}}$ dependence factorizes.
- Only leading-twist.
- Full DGLAP evolution in Pomeron structure function.
- Extract universal DPDFs.
- Only applies to inclusive DDIS.

Perturbative $\mathbb{P}$ contribution

- $\mathbb{P}$ is ~~purely~~ partly perturbative, i.e. a gluon ladder.
- Q^2 dependence predicted.
- β dependence predicted.
- $x_{\mathbb{P}}$ dependence given by square of skewed gluon density ~~(or dipole cross section)~~.
- $x_{\mathbb{P}}$ dependence doesn't factorize.
- Goes beyond leading-twist.
- Full DGLAP evolution in Pomeron structure function.
- Extract universal DPDFs.
- Also explains exclusive processes.

Perturbative Pomeron contribution to DDIS

- Generalize $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ to arbitrary number of parton emissions [Ryskin,'90; Levin–Wüsthoff,'94].
- Work in Leading Logarithmic Approximation (LLA) $\Rightarrow$ virtualities of t-channel partons are strongly ordered: $\mu_0^2 \ll \ldots \ll \mu^2 \ll \ldots \ll Q^2$, i.e. pQCD Pomeron is a DGLAP ladder rather than a BFKL ladder.



- **New feature:** integral over scale μ^2 (starting scale for DGLAP evolution of Pomeron PDFs).

$$F_2^{\mathrm{D}(3)} = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, F_2^{\mathbb{P}}(\beta, Q^2; \mu^2)$$

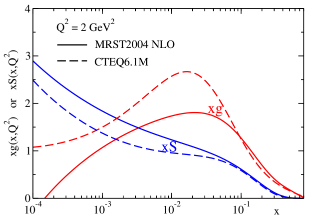
$$f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) = \frac{1}{x_{\mathbb{P}} B_D}\left[R_g \frac{\alpha_S(\mu^2)}{\mu}\, x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)\right]^2$$

$$F_2^{\mathbb{P}}(\beta, Q^2; \mu^2) = \sum_{a=q,g} C_{2,a} \otimes a^{\mathbb{P}}$$

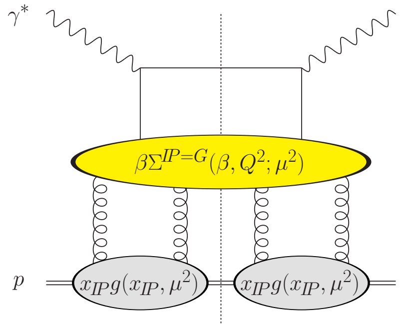
(B_D from t-integration, R_g from skewness)

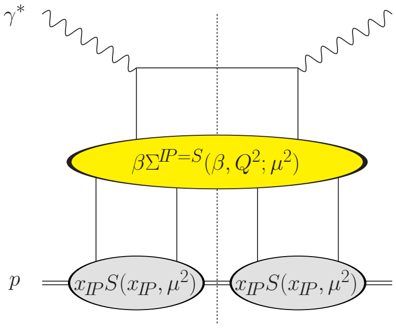
- Pomeron PDFs $a^{\mathbb{P}}(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale μ^2 up to Q^2.
- For $\mu^2 < \mu_0^2 \sim 1$ GeV^2, replace lower parton ladder with usual Regge pole contribution. Take $\alpha_{\mathbb{P}}(0) \simeq 1.08$ (or fit) and fit Pomeron PDFs DGLAP-evolved from an input scale μ_0^2.
- **Important:** scale that controls $x_{\mathbb{P}}$ dependence, e.g. effective $\alpha_{\mathbb{P}}(0)$, is μ^2 not Q^2!

Gluonic and sea-quark Pomeron



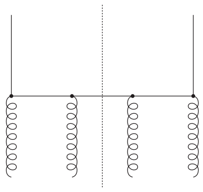
- At low scales, sea-quark density of the proton dominates over gluon density at small $x \Rightarrow$ need to account for sea-quark density in perturbative Pomeron flux factor.





- Pomeron structure function $F_2^{\mathbb{P}}(\beta, Q^2; \mu^2)$ calculated from quark singlet $\Sigma^{\mathbb{P}}(z, Q^2; \mu^2)$ and gluon $g^{\mathbb{P}}(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale μ^2 up to Q^2.
- Input Pomeron PDFs $\Sigma^{\mathbb{P}}(z, \mu^2; \mu^2)$ and $g^{\mathbb{P}}(z, \mu^2; \mu^2)$ to DGLAP evolution are Pomeron-to-parton splitting functions.

LO Pomeron-to-parton splitting functions



- **Notation:** '$\mathbb{P} = G$' means gluonic Pomeron, '$\mathbb{P} = S$' means sea-quark Pomeron, '$\mathbb{P} = GS$' means interference between these.
- LO Pomeron-to-parton splitting functions are [hep-ph/0504132]:

$$z\Sigma^{\mathbb{P}=G}(z, \mu^2; \mu^2) = P_{q,\mathbb{P}=G}(z) = z^3\,(1-z),$$

$$zg^{\mathbb{P}=G}(z, \mu^2; \mu^2) = P_{g,\mathbb{P}=G}(z) = \frac{9}{16}\,(1+z)^2\,(1-z)^2,$$

$$z\Sigma^{\mathbb{P}=S}(z, \mu^2; \mu^2) = P_{q,\mathbb{P}=S}(z) = \frac{4}{81}\,z\,(1-z),$$

$$zg^{\mathbb{P}=S}(z, \mu^2; \mu^2) = P_{g,\mathbb{P}=S}(z) = \frac{1}{9}\,(1-z)^2,$$

$$z\Sigma^{\mathbb{P}=GS}(z, \mu^2; \mu^2) = P_{q,\mathbb{P}=GS}(z) = \frac{2}{9}\,z^2\,(1-z),$$

$$zg^{\mathbb{P}=GS}(z, \mu^2; \mu^2) = P_{g,\mathbb{P}=GS}(z) = \frac{1}{4}\,(1+2z)\,(1-z)^2$$

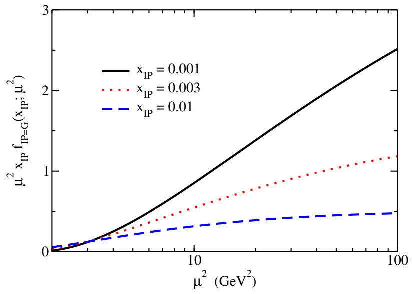
Evolve these input Pomeron PDFs from μ^2 up to Q^2 using NLO DGLAP evolution.

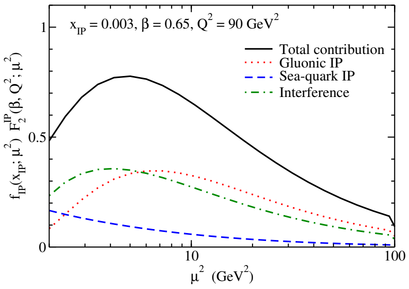
Contribution to $F_2^{\mathrm{D}(3)}$ as a function of μ^2

$$F_2^{\mathrm{D}(3)} = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, F_2^{\mathbb{P}}(\beta, Q^2;\mu^2)$$

$$f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) = \frac{1}{x_{\mathbb{P}} B_D}\left[R_g \frac{\alpha_S(\mu^2)}{\mu}\, x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)\right]^2$$

- Naïvely, $f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sim 1/\mu^2$, so contributions from large μ^2 are strongly suppressed.
- But $x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2) \sim (\mu^2)^\gamma$, where γ is the anomalous dimension. In BFKL limit $\gamma \simeq 0.5$, so $f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sim$ constant.
- HERA domain is in an intermediate region: γ is not small, but is less than 0.5.
- Upper plot: $\mu^2 x_{\mathbb{P}} f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)$ is not flat for small $x_{\mathbb{P}}$. Lower plot: integrand as a function of μ^2 (using MRST2004F3 NLO PDFs) $\Rightarrow$ large contribution from large μ^2.
- Recall that fits using 'Regge factorization' include contributions from $\mu^2 \leq Q_0^2$ in the input densities, but neglect all contributions from $\mu^2 > Q_0^2$.





Inhomogeneous evolution of DPDFs

$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}},$$

$$\text{where } a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2) = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, a^{\mathbb{P}}(z, Q^2;\mu^2)$$

$$\Rightarrow \frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} + \underbrace{f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)\, P_{a\mathbb{P}}(z)}_{\text{Extra inhomogeneous term}}$$

- Inhomogeneous evolution of DPDFs is **not a new idea**:

  "We introduce a diffractive dissociation structure function and show that it obeys the DGLAP evolution equation, ***but****, with an additional inhomogeneous term."*
  [Levin–Wüsthoff,'94]

Pomeron structure is analogous to photon structure

Photon structure function

$$F_2^{\gamma}(x_{\mathrm{Bj}}, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\gamma}}_{\text{Resolved photon}} + \underbrace{C_{2,\gamma}}_{\text{Direct photon}}$$

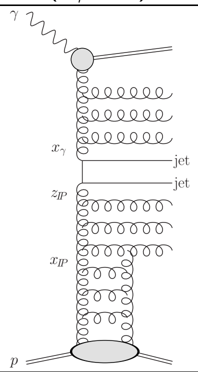
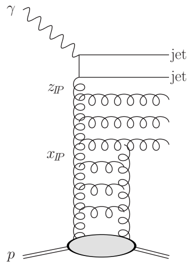
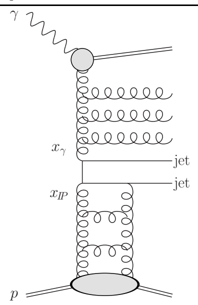
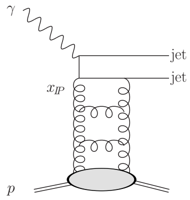
$$\text{where } \frac{\partial a^{\gamma}(x, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\gamma}}_{\text{DGLAP term}} + \underbrace{P_{a\gamma}(x)}_{\text{Inhomogeneous term}}$$

Diffractive structure function

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}}}_{\text{Resolved Pomeron}} + \underbrace{C_{2,\mathbb{P}}}_{\text{Direct Pomeron}}$$

$$\text{where } \frac{\partial a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} + \underbrace{P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)}_{\text{Inhomogeneous term}}$$

Dijets in diffractive photoproduction

	Resolved photon ($x_\gamma < 1$)	Direct photon ($x_\gamma = 1$)
Resolved Pomeron ($z_{\mathbb{P}} < 1$)		
Direct Pomeron ($z_{\mathbb{P}} = 1$)		

Analysis of H1 LRG data

Motivation: What impact do the perturbative Pomeron terms have on the H1 2006 DPDF analysis?

- Take input quark singlet and gluon densities at $Q_0^2 = 2$ GeV^2 in the form:

$$z\Sigma^{\rm D}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, A_q\, z^{B_q}(1-z)^{C_q},$$
$$zg^{\rm D}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, A_g\, z^{B_g}(1-z)^{C_g}.$$

- $f_{\mathbb{P}}(x_{\mathbb{P}})$ as in the H1 2006 fit with $\alpha_{\mathbb{P}}(0)$, A_a, B_a, and C_a $(a = q, g)$ as free parameters.
- Work in Fixed Flavor Number Scheme: no heavy quark DPDFs.
- Treatment of secondary Reggeon as in H1 2006 fit, i.e. using pion PDFs, but using GRV NLO instead of Owens LO. (N.B.: No good reason that the $\mathbb{R}$ PDFs should be same as pion PDFs.)
- Fit H1 LRG data binned at fixed $x_{\mathbb{P}}$ values with cut $M_X \geq 2$ GeV. Will study effect of cut $Q^2 \geq Q^2_{\rm min}$ on fitted data.
- Statistical and systematic experimental errors added in quadrature.
- Two types of fits:
  - **"Regge"** = 'Regge factorization' approach (i.e. **no** $C_{2,\mathbb{P}}$ or $P_{a\mathbb{P}}$) $\simeq$ H1 2006 Fit A.
  - **"pQCD"** = 'perturbative QCD' approach **with** LO $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$.
- Use MRST2004F3 NLO PDFs with $\Lambda_{\rm QCD}^{(n_f=3)} = 407$ MeV.

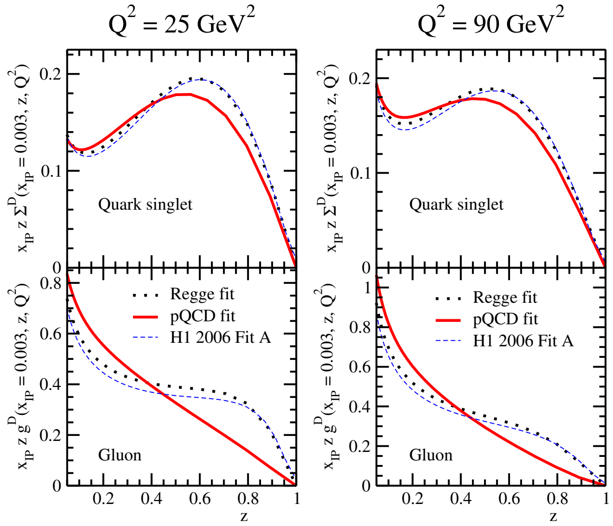
Stability with respect to $Q^2_{\rm min}$ variation

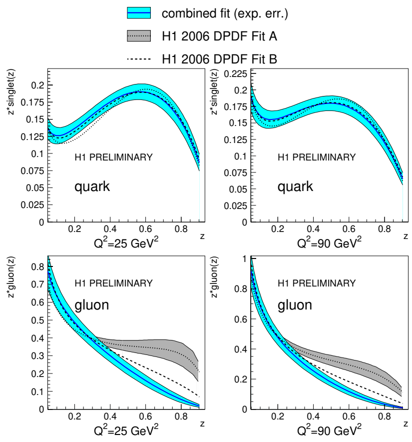
- Stability analysis following MRST [hep-ph/0308087].

$Q^2_{\rm min}$ (GeV^2)	3.5	5.0	6.5	8.5	12	15
Number of data points	266	239	214	190	164	141
$\chi^2(Q^2 \geq 3.5$ GeV$^2)$	272					
	262					
$\chi^2(Q^2 \geq 5$ GeV$^2)$	233	222				
	225	219				
$\chi^2(Q^2 \geq 6.5$ GeV$^2)$	208	186	174			
	205	200	189			
$\chi^2(Q^2 \geq 8.5$ GeV$^2)$	178	155	144	142		
	181	174	156	153		
$\chi^2(Q^2 \geq 12$ GeV$^2)$	156	136	124	123	122	
	162	155	139	134	134	
$\chi^2(Q^2 \geq 15$ GeV$^2)$	133	111	100	98	97	96
	138	130	112	106	104	103
Stability measure Δ_i^{i+1}	0.40	0.45	0.07	0.03	0.04	
	0.20	0.42	0.14	0.03	0.06	

- Both Regge and pQCD fits stable for $Q^2_{\rm min} \gtrsim 6.5$ GeV^2. To compare directly with H1 2006 fits, take $Q^2_{\rm min} = 8.5$ GeV^2.

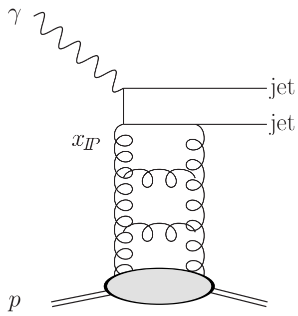
DPDFs with $Q^2_{\rm min} = 8.5$ GeV^2 compared to H1 DPDFs

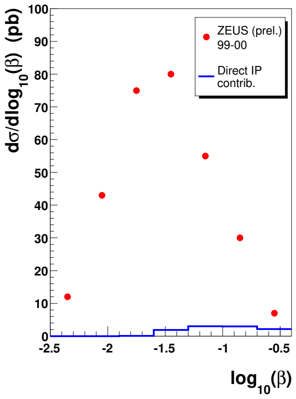




- Regge fit $\simeq$ H1 2006 Fit A.
- pQCD fit closer to H1 combined fit than H1 2006 Fit A **without** including dijet data in fit $\Rightarrow$ should describe dijet data better than H1 2006 Fit A.
- Suggests tension between inclusive DDIS and diffractive dijet data is alleviated by inclusion of perturbative Pomeron terms.

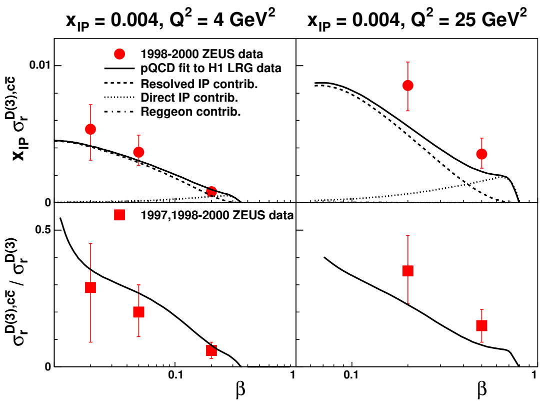
Direct Pomeron contribution to dijet production

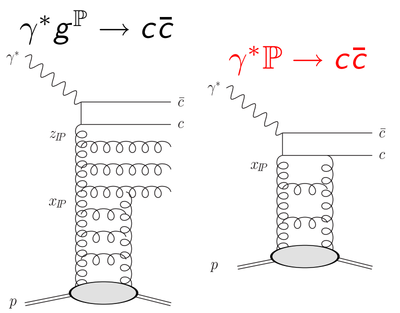




- Direct Pomeron contribution ($z_{\mathbb{P}} = 1$) calculated with ZEUS (prel.) kinematic cuts: 31% of data in largest β bin.
- Exclusive diffractive dijet contribution to inclusive diffractive dijet production is small in the HERA kinematic regime.
- H1 combined fit is to dijet data with $z_{\mathbb{P}} < 0.9$ integrated over β. Therefore, safe to neglect direct Pomeron contribution and include only the resolved Pomeron contribution calculated using NLOJET++.
- Searches for exclusive dijets in progress at HERA by H1 and ZEUS.
- Alternative calculations for exclusive dijets using LO collinear factorization by Braun and Ivanov [hep-ph/0505263].

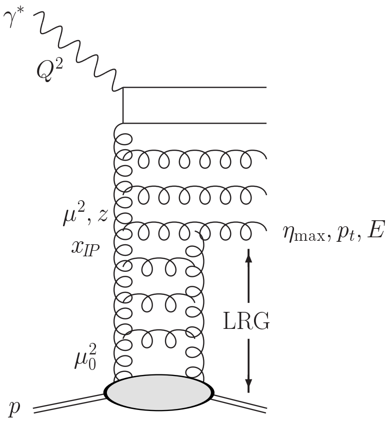
Predictions for diffractive charm production





- Direct Pomeron contribution, i.e. $\gamma^*\mathbb{P} \to c\bar{c}$ ($z_{\mathbb{P}} = 1$), is significant at moderate/high β.
- If the direct Pomeron contribution is neglected, the diffractive gluon density would need to be artificially large to fit the charm data.

Comment on the LRG method



- **LRG method:** event selection using cut on maximum (pseudo)rapidity $\eta_{\max} < \eta_{\rm cut} = 3.3$ [H1, hep-ex/0606004].
- Kinematics of $\mathbb{P}$ remnant:

$$E = p_t \cosh \eta_{\max} \simeq (1-z)x_{\mathbb{P}}E_p$$

$$\boxed{\Rightarrow p_t > (1-z)x_{\mathbb{P}}E_p \,\mathrm{sech}\, \eta_{\rm cut}.}$$

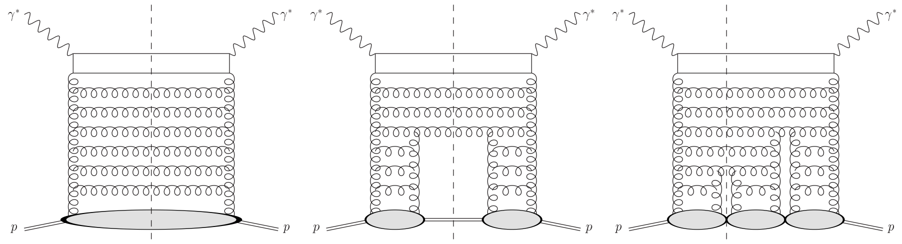
- Therefore, strong cut on $\eta_{\max}$ increases relative contribution to DDIS from perturbative Pomeron, i.e. large virtuality $\mu^2 \simeq p_t^2/(1-z) \gtrsim 1$ GeV^2.
- Originally discussed by J. Ellis and G. Ross [hep-ph/9604360, hep-ph/9812385].
- In recent H1 measurements, effect of cut on $\eta_{\max}$ is compensated as part of acceptance corrections using RAPGAP event generator.
  - Pomeron remnant p_t [H1, hep-ex/0012051] and $\eta_{\max}$ distributions are well described by RAPGAP.
  - Good agreement of LRG data with leading-proton data.

  Gives confidence that procedure is correct (although uncertainty due to acceptance correction for cut on $\eta_{\max}$ is dominant uncertainty at high $x_{\mathbb{P}}$).
- Interesting experimental possibility to enhance perturbative Pomeron contribution by event selection with a strong cut on $\eta_{\max}$.

Further corrections to DPDF evolution

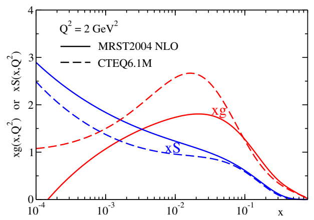
- NNLO parton-to-parton splitting functions (known).
- NLO Pomeron-to-parton splitting functions (unknown).
- Absorptive corrections. Schematically,

$$\frac{\partial g^{\mathrm{D}}}{\partial \ln Q^2} = P_{gg} \otimes g^D + P_{g\mathbb{P}} \otimes g^2 - 4P_{g\mathbb{P}} \otimes g g^D + \ldots .$$



Possible that further corrections will stabilize the results of the fit with respect to the Q^2_{min} cut.

Non-linear evolution of inclusive PDFs



- Regge theory expectation for small-x PDFs at low scales $Q \lesssim Q_0 \sim 1$ GeV is that:

$$xg \sim x^{-\lambda_g} \text{ and } xS \sim x^{-\lambda_S},$$

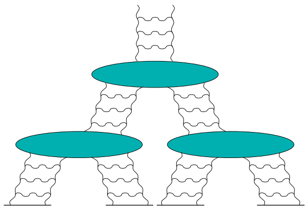
  with $\lambda_g = \lambda_S = \lambda_{\mathrm{soft}} \simeq 0.08$, then at higher $Q^2 \gtrsim 1$ GeV, QCD evolution should take over, increasing λ_g and λ_S.
- Current PDF sets exhibit a very different behavior.

Gribov–Levin–Ryskin ('83) and Mueller–Qiu ('86) (GLRMQ):

$$\frac{\partial xg(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_S}{2\pi} \sum_{a'=q,g} P_{ga'} \otimes a' \; - \; \frac{9}{2} \frac{\alpha_S^2(Q^2)}{R^2\, Q^2} \int_x^1 \frac{\mathrm{d}x'}{x'} \left[x' g(x',Q^2)\right]^2$$

(BFKL-based refinements: Balitsky–Kovchegov and JIMWLK. See review by Jalilian-Marian and Kovchegov [hep-ph/0505052].)

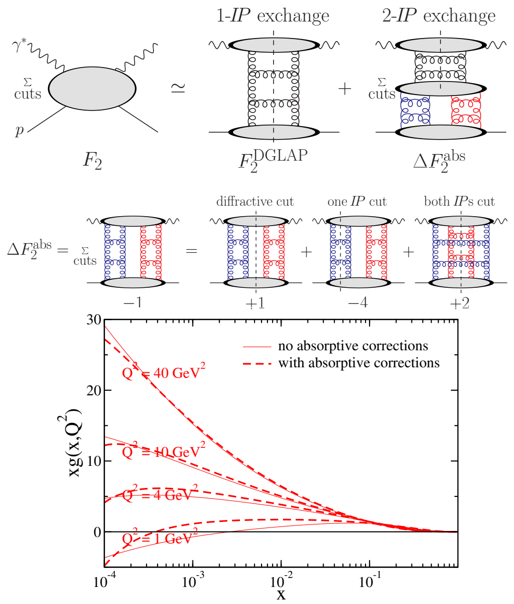
- Are the input MRST/CTEQ gluon densities at small-x and low Q^2 forced to be artificially small in order to *mimic* the neglected screening corrections?



Non-linear evolution of inclusive PDFs

Martin, Ryskin, G.W. [hep-ph/0406225, hep-ph/0508093]:

$$\boxed{\frac{\partial a(x,Q^2)}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a' \;-\; \int_x^1 \mathrm{d}x_{\mathbb{P}}\; P_{a\mathbb{P}}(x/x_{\mathbb{P}})\, f_{\mathbb{P}}(x_{\mathbb{P}};Q^2).}$$



- Application of DDIS formalism to calculate shadowing corrections to inclusive DIS via Abramovsky–Gribov–Kancheli (AGK) cutting rules.
- Inhomogeneous evolution of DPDFs $\Rightarrow$ non-linear evolution of inclusive PDFs.
- More precise version of Gribov–Levin–Ryskin–Mueller–Qiu (GLRMQ) equation derived.
- Fit HERA F_2 data similar to MRST2001 NLO fit. Small-x gluon slightly enhanced at low scales.

Outline

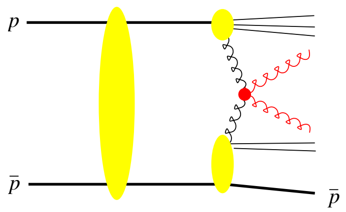
1. Introduction
2. Exclusive diffraction in *ep* collisions
   - Exclusive processes within collinear factorization
   - Exclusive processes within the dipole picture
     - Dipole picture in the non-forward direction
     - Impact parameter dependent dipole cross sections
     - Description of HERA vector meson data
     - Description of HERA DVCS data
3. Inclusive diffraction in *ep* collisions
   - Diffractive DIS kinematics and structure functions
   - Collinear factorization in DDIS
   - Perturbative Pomeron contribution to DDIS
   - Analysis of HERA DDIS data
   - Non-linear evolution of inclusive PDFs
4. Diffraction in pp and $p\bar{p}$ collisions
   - Factorization breaking in diffractive hadron–hadron collisions
   - Exclusive diffractive Higgs production at LHC
5. Summary and outlook

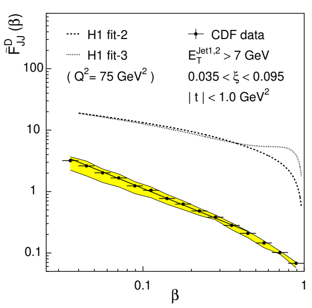
Factorization breaking in diffractive pp and $p\bar{p}$ collisions

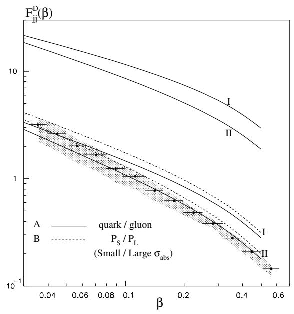
- Factorization is broken in diffractive hadron–hadron collisions by (soft) interaction between spectator partons of the colliding hadrons.
- Can be accounted for with suppression factor S^2 calculable from eikonal models with parameters fitted to soft hadron–hadron data [review by Gotsman *et al.*, `hep-ph/0511060`].
- Consider diffractive dijet production at Tevatron. Diffractive structure function of the antiproton:

$$\tilde{F}^{\mathrm{D}}_{JJ}(\beta) = \frac{1}{\xi_{\max} - \xi_{\min}} \int_{\xi_{\min}}^{\xi_{\max}} \mathrm{d}\xi \left[\beta g^{\mathrm{D}}(\xi, \beta, Q^2) + \frac{4}{9} \beta \Sigma^{\mathrm{D}}(\xi, \beta, Q^2) \right],$$

  measured by CDF [PRL **84**, 5043 (2000)].
- Prediction using DPDFs from HERA agree with $\tilde{F}^{\mathrm{D}}_{JJ}$ data when S^2 factor included [Kaidalov *et al.*, `hep-ph/0105145`].

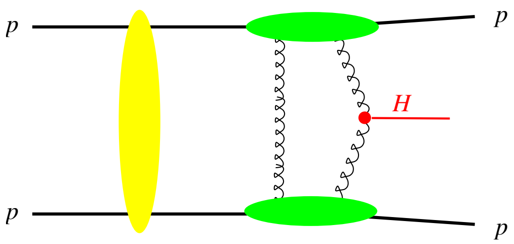






Exclusive diffractive Higgs production at LHC

Khoze, Martin, Ryskin (KMR) (+ Kaidalov, Stirling) ['97–].
Review by J. Forshaw [`hep-ph/0508274`].



$$\sigma(pp \to p + H + p) \sim 3 \text{ fb}$$
$$\sim 10^{-4}\, \sigma(pp \to HX),$$

for a Standard Model Higgs with $M_H \simeq 120$ GeV.

Advantages:

1. Tag outgoing protons: measure Higgs mass with resolution ~ 1 GeV.
2. Investigate quantum numbers of produced states.
3. Clean environment with low background.
4. Regions of SUSY parameter space where exclusive Higgs production enhanced compared to inclusive Higgs production.

FP420 proposal to add forward proton tagging detectors 420 m from the interaction points of the ATLAS/CMS experiments [see `http://www.fp420.com/`].

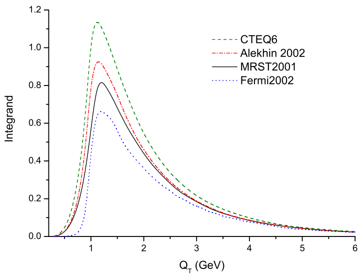
Main ingredients of KMR approach

$$\boxed{\sigma(pp \to p + H + p) \sim \frac{S^2}{B_D^2} \left| N \int \frac{\mathrm{d}Q_T^2}{Q_T^4} \, f_g(x_1, x_1', Q_T^2, \mu^2) \, f_g(x_2, x_2', Q_T^2, \mu^2) \right|^2}$$

where $S^2 \simeq 0.026$, $B_D \simeq 4\ \mathrm{GeV}^{-2}$, $\mu \simeq M_H/2$, and

$$\boxed{f_g(x, x', Q_T^2, \mu^2) = R_g \frac{\partial}{\partial \ln Q_T^2} \left[\sqrt{T_g(Q_T^2, \mu^2)} \, xg(x, Q_T^2) \right],}$$

with R_g = skewness. The Sudakov factor, $T_g(Q_T^2, \mu^2)$, suppresses the infrared Q_T region:



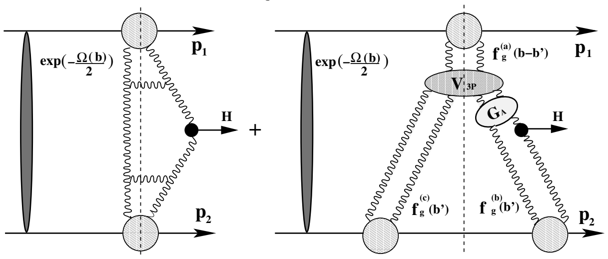
- Plot by J. Forshaw [hep-ph/0508274].
- Integrand dominated by $Q_T \sim 1$–2 GeV $\Rightarrow$ pQCD applicable (just!).
- Calculations implemented in ExHuME Monte Carlo program [hep-ph/0502077].

Check KMR approach at HERA and Tevatron

- Skewed unintegrated gluon densities, $f_g(x, x', Q_T^2, \mu^2)$, give reasonable description of exclusive diffractive vector meson production at HERA [Martin–Ryskin–Teubner, hep-ph/9912551].
- Predictions for the suppression factor S^2 can be tested at Tevatron, and for the resolved photon contribution to photoproduction at HERA (in both diffraction and with leading neutrons).
- Searches in progress at Tevatron for exclusive dijet ($p\bar{p} \to p + jj + \bar{p}$) and diphoton ($p\bar{p} \to p + \gamma\gamma + \bar{p}$) production [$\to$ talk by C. Mesropian].

Hard rescattering corrections to exclusive Higgs production

Bartels–Bondarenko–Kutak–Motyka [hep-ph/0601128] find that hard rescattering corrections give large negative contribution, using pQCD with infrared behavior stabilized by saturation scale:



Rebuttal by Khoze–Martin–Ryskin [hep-ph/0602247], who argue that hard rescattering corrections must be small because:

- Similar effect not observed for leading neutron production at HERA.
- Rapidity interval at LHC not large enough for hard rescattering corrections.
- LO pQCD approach invalid: no evidence in data that saturation scale is large enough to provide infrared cutoff, gluon density not constrained at very small x.
- If first hard rescattering corrections important, would need to consider additional corrections and would need to recalculate S^2.

Better theoretical understanding needed to clarify this issue.

Outline

Summary

Exclusive diffraction in ep collisions

- Exclusive diffractive processes are well described within the **dipole picture** using an impact parameter dependent (Glauber–Mueller) dipole cross section.
- t-distributions show that the proton shape in the transverse plane takes a **universal Gaussian form** in the limit of small dipole sizes, with $\sqrt{\langle b^2 \rangle} = 0.56$ fm (cf. the proton charge radius of 0.87 fm).

Inclusive diffraction in ep collisions

- Perturbative Pomeron contribution leads to an **inhomogeneous** evolution equation for the diffractive PDFs, analogous to the evolution equation for the photon PDFs.
- Evidence of instability in the fits for $Q^2 \lesssim 6.5$ GeV^2: further theoretical corrections such as NLO Pomeron-to-parton splitting functions or absorptive corrections may help.

Diffraction in pp and $p\bar{p}$ collisions

- Factorization **broken** in hadron–hadron collisions, but can be accounted for with an extra suppression factor calculable from eikonal models of soft interactions.
- Case for installing proton taggers at the LHC to detect exclusive diffractive Higgs production. Calculations can be checked at HERA and Tevatron.

Outlook

Diffraction in ep collisions at a future EIC

- Cleanest measurements of diffraction by tagging outgoing proton. Requires proper integration of forward detectors.
- Measure t-distributions of exclusive vector meson production and DVCS to high accuracy to give information about the proton shape in the transverse plane and the small-x gluon density.
- Precise measurements of $F_L^{\mathrm{D}(3)}$ could clarify role of higher-twist contributions in diffractive DIS. (First measurements expected from HERA low-energy running in 2007.)
- Search for exclusive diffractive dijet production.
- Strong rapidity cut requirement could enhance perturbative Pomeron contribution. Interesting to measure transverse momentum of Pomeron remnant.
- Diffractive DIS offers a more direct way to study perturbative multi-ladder diagrams than inclusive DIS.