Theory of diffractive structure functions

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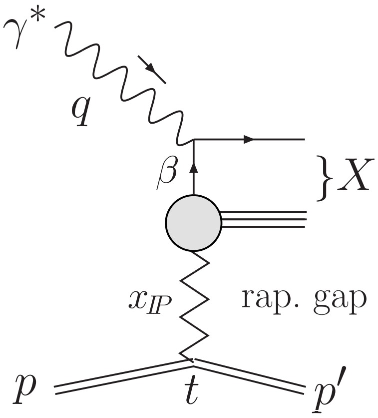
(see `hep-ph/0511333`, and references therein)

Outline

- Diffractive Deep-Inelastic Scattering (DDIS) is characterised by a Large Rapidity Gap (LRG) due to 'Pomeron' (vacuum quantum number) exchange.
- How do we extract Diffractive Parton Density Functions (DPDFs) from DDIS data?

1. Introduction
2. 'Regge factorisation' approach to DPDFs
3. How to reconcile two-gluon exchange with DPDFs?
4. Pomeron structure is analogous to photon structure
5. Description of DDIS data
6. Comments on the experimental methods
7. Conclusions

Diffractive DIS kinematics



- $q^2 \equiv -Q^2$
- $W^2 \equiv (q+p)^2 = -Q^2 + 2\,p\cdot q$
  $\Rightarrow \quad x_B \equiv \frac{Q^2}{2\,p\cdot q} = \frac{Q^2}{Q^2+W^2}$ (fraction of proton's momentum carried by struck quark)
- $t \equiv (p-p')^2 \approx 0$, $(p-p') \approx x_{\mathbb{P}}\,p$
- $M_X^2 \equiv (q+p-p')^2 = -Q^2 + x_{\mathbb{P}}(Q^2+W^2)$
  $\Rightarrow \quad x_{\mathbb{P}} = \frac{Q^2+M_X^2}{Q^2+W^2}$
  (fraction of proton's momentum carried by Pomeron)
- $\beta \equiv \frac{x_B}{x_{\mathbb{P}}} = \frac{Q^2}{Q^2+M_X^2}$ (fraction of Pomeron's momentum carried by struck quark)

Diffractive structure function $F_2^{\mathrm{D}(3)}$

- Diffractive cross section (integrated over t):

$$\frac{\mathrm{d}^3\sigma^{\mathrm{D}}}{\mathrm{d}x_{\mathbb{P}}\,\mathrm{d}\beta\,\mathrm{d}Q^2} = \frac{2\pi\alpha_{\mathrm{em}}^2}{\beta\,Q^4}\left[1+(1-y)^2\right]\,\sigma_r^{\mathrm{D}(3)}(x_{\mathbb{P}},\beta,Q^2),$$

  where $y = Q^2/(x_B s)$, $s = 4E_eE_p$, and

$$\sigma_r^{\mathrm{D}(3)} = F_2^{\mathrm{D}(3)} - \frac{y^2}{1+(1-y)^2}\,F_L^{\mathrm{D}(3)} \approx F_2^{\mathrm{D}(3)}(x_{\mathbb{P}},\beta,Q^2),$$

  for small y or assuming that $F_L^{\mathrm{D}(3)} \ll F_2^{\mathrm{D}(3)}$
- Measurements of $\sigma_r^{\mathrm{D}(3)} \Rightarrow$ *diffractive* parton distribution functions (DPDFs)
  $a^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2) = zq^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2)$ or $zg^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2)$,
  where $\beta \le z \le 1$, cf. $x_B \le x \le 1$ in DIS.

Recent measurements of DDIS using three methods

1. Detect leading proton. No proton dissociation background, but low statistics. Both Pomeron ($\mathbb{P}$) and secondary Reggeon ($\mathbb{R}$) contributions. [ZEUS: Eur. Phys. J. C **38** (2004) 43, `H1prelim-01-112`]
2. Look for Large Rapidity Gap (LRG). (Non-diffractive contribution is exponentially suppressed as a function of the gap size.) Proton dissociation background ($M_Y < 1.6$ GeV). Both $\mathbb{P}$ and $\mathbb{R}$ contributions. [`H1prelim-02-012`, `H1prelim-02-112`, `H1prelim-03-011`]
3. Use "M_X method". Subtract non-diffractive contribution in each (W, Q^2) bin by fitting (in a limited range of $\ln M_X^2$):

$$\frac{\mathrm{d}N}{\mathrm{d}\ln M_X^2} = D + \underbrace{c\exp(b\ln M_X^2)}_{\text{non-diffractive}}$$

   Proton dissociation background ($M_Y < 2.3$ GeV). Only $\mathbb{P}$ contribution since $\mathbb{R}$ contribution is subtracted as part of non-diffractive contribution. Comment on this method later. [ZEUS: Nucl. Phys. B **713** (2005) 3]

Imminent release of new H1 leading-proton and LRG data: see talk by P. Newman. New ZEUS leading-proton, LRG and M_X data in progress.

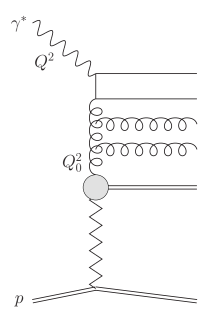
Collinear factorisation in DDIS

$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} + \mathcal{O}(1/Q), \tag{1}$$

where $C_{2,a}$ are the **same** coefficient functions as in inclusive DIS and where the DPDFs $a^{\mathrm{D}} = zq^{\mathrm{D}}$ or zg^{D} satisfy DGLAP evolution in Q^2:

$$\frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}} \tag{2}$$

"The factorisation theorem applies when Q is made large while x_B, $x_{\mathbb{P}}$, and t are held fixed." [Collins,'98]



- Says **nothing** about the mechanism for diffraction: information about the diffractive exchange ('Pomeron') needs to be parameterised at an input scale Q_0 and fit to data. Will show later that assuming a 'QCD Pomeron' we need to modify both (1) and (2).
- Factorisation should also hold for final states (jets etc.) in DDIS, but is broken in hadron–hadron collisions, although hope that same formalism can be applied with extra suppression factor calculable from eikonal models.
- **LO diffractive dijet photoproduction:** resolved photon contribution should be suppressed, but direct photon contribution unsuppressed. Complications at NLO [Klasen–Kramer,'05].

H1 2002 (prel.) extraction of DPDFs (ZEUS similar)

- Assume Regge factorisation [Ingelman–Schlein,'85]:

$$\boxed{a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, a^{\mathbb{P}}(z, Q^2)}$$

- Pomeron flux factor from Regge phenomenology:

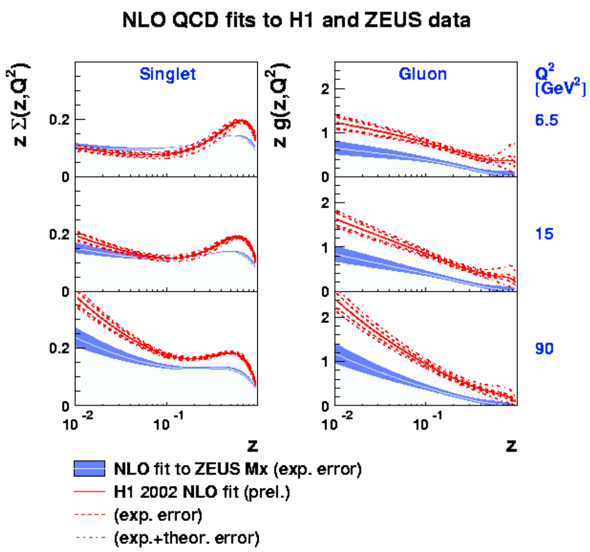
$$\boxed{f_{\mathbb{P}}(x_{\mathbb{P}}) = \int_{t_{\mathrm{cut}}}^{t_{\mathrm{min}}} \mathrm{d}t\; \mathrm{e}^{B_{\mathbb{P}}\, t}\; x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)}} \qquad (\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t)$$

"Regge factorisation relates the power of $x_{\mathbb{P}}$ measured in DDIS to the power of s measured in hadron–hadron elastic scattering." [Collins,'98]

- Fit to H1 LRG (prel.) data gives $\alpha_{\mathbb{P}}(0) = 1.17 > 1.08$, the value of the 'soft Pomeron' [Donnachie–Landshoff,'92]. By Collins' definition, Regge factorisation is broken. H1/ZEUS meaning of 'Regge factorisation' is that the $x_{\mathbb{P}}$ dependence factorises as a power law, with the power independent of β and Q^2 (also broken, see later).
- Pomeron PDFs $a^{\mathbb{P}}(z, Q^2) = z\Sigma^{\mathbb{P}}(z, Q^2)$ or $zg^{\mathbb{P}}(z, Q^2)$ are DGLAP-evolved from arbitrary inputs at $Q_0^2 = 3$ GeV^2:

$$a^{\mathbb{P}}(z, Q_0^2) = \left[A_a + B_a(2z-1) + C_a\left(2(2z-1)^2 - 1\right)\right]^2 \exp(-0.01/(1-z))$$

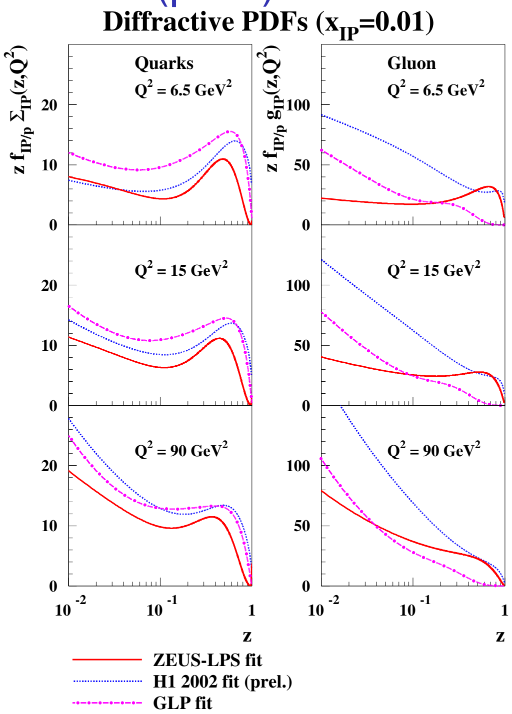
H1 LRG (prel.) vs. ZEUS M_X DPDFs



Fits and plot by F.-P. Schilling (H1). Presented at DIS 2005.

- Same procedure used to fit H1 LRG (prel.) and ZEUS M_X data. (ZEUS M_X data scaled by a constant factor to account for different amount of proton dissociation.)
- Gluon from ZEUS M_X fit $\sim$ factor two smaller than gluon from H1 LRG data, due to different Q^2 dependence of the data sets. Predictions from H1 2002 fit give good agreement with (LRG) DDIS dijet and D^* production data.
- N.B. 2-loop α_S fixed by $\Lambda_{\mathrm{QCD}} = 200$ MeV for 4 flavours. Gives α_S values much smaller than world average $\Rightarrow$ H1 2002 gluon artificially enhanced. Will be corrected for H1 publication.

H1 LRG (prel.) vs. ZEUS M_X vs. ZEUS LPS DPDFs



Plot by T. Tawara (ZEUS).

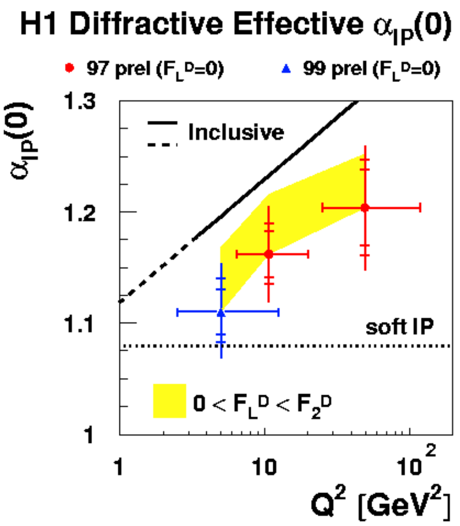
- No correction made for different amounts of proton dissociation.
- GLP = Groys–Levy–Proskuryakov (ZEUS) fit to ZEUS M_X data, gives much too low prediction for ZEUS (LRG) DDIS dijets.
- ZEUS LPS fit describes dijets well, but:

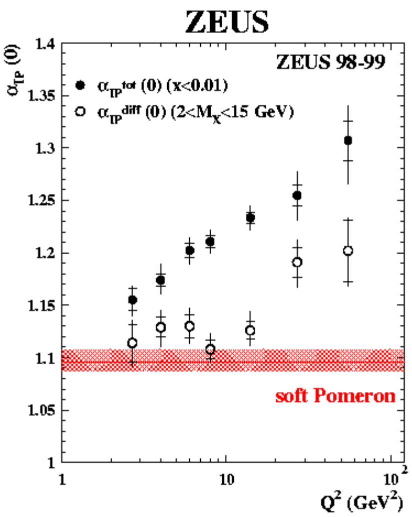
  "The shape of the fitted PDFs changes significantly depending on the functional form of the initial parameterisation, a consequence of the relatively large statistical uncertainties of the present sample. ***Therefore, these data cannot constrain the shapes of the PDFs."***

  [ZEUS: Eur. Phys. J. C **38** *(2004) 43]*

- See also talk by A. Bonato.

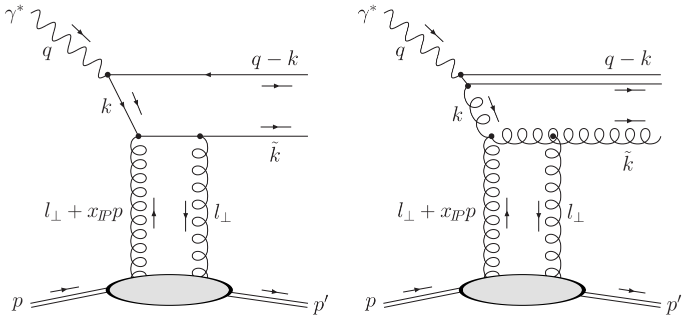
Q^2 dependence of effective Pomeron intercept





- Recall that 'Regge factorisation' fits assume that $\alpha_{\mathbb{P}}(0)$, which controls the $x_{\mathbb{P}}$ dependence, is independent of β and Q^2.
- $\alpha_{\mathbb{P}}(0)$ clearly rises with Q^2, but is smaller than in inclusive DIS, indicating that the $x_{\mathbb{P}}$ dependence is controlled by some scale $\mu^2 < Q^2$.
- $\alpha_{\mathbb{P}}(0) > 1.08$ [Donnachie–Landshoff,'92] indicating that the Pomeron in DDIS is **not** the 'soft' Pomeron exchanged in hadron–hadron collisions $\Rightarrow$ should use pQCD instead of Regge phenomenology. In pQCD, Pomeron exchange can be described by two-gluon exchange.

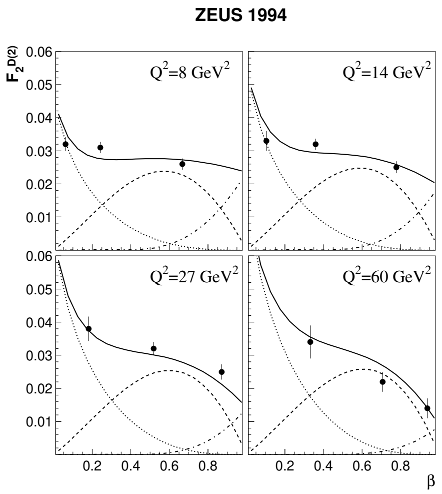
How to reconcile two-gluon exchange with DPDFs?



Two-gluon exchange calculations are the basis for the colour dipole model description of DDIS.

- Right: $x_{\mathbb{P}} F_2^{\mathrm{D}(3)}$ for $x_{\mathbb{P}} = 0.0042$ as a function of β [Golec-Biernat–Wüsthoff,'99].
  - dotted lines: $\gamma_T^* \to q\bar{q}g$,
  - dashed lines: $\gamma_T^* \to q\bar{q}$,
  - dot-dashed lines: $\gamma_L^* \to q\bar{q}$,

  important at low, medium, and high β respectively.
- $\gamma_L^* \to q\bar{q}$ is higher-twist, but DPDFs only include leading-twist contributions, therefore H1/ZEUS DPDFs are artificially large at high z.



Comparison of two approaches

‘Regge factorisation’ approach	Two-gluon exch. (e.g. dipole model)
• $\mathbb{P}$ is purely non-perturbative, i.e. a Regge pole.	• $\mathbb{P}$ is purely perturbative, i.e. a gluon ladder.
• Q^2 dependence given by DGLAP.	• Q^2 dependence predicted.
• Need to fit β dependence.	• β dependence predicted.
• $x_{\mathbb{P}}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.	• $x_{\mathbb{P}}$ dependence given by square of skewed gluon distribution (or dipole cross section).
• Only leading-twist.	• Goes beyond leading-twist.
• Full DGLAP evolution in Pomeron structure function.	• Only $q\bar{q}$ and $q\bar{q}g$ final states as products of photon dissociation.
• Extract universal DPDFs.	• No concept of DPDFs.
• $x_{\mathbb{P}}$ dependence factorises.	• $x_{\mathbb{P}}$ dependence doesn’t factorise.
• Only applies to inclusive DDIS.	• Also explains exclusive processes.

- In reality, both non-perturbative and perturbative Pomeron contributions to inclusive DDIS. Want to combine advantages of both approaches while eliminating the limitations. Improve two-gluon exchange calculations by introducing DGLAP evolution in ‘Pomeron structure function’ allowing universal DPDFs to be extracted.

Combination of two approaches

- Inclusive DDIS consists of both non-perturbative and perturbative Pomeron contributions.

Non-perturbative $\mathbb{P}$ contribution

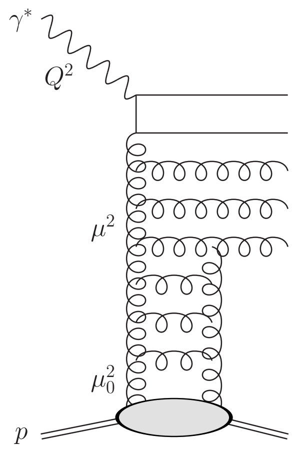
- $\mathbb{P}$ is ~~purely~~ partly non-perturbative, i.e. a Regge pole.
- Q^2 dependence given by DGLAP.
- Need to fit β dependence.
- $x_{\mathbb{P}}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.
- Only leading-twist.
- Full DGLAP evolution in Pomeron structure function.
- Extract universal DPDFs.
- $x_{\mathbb{P}}$ dependence factorises.
- Only applies to inclusive DDIS.

Perturbative $\mathbb{P}$ contribution

- $\mathbb{P}$ is ~~purely~~ partly perturbative, i.e. a gluon ladder.
- Q^2 dependence predicted.
- β dependence predicted.
- $x_{\mathbb{P}}$ dependence given by square of skewed gluon distribution ~~(or dipole cross section)~~.
- Goes beyond leading-twist.
- Full DGLAP evolution in Pomeron structure function.
- Extract universal DPDFs.
- $x_{\mathbb{P}}$ dependence doesn't factorise.
- Also explains exclusive processes.

The QCD Pomeron is a parton ladder

- Generalise $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ to arbitrary number of parton emissions [Ryskin,'90; Levin–Wüsthoff,'94].
- Work in Leading Logarithmic Approximation (LLA) $\Rightarrow$ virtualities of t-channel partons are strongly ordered: $\mu_0^2 \ll \ldots \ll \mu^2 \ll \ldots \ll Q^2$.
- **New feature:** integral over scale μ^2 (starting scale for DGLAP evolution of Pomeron PDFs).



$$F_2^{\mathrm{D}(3)} = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, F_2^{\mathbb{P}}(\beta,Q^2;\mu^2)$$

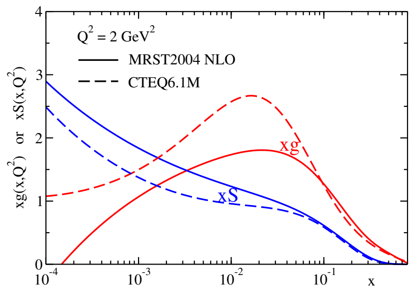
$$f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) = \frac{1}{x_{\mathbb{P}} B_D}\left[R_g \frac{\alpha_S(\mu^2)}{\mu}\, x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)\right]^2$$

$$F_2^{\mathbb{P}}(\beta,Q^2;\mu^2) = \sum_{a=q,g} C_{2,a} \otimes a^{\mathbb{P}}$$

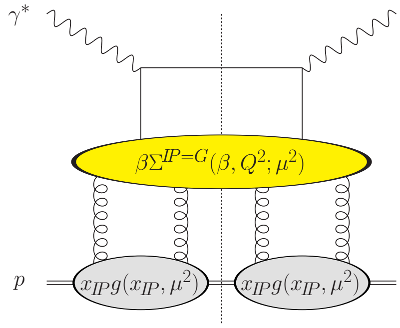
B_D from t-integration, R_g from skewedness [Shuvaev *et al.*,'99]

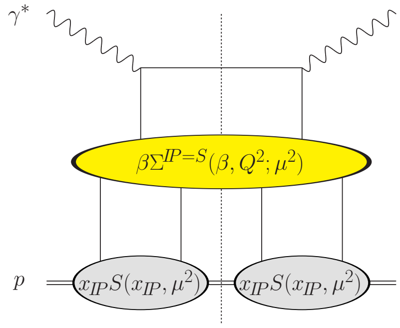
- Pomeron PDFs $a^{\mathbb{P}}(z,Q^2;\mu^2)$ DGLAP-evolved from an input scale μ^2 up to Q^2.
- For $\mu^2 < \mu_0^2 \sim 1$ GeV^2, replace lower parton ladder with usual Regge pole contribution. Take $\alpha_{\mathbb{P}}(0) \simeq 1.08$ (or fit) and fit Pomeron PDFs DGLAP-evolved from an input scale μ_0^2.

Gluonic and sea-quark Pomeron



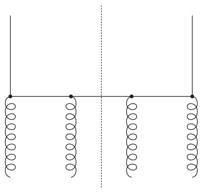
- At low scales, sea-quark density of the proton dominates over gluon density at small $x \Rightarrow$ need to account for sea-quark density in perturbative Pomeron flux factor.





- Pomeron structure function $F_2^{\mathbb{P}}(\beta, Q^2; \mu^2)$ calculated from quark singlet $\Sigma^{\mathbb{P}}(z, Q^2; \mu^2)$ and gluon $g^{\mathbb{P}}(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale μ^2 up to Q^2.
- Input Pomeron PDFs $\Sigma^{\mathbb{P}}(z, \mu^2; \mu^2)$ and $g^{\mathbb{P}}(z, \mu^2; \mu^2)$ are Pomeron-to-parton splitting functions.

LO Pomeron-to-parton splitting functions



- LO Pomeron-to-parton splitting functions calculated in Eur. Phys. J. C **44** (2005) 69.
- **Notation:** '$\mathbb{P} = G$' means gluonic Pomeron, '$\mathbb{P} = S$' means sea-quark Pomeron, '$\mathbb{P} = GS$' means interference between these.

$$
\begin{aligned}
z\Sigma^{\mathbb{P}=G}(z, \mu^2; \mu^2) &= P_{q,\mathbb{P}=G}(z) = z^3\,(1-z), \\
zg^{\mathbb{P}=G}(z, \mu^2; \mu^2) &= P_{g,\mathbb{P}=G}(z) = \frac{9}{16}\,(1+z)^2\,(1-z)^2, \\
z\Sigma^{\mathbb{P}=S}(z, \mu^2; \mu^2) &= P_{q,\mathbb{P}=S}(z) = \frac{4}{81}\,z\,(1-z), \\
zg^{\mathbb{P}=S}(z, \mu^2; \mu^2) &= P_{g,\mathbb{P}=S}(z) = \frac{1}{9}\,(1-z)^2, \\
z\Sigma^{\mathbb{P}=GS}(z, \mu^2; \mu^2) &= P_{q,\mathbb{P}=GS}(z) = \frac{2}{9}\,z^2\,(1-z), \\
zg^{\mathbb{P}=GS}(z, \mu^2; \mu^2) &= P_{g,\mathbb{P}=GS}(z) = \frac{1}{4}\,(1+2z)\,(1-z)^2
\end{aligned}
$$

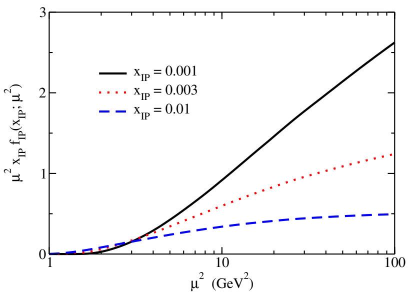
Evolve these input Pomeron PDFs from μ^2 up to Q^2 using NLO DGLAP evolution.

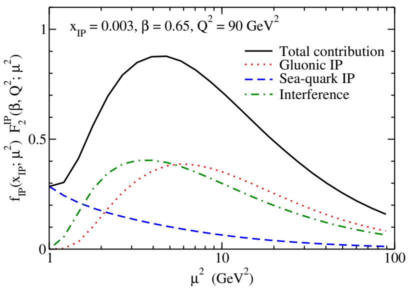
Contribution to $F_2^{\mathrm{D}(3)}$ as a function of μ^2

$$F_2^{\mathrm{D}(3)} = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\; f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\; F_2^{\mathbb{P}}(\beta, Q^2; \mu^2)$$

$$f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) = \frac{1}{x_{\mathbb{P}} B_D}\left[R_g \frac{\alpha_S(\mu^2)}{\mu}\; x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)\right]^2$$

- Naïvely, $f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sim 1/\mu^2$, so contributions from large μ^2 are strongly suppressed.
- But $x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2) \sim (\mu^2)^{\gamma}$, where γ is the anomalous dimension. In BFKL limit $\gamma \simeq 0.5$, so $f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sim$ constant.
- HERA domain is in an intermediate region: γ is not small, but is less than 0.5.
- Upper plot: $\mu^2 x_{\mathbb{P}} f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)$ is not flat for small $x_{\mathbb{P}}$. Lower plot: integrand as a function of μ^2 (using MRST2001 NLO PDFs) $\Rightarrow$ large contribution from large μ^2.
- Recall that fits using 'Regge factorisation' include contributions from $\mu^2 \leq Q_0^2$ in the input distributions, but neglect all contributions from $\mu^2 > Q_0^2$, where typically $Q_0^2 \approx 3$ GeV^2.





Inhomogeneous evolution of DPDFs

$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}},$$

$$\text{where } a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2) = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\; f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\; a^{\mathbb{P}}(z, Q^2; \mu^2)$$

$$\begin{aligned}
\Longrightarrow \frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} &= \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\; f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \frac{\partial a^{\mathbb{P}}}{\partial \ln Q^2} \;+\; f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, a^{\mathbb{P}}(z, Q^2; \mu^2)\Big|_{\mu^2 = Q^2} \\
&= \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\; f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathbb{P}} \;+\; f_{\mathbb{P}}(x_{\mathbb{P}};Q^2)\, a^{\mathbb{P}}(z, Q^2; Q^2) \\
&= \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} \;+\; \underbrace{f_{\mathbb{P}}(x_{\mathbb{P}};Q^2)\, P_{a\mathbb{P}}(z)}_{\text{Extra inhomogeneous term}}
\end{aligned}$$

Inhomogeneous evolution of DPDFs is **not a new idea**:

> *"We introduce a diffractive dissociation structure function and show that it obeys the DGLAP evolution equation, **but**, with an additional inhomogeneous term." [Levin–Wüsthoff,'94]*

Pomeron structure is analogous to photon structure

Photon structure function

$$F_2^{\gamma}(x_B, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\gamma}}_{\text{Resolved photon}} + \underbrace{C_{2,\gamma}}_{\text{Direct photon}}$$

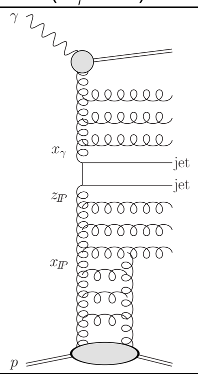
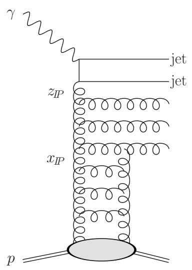
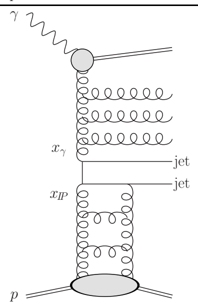
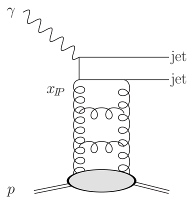
$$\text{where } \frac{\partial a^{\gamma}(x, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\gamma}}_{\text{DGLAP term}} + \underbrace{P_{a\gamma}(x)}_{\text{Inhomogeneous term}}$$

Diffractive structure function

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}}}_{\text{Resolved Pomeron}} + \underbrace{C_{2,\mathbb{P}}}_{\text{Direct Pomeron}}$$

$$\text{where } \frac{\partial a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} + \underbrace{P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)}_{\text{Inhomogeneous term}}$$

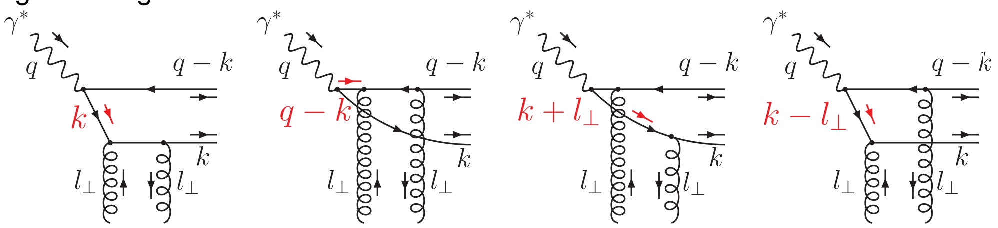
Dijets in diffractive photoproduction

	Resolved photon ($x_\gamma < 1$)	Direct photon ($x_\gamma = 1$)
Resolved Pomeron ($z_{\mathbb{P}} < 1$)		
Direct Pomeron ($z_{\mathbb{P}} = 1$)		

- Direct Pomeron contributions ($z_{\mathbb{P}} = 1$) are neglected in 'Regge factorisation' analyses.

Need for NLO calculations

- NLO analysis of DDIS data is not yet possible.
- Need $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$ at NLO. Should be calculable with usual methods, e.g. LO diagrams are:



Dimensional regularisation: work in $4-2\varepsilon$ dimensions, collinear singularity appears as $1/\varepsilon$ pole multiplied by $P_{q\mathbb{P}}$, subtract in e.g. $\overline{MS}$ factorisation scheme to leave finite remainder $C_{2,\mathbb{P}}$.

- Here, present **simplified analysis**: take NLO $C_{2,a}$ and $P_{aa'}$ ($a, a' = q, g$), but LO $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$.
    - Work in Fixed Flavour Number Scheme (no charm DPDF), with charm production via NLO $\gamma^* g^{\mathbb{P}} \to c\bar{c}$ [Riemersma *et al.*,'95] and LO $\gamma^*\mathbb{P} \to c\bar{c}$ [Levin–Martin–Ryskin–Teubner,'97].
    - For light quarks, include LO $\gamma_L^*\mathbb{P} \to q\bar{q}$ (higher-twist), but not $\gamma_T^*\mathbb{P} \longrightarrow q\bar{q}$. [The latter could alternatively be included using $C_{T,\mathbb{P}} = F_{T,q\bar{q}}^{\mathrm{D}(3)} - F_{T,q\bar{q}}^{\mathrm{D}(3)}\Big|_{\mu^2 \ll Q^2}$. This subtraction defines a choice of factorisation scheme.]

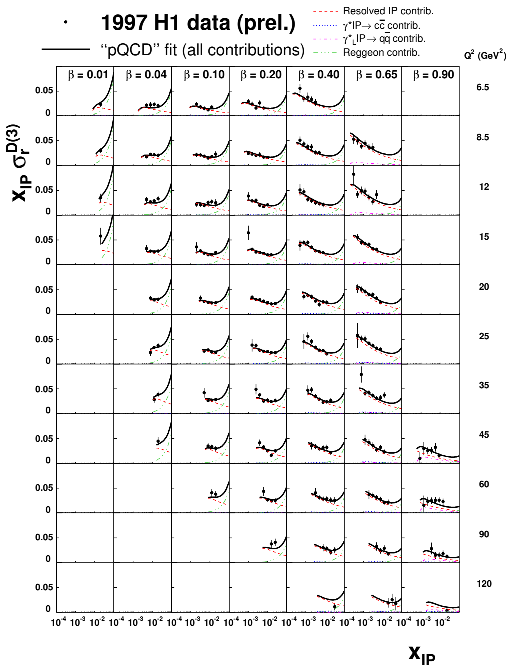
Description of DDIS data

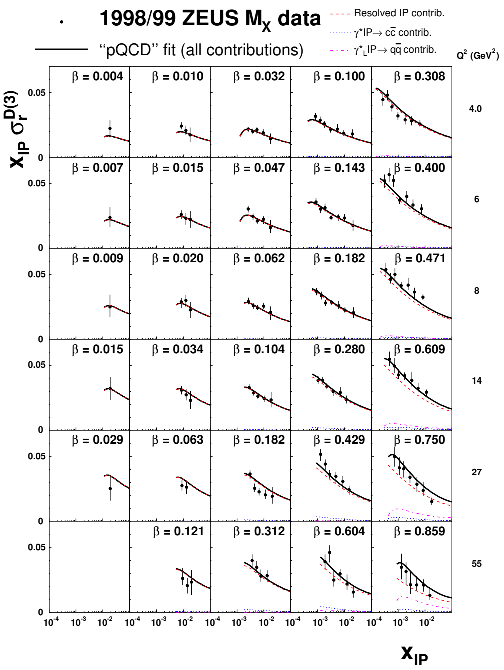
- Take input quark singlet and gluon densities at $Q_0^2 = 3$ GeV^2 in the form:

$$z\Sigma^{\mathrm{D}}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\; C_q\, z^{A_q}(1-z)^{B_q},$$
$$zg^{\mathrm{D}}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\; C_g\, z^{A_g}(1-z)^{B_g}.$$

- Take $f_{\mathbb{P}}(x_{\mathbb{P}})$ as in the H1 2002 (prel.) fit with $\alpha_{\mathbb{P}}(0)$, C_a, A_a, and B_a ($a = q, g$) as free parameters.
- Treatment of secondary Reggeon as in H1 2002 fit, i.e. using pion PDFs. (N.B. No good reason that the $\mathbb{R}$ PDFs should be same as pion PDFs.)
- Fit H1 LRG (prel.) and ZEUS M_X data separately with cuts $M_X > 2$ GeV and $Q^2 > 3$ GeV^2. Allow overall normalisation factors of 1.10 and 1.43 respectively to account for proton dissociation.
- Statistical and systematic experimental errors added in quadrature.
- Two types of fits:
    - **"Regge"** = 'Regge factorisation' approach (i.e. **no** $C_{2,\mathbb{P}}$ or $P_{a\mathbb{P}}$) as H1/ZEUS do.
    - **"pQCD"** = 'perturbative QCD' approach **with** LO $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$.

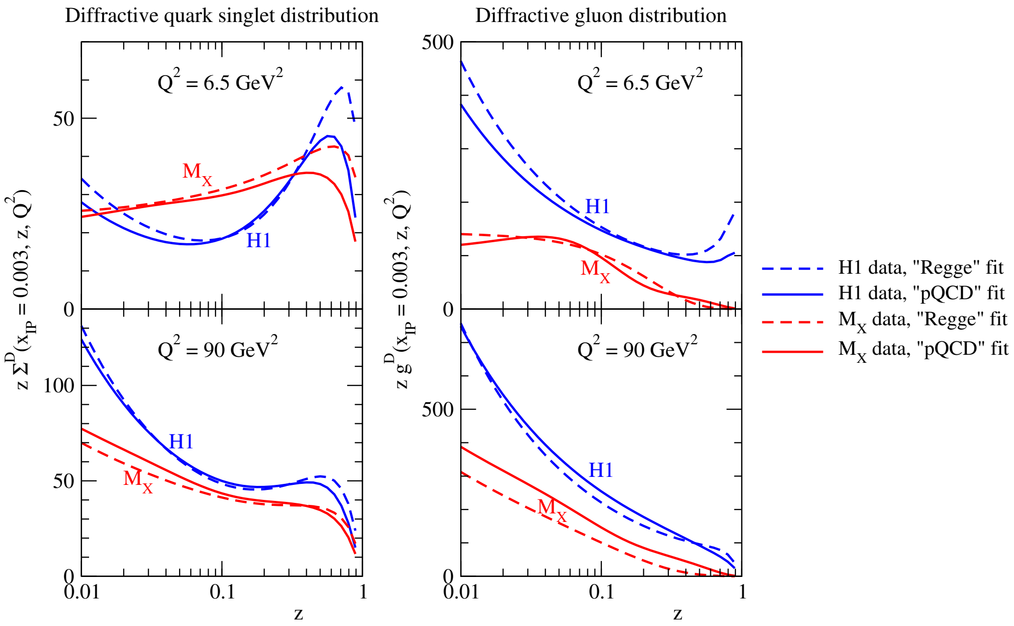
“pQCD” fits to H1 LRG (prel.) and ZEUS M_X data





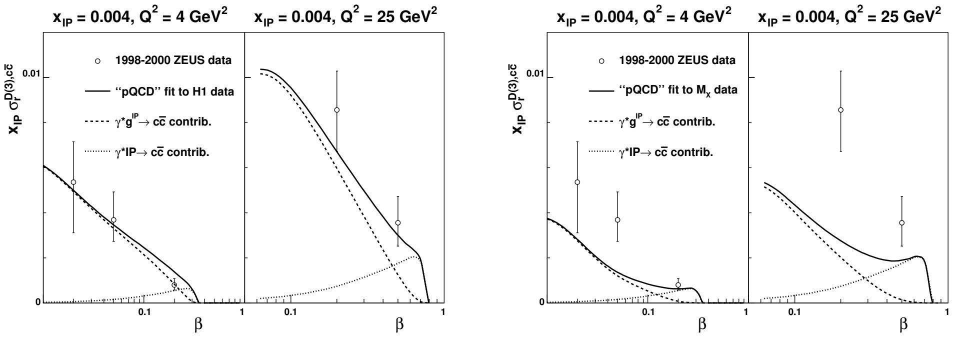
- $\chi^2/\mathrm{d.o.f.} = 0.71$ (0.75 for “Regge” fit)
- $\chi^2/\mathrm{d.o.f.} = 0.84$ (0.76 for “Regge” fit)

DPDFs from fits to H1 LRG (prel.) and ZEUS M_X data

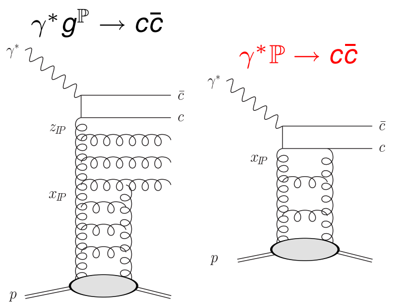


- “pQCD” DPDFs are smaller at large z due to inclusion of the higher-twist $\gamma_L^* \mathbb{P} \to q\bar{q}$.
- “pQCD” DPDFs have slightly more rapid evolution due to the inhomogeneous term.
- Difference between fits to H1 LRG (prel.) and ZEUS M_X data not resolved.

Predictions for diffractive charm production

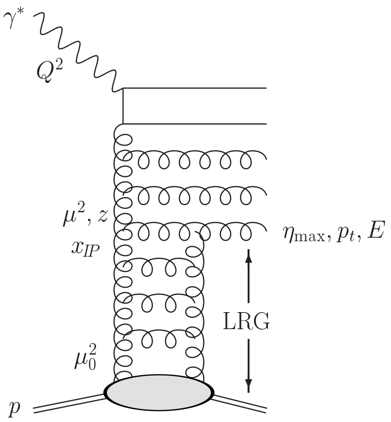


- ZEUS charm data measured using LRG method (as are all final-state DDIS data).
- "pQCD" DPDFs from H1 LRG data (left) give good description, those from ZEUS M_X data (right) too small at low β.



- Direct Pomeron contribution, i.e. $\gamma^* \mathbb{P} \to c\bar{c}$ ($z_{\mathbb{P}} = 1$), is significant at moderate/high β. These charm data points are included in the ZEUS LPS fit [ZEUS: Eur. Phys. J. C **38** (2004) 43], but only the $\gamma^* g^{\mathbb{P}} \to c\bar{c}$ contribution was included and not the $\gamma^* \mathbb{P} \to c\bar{c}$ contribution. Therefore, diffractive gluon from ZEUS LPS fit needed to be artificially large to fit the charm data.

Comment on the LRG method[1]



- **LRG method:** event selection using cut on maximum (pseudo)rapidity $\eta_{\max} < \eta_{\rm cut} = 3.2$ [H1prelim-01-112].
- Kinematics of $\mathbb{P}$ remnant:

$$E = p_t \cosh \eta_{\max} \simeq (1 - z) x_{\mathbb{P}} E_p$$

$$\boxed{\Rightarrow p_t > (1 - z) x_{\mathbb{P}} E_p \,\mathrm{sech}\, \eta_{\rm cut}.}$$

- Therefore, strong cut on $\eta_{\max}$ increases relative contribution to DDIS from perturbative Pomeron, i.e. large virtuality $\mu^2 \simeq p_t^2/(1-z) \gtrsim 1$ GeV^2.

- Originally discussed by J. Ellis and G. Ross [Phys. Lett. B **384** (1996) 293].
- In recent H1 measurements, effect of cut on $\eta_{\max}$ is compensated as part of acceptance corrections using RAPGAP event generator.
  - $\mathbb{P}$ remnant p_t [H1, Eur. Phys. J. C **20** (2001) 29] and $\eta_{\max}$ distributions are well-described by RAPGAP.
  - Good agreement of LRG data with leading-proton data.

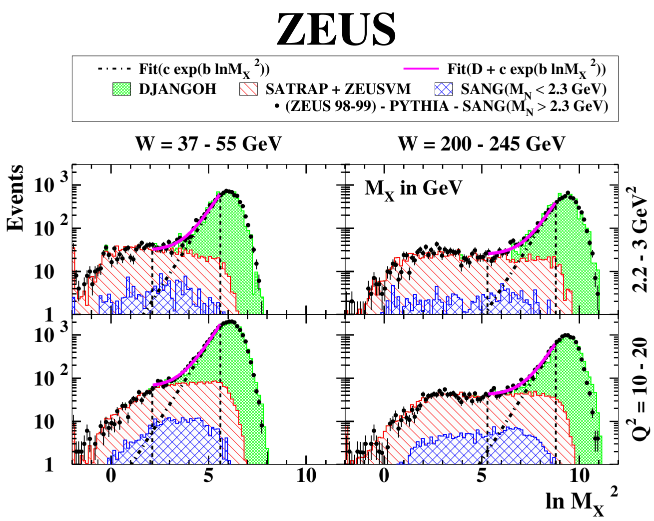
  Gives confidence that procedure is correct (although uncertainty due to acceptance correction for cut on $\eta_{\max}$ is dominant uncertainty at high $x_{\mathbb{P}}$).

[1] Thanks to M. Arneodo, H. Lim, and especially P. Newman for discussions.

Comment on the "M_X method"

- **Reminder:** The M_X method subtracts non-diffractive events in each (W,Q^2) bin by fitting (in a limited range of $\ln M_X^2$):

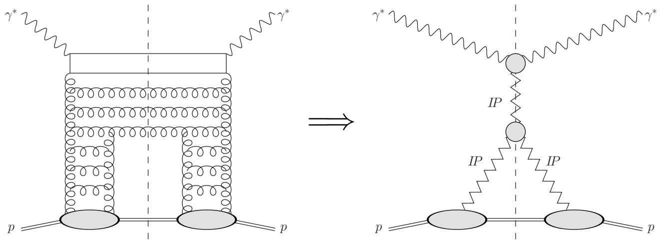
$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}\ln M_X^2} = D + \underbrace{c\exp(b\ln M_X^2)}_{\text{non-diffractive}}}$$



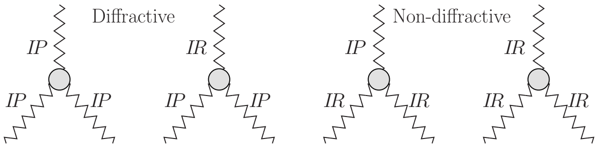
ZEUS: Nucl. Phys. B **713** (2005) 3

Regge theory derivation of the M_X method

- Replace pQCD ladders by "effective" Regge trajectories, e.g.



- For $W^2 \gg M_X^2$ and $Q^2 \gg t$, consider triple Regge diagrams:



$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln x_{\mathbb{P}}} = \frac{|g_{\mathbb{P}}(\bar{t})|^2}{16\pi^2}\, x_{\mathbb{P}}^{2-2\alpha_{\mathbb{P}}(\bar{t})} \left[\mathcal{A}_{\mathbb{PPP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{PPR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right]$$
$$+ \frac{|g_{\mathbb{R}}(\bar{t})|^2}{16\pi^2}\, x_{\mathbb{P}}^{2-2\alpha_{\mathbb{R}}(\bar{t})} \left[\mathcal{A}_{\mathbb{RRP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{RRR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right]$$
$$+ \mathbb{PRP} + \mathbb{RPP} + \mathbb{PRR} + \mathbb{RPR},$$

where $\alpha_{\mathbb{P}}(0) \approx 1.1$–$1.2$, $\alpha_{\mathbb{R}}(0) \lesssim 0.5$, and $\bar{t}$ is some average value of t.

Regge theory derivation of the M_X method

- Since $x_{\mathbb{P}} = (M_X^2 + Q^2)/(W^2 + Q^2)$ and $\beta = Q^2/(M_X^2 + Q^2)$, rewrite as

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln(M_X^2+Q^2)} =& A_{\mathbb{PPP}}(\bar{t})(W^2)^{2\alpha_{\mathbb{P}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{P}}(\bar{t})}\\ &+A_{\mathbb{PPR}}(\bar{t})(W^2)^{2\alpha_{\mathbb{P}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{P}}(\bar{t})}\\ &+A_{\mathbb{RRP}}(\bar{t})(W^2)^{2\alpha_{\mathbb{R}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{R}}(\bar{t})}\\ &+A_{\mathbb{RRR}}(\bar{t})(W^2)^{2\alpha_{\mathbb{R}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{R}}(\bar{t})}\\ &+\mathbb{PRP}+\mathbb{RPP}+\mathbb{PRR}+\mathbb{RPR}.\end{aligned}$$

- The M_X method neglects the possible interference terms and further assumes that $M_X^2 \gg Q^2$ ($\Rightarrow \beta \ll 1$, so $\mathbb{PPR}$ and $\mathbb{RRR}$ contributions are negligible), and $\alpha_{\mathbb{P}}(0) \approx \alpha_{\mathbb{P}}(\bar{t}) \approx 1$. Then

$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln M_X^2} = D + c(M_X^2)^b = D + c\exp(b\ln M_X^2),$$

where $b = 1 + \alpha_{\mathbb{P}}(0) - 2\alpha_{\mathbb{R}}(\bar{t})$.
- Therefore, the subtraction of non-diffractive events made in the M_X method is based on an over-simplified formula, which cannot be justified for $Q^2 \gtrsim M_X^2$. This *might* explain the different Q^2 dependence of the ZEUS M_X data observed w.r.t. the H1 LRG (prel.) data.

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Summary

- Diffractive DIS is more complicated to analyse than inclusive DIS.
- Need to include separate contributions from perturbative Pomeron (calculable) and non-perturbative Pomeron (need to fit to data).
- Collinear factorisation holds, but we need to account for the direct Pomeron coupling:

$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} + C_{2,\mathbb{P}}$$

$$\frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}} + P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)$$

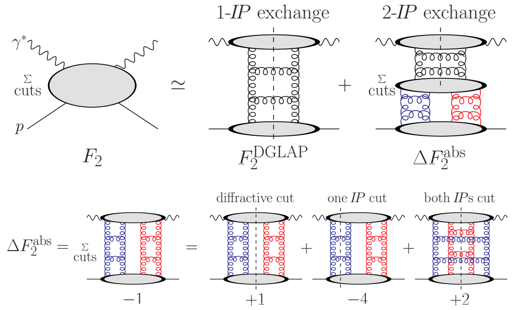
  Analogous to the photon case. Direct Pomeron contribution should also be included when calculating jet or heavy quark production.
- **Experimental methods:** leading-proton and LRG methods seem compatible, but M_X method not theoretically justified.
- Premature to make *precise* claims about factorisation breaking based on existing diffractive PDFs. Need to have good understanding of $\gamma^* p$ (HERA) before extending, in turn, to γp (HERA), $p\bar{p}$ (Tevatron) and pp (LHC).

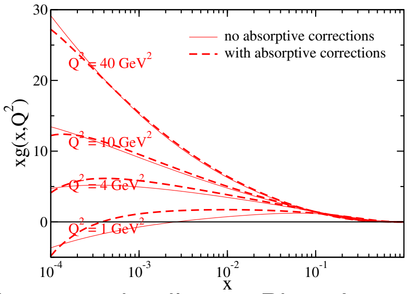
Outlook

- Need direct Pomeron terms ($C_{2,\mathbb{P}}$) and Pomeron-to-parton splitting functions ($P_{a\mathbb{P}}$, $a = q, g$) at NLO for a full NLO analysis of data. (Possibly large π^2-enhanced virtual loop corrections similar to those found in the Drell–Yan process.)
- Possible to extend formalism to secondary Reggeon component. Perturbative contribution would depend on the square of the (skewed) valence-quark distribution of the proton. (Should also consider possibility of Pomeron–Reggeon interference.)
- **Usual problems in any PDF determination:** need to study sensitivity to arbitrary choices made in fit, e.g. form of input parameterisation, starting scale Q_0, kinematic cuts on fitted data, heavy flavour treatment, α_S choice etc. This has not been done for diffractive PDFs in detail: new precise data will help reduce the uncertainties.
- Inclusion of jet and heavy quark diffractive DIS data, and possibly $F_L^{\mathrm{D}(3)}$ if it is measured, would help to constrain the diffractive PDFs further, but **only meaningful** if the correct theoretical framework is used (i.e. need to include direct Pomeron contributions).
- Revisit in more detail with improved calculations after the publication of the new H1 and ZEUS diffractive DIS data.

Appendix: Non-linear evolution of inclusive PDFs

$$\boxed{\frac{\partial a(x,Q^2)}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a' \; - \; \int_x^1 \mathrm{d}x_{\mathbb{P}}\; P_{a\mathbb{P}}(x/x_{\mathbb{P}})\, f_{\mathbb{P}}(x_{\mathbb{P}};Q^2).}$$





- Interesting application of DDIS formalism to calculate shadowing corrections to inclusive DIS via Abramovsky–Gribov–Kancheli (AGK) cutting rules.
- Inhomogeneous evolution of DPDFs ⇒ non-linear evolution of inclusive PDFs.
- More precise version of Gribov–Levin–Ryskin–Mueller–Qiu (GLRMQ) equation derived.
- Fit HERA F_2 data similar to MRST2001 NLO fit. Small-x gluon enhanced at low scales.

For more details see Phys. Lett. B **627** (2005) 97 (`hep-ph/0508093`).