Theoretical uncertainties in diffractive parton densities

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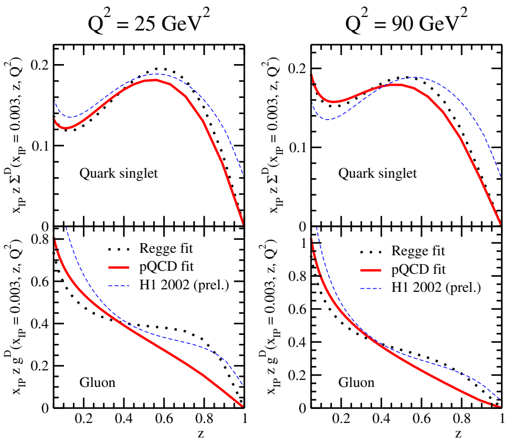
Outline

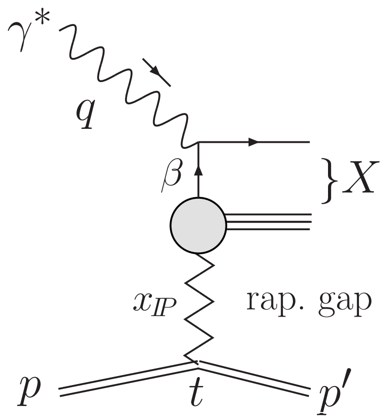
- Diffractive Deep-Inelastic Scattering (DDIS) is characterised by a Large Rapidity Gap (LRG) due to ‘Pomeron’ (vacuum quantum number) exchange.
- How do we extract Diffractive Parton Density Functions (DPDFs) from DDIS data?
- How ‘wrong’ are the H1 2006 DPDFs due to the oversimplified theory used in their fits?

For the impatient, here’s the answer (right), where:

- “Regge fit” $\simeq$ H1 2006 Fit A.
- “pQCD fit” is the subject of this talk.



Diffractive DIS kinematics



- $q^2 \equiv -Q^2$
- $W^2 \equiv (q+p)^2 = -Q^2 + 2\,p \cdot q$
  $\Rightarrow \quad x_B \equiv \frac{Q^2}{2\,p\cdot q} = \frac{Q^2}{Q^2+W^2}$ (fraction of proton's momentum carried by struck quark)
- $t \equiv (p-p')^2 \approx 0$, $(p-p') \approx x_{\mathbb{P}}\, p$
- $M_X^2 \equiv (q+p-p')^2 = -Q^2 + x_{\mathbb{P}}(Q^2+W^2)$
  $\Rightarrow \quad x_{\mathbb{P}} = \frac{Q^2+M_X^2}{Q^2+W^2}$
  (fraction of proton's momentum carried by Pomeron)
- $\beta \equiv \frac{x_B}{x_{\mathbb{P}}} = \frac{Q^2}{Q^2+M_X^2}$ (fraction of Pomeron's momentum carried by struck quark)

Diffractive reduced cross section $\sigma_r^{\mathrm{D}(3)}$

- Diffractive cross section (integrated over t):

$$\frac{\mathrm{d}^3\sigma^{\mathrm{D}}}{\mathrm{d}x_{\mathbb{P}}\,\mathrm{d}\beta\,\mathrm{d}Q^2} = \frac{2\pi\alpha_{\mathrm{em}}^2}{\beta\, Q^4}\left[1+(1-y)^2\right]\,\sigma_r^{\mathrm{D}(3)}(x_{\mathbb{P}},\beta,Q^2),$$

where $y = Q^2/(x_B s)$, $s = 4E_eE_p$, and

$$\sigma_r^{\mathrm{D}(3)} = F_2^{\mathrm{D}(3)} - \frac{y^2}{1+(1-y)^2}\,F_L^{\mathrm{D}(3)} \approx F_2^{\mathrm{D}(3)}(x_{\mathbb{P}},\beta,Q^2),$$

for small y or assuming that $F_L^{\mathrm{D}(3)} \ll F_2^{\mathrm{D}(3)}$

- Measurements of $\sigma_r^{\mathrm{D}(3)} \Rightarrow$ *diffractive* parton density functions (DPDFs)
  $a^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2) = zq^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2)$ or $zg^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2)$,
  where $\beta \le z \le 1$, cf. $x_B \le x \le 1$ in DIS.

Leading-twist collinear factorisation in DDIS

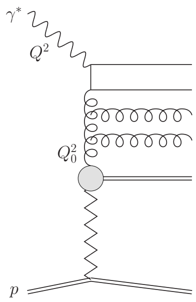
$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} + \mathcal{O}(1/Q), \qquad (1)$$

where $C_{2,a}$ are the **same** coefficient functions as in inclusive DIS and where the DPDFs $a^{\mathrm{D}} = zq^{\mathrm{D}}$ or zg^{D} satisfy DGLAP evolution in Q^2:

$$\frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}} \qquad (2)$$

"The factorisation theorem applies when Q is made large while x_B, $x_{\mathbb{P}}$, and t are held fixed." [Collins,'98]

- Says **nothing** about the mechanism for diffraction: information about the diffractive exchange ('Pomeron') needs to be parameterised at an input scale Q_0 and fit to data. Will show later that assuming a 'QCD Pomeron' we need to modify both (1) and (2).
- Factorisation should also hold for final states (jets etc.) in DDIS, but is broken in hadron–hadron collisions, although hope that same formalism can be applied with extra suppression factor calculable from eikonal models.
- **LO diffractive dijet photoproduction:** resolved photon contribution should be suppressed, but direct photon contribution unsuppressed. Complications at NLO [Klasen–Kramer,'05].



H1 2006 extraction of DPDFs

- Assume Regge factorisation [Ingelman–Schlein,'85]:

$$\boxed{a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, a^{\mathbb{P}}(z, Q^2)} \qquad (3)$$

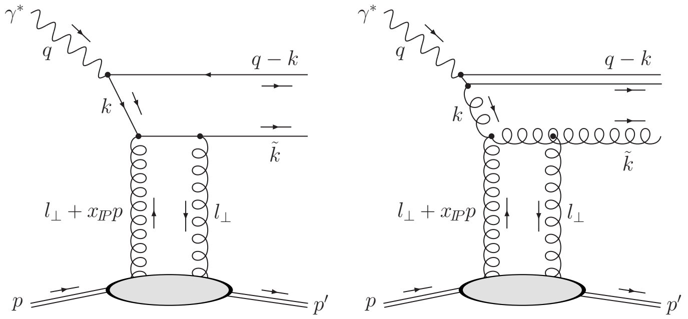
- Pomeron flux factor from Regge phenomenology:

$$\boxed{f_{\mathbb{P}}(x_{\mathbb{P}}) = \int_{t_{\mathrm{cut}}}^{t_{\mathrm{min}}} \mathrm{d}t\; \mathrm{e}^{B_{\mathbb{P}}\, t}\, x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)}} \qquad (\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t)$$

"Regge factorisation relates the power of $x_{\mathbb{P}}$ measured in DDIS to the power of s measured in hadron–hadron elastic scattering." [Collins,'98]

- Pomeron PDFs $a^{\mathbb{P}}(z, Q^2) = z\Sigma^{\mathbb{P}}(z, Q^2)$ or $zg^{\mathbb{P}}(z, Q^2)$ are DGLAP-evolved from arbitrary inputs at some scale Q_0^2, with the input parameters fitted to data.
- Fit to H1 FPS data gives $\alpha_{\mathbb{P}}(t) = 1.11 + 0.06\, t$. Fit to H1 LRG data gives $\alpha_{\mathbb{P}}(0) = 1.12$ if $\alpha'_{\mathbb{P}} = 0.06$, or $\alpha_{\mathbb{P}}(0) = 1.15$ if $\alpha'_{\mathbb{P}} = 0.25$. So the Pomeron in DDIS is **not** the universal 'soft Pomeron' [Donnachie–Landshoff,'92] with $\alpha_{\mathbb{P}}(t) = 1.08 + 0.25\, t$. By Collins' definition, Regge factorisation is broken. H1/ZEUS assume that the $x_{\mathbb{P}}$ dependence factorises as eq.(3) regardless, with the fitted $\alpha_{\mathbb{P}}(0)$ independent of β and Q^2 (also broken, see later).
- Breaking of Regge factorisation with $\alpha_{\mathbb{P}}(0) > 1.08$ suggests a significant perturbative QCD (pQCD) contribution to diffractive DIS. In pQCD, Pomeron exchange can be described by two-gluon exchange.

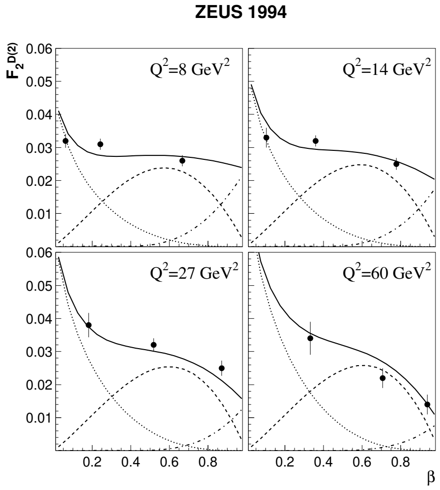
How to reconcile two-gluon exchange with DPDFs?



Two-gluon exchange calculations are the basis for the colour dipole model description of DDIS.

- Right: $x_{\mathbb{P}} F_2^{\mathrm{D}(3)}$ for $x_{\mathbb{P}} = 0.0042$ as a function of β [Golec-Biernat–Wüsthoff,'99].
  - dotted lines: $\gamma_T^* \to q\bar{q}g$,
  - dashed lines: $\gamma_T^* \to q\bar{q}$,
  - dot-dashed lines: $\gamma_L^* \to q\bar{q}$,

  important at low, medium, and high β respectively.
- $\gamma_T^* \to q\bar{q}g$ and $\gamma_T^* \to q\bar{q}$ are partly higher-twist, $\gamma_L^* \to q\bar{q}$ is **purely** higher-twist, but H1/ZEUS DPDFs only include leading-twist contributions.



Comparison of two approaches

‘Regge factorisation’ approach	Two-gluon exch. (e.g. dipole model)
$\mathbb{P}$ is purely non-perturbative, i.e. a Regge pole.	$\mathbb{P}$ is purely perturbative, i.e. a gluon ladder.
Q^2 dependence given by DGLAP.	Q^2 dependence predicted.
Need to fit β dependence.	β dependence predicted.
$x_{\mathbb{P}}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.	$x_{\mathbb{P}}$ dependence given by square of skewed gluon distribution (or dipole cross section).
Only leading-twist.	Goes beyond leading-twist.
Full DGLAP evolution in Pomeron structure function.	Only $q\bar{q}$ and $q\bar{q}g$ final states as products of photon dissociation.
Extract universal DPDFs.	No concept of DPDFs.
$x_{\mathbb{P}}$ dependence factorises.	$x_{\mathbb{P}}$ dependence doesn't factorise.
Only applies to inclusive DDIS.	Also explains exclusive processes.

- In reality, both non-perturbative and perturbative Pomeron contributions to inclusive DDIS. Want to combine advantages of both approaches while eliminating the limitations. Improve two-gluon exchange calculations by introducing DGLAP evolution in ‘Pomeron structure function’ allowing universal DPDFs to be extracted.

Combination of two approaches

- Inclusive DDIS consists of both non-perturbative and perturbative Pomeron contributions.

Non-perturbative $\mathbb{P}$ contribution

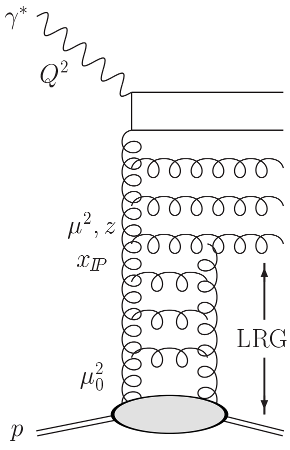
- $\mathbb{P}$ is ~~purely~~ partly non-perturbative, i.e. a Regge pole.
- Q^2 dependence given by DGLAP.
- Need to fit β dependence.
- $x_{\mathbb{P}}$ dependence taken as a power law, with the power either taken from soft hadron data or fitted.
- Only leading-twist.
- Full DGLAP evolution in Pomeron structure function.
- Extract universal DPDFs.
- $x_{\mathbb{P}}$ dependence factorises.
- Only applies to inclusive DDIS.

Perturbative $\mathbb{P}$ contribution

- $\mathbb{P}$ is ~~purely~~ partly perturbative, i.e. a gluon ladder.
- Q^2 dependence predicted.
- β dependence predicted.
- $x_{\mathbb{P}}$ dependence given by square of skewed gluon distribution ~~(or dipole cross section)~~.
- Goes beyond leading-twist.
- Full DGLAP evolution in Pomeron structure function.
- Extract universal DPDFs.
- $x_{\mathbb{P}}$ dependence doesn't factorise.
- Also explains exclusive processes.

The QCD Pomeron is a parton ladder

- Generalise $\gamma^* \to q\bar{q}$ and $\gamma^* \to q\bar{q}g$ to arbitrary number of parton emissions [Ryskin,'90; Levin–Wüsthoff,'94].
- Work in Leading Logarithmic Approximation (LLA) $\Rightarrow$ virtualities of t-channel partons are strongly ordered: $\mu_0^2 \ll \ldots \ll \mu^2 \ll \ldots \ll Q^2$, i.e. QCD Pomeron is a DGLAP ladder rather than a BFKL ladder.
- **New feature:** integral over scale μ^2 (starting scale for DGLAP evolution of Pomeron PDFs).



$$F_2^{\mathrm{D}(3)} = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, F_2^{\mathbb{P}}(\beta, Q^2; \mu^2)$$

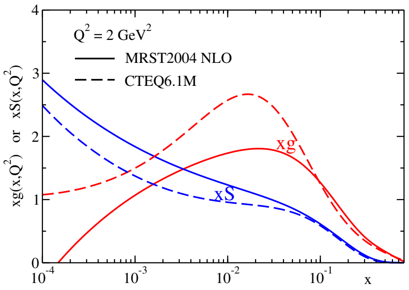
$$f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) = \frac{1}{x_{\mathbb{P}} B_D}\left[R_g \frac{\alpha_S(\mu^2)}{\mu}\, x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)\right]^2$$

$$F_2^{\mathbb{P}}(\beta, Q^2; \mu^2) = \sum_{a=q,g} C_{2,a} \otimes a^{\mathbb{P}}$$

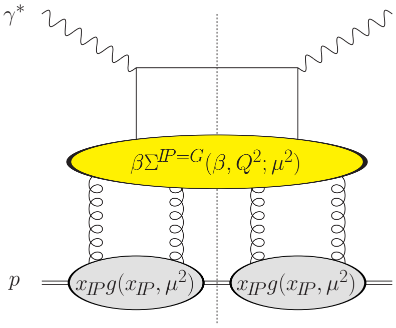
B_D from t-integration, R_g from skewedness [Shuvaev *et al.*,'99]

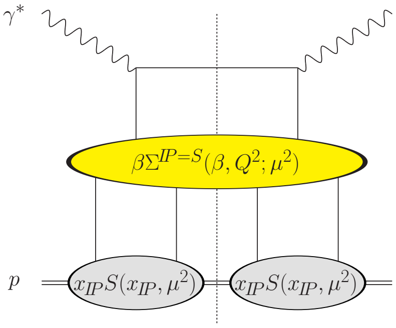
- Pomeron PDFs $a^{\mathbb{P}}(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale μ^2 up to Q^2.
- For $\mu^2 < \mu_0^2 \sim 1$ GeV^2, replace lower parton ladder with usual Regge pole contribution. Take $\alpha_{\mathbb{P}}(0) \simeq 1.08$ (or fit) and fit Pomeron PDFs DGLAP-evolved from an input scale μ_0^2.

Gluonic and sea-quark Pomeron



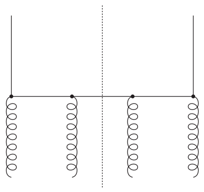
- At low scales, sea-quark density of the proton dominates over gluon density at small x $\Rightarrow$ need to account for sea-quark density in perturbative Pomeron flux factor.





- Pomeron structure function $F_2^{\mathbb{P}}(\beta, Q^2; \mu^2)$ calculated from quark singlet $\Sigma^{\mathbb{P}}(z, Q^2; \mu^2)$ and gluon $g^{\mathbb{P}}(z, Q^2; \mu^2)$ DGLAP-evolved from an input scale μ^2 up to Q^2.
- Input Pomeron PDFs $\Sigma^{\mathbb{P}}(z, \mu^2; \mu^2)$ and $g^{\mathbb{P}}(z, \mu^2; \mu^2)$ to DGLAP evolution are Pomeron-to-parton splitting functions.

LO Pomeron-to-parton splitting functions



- LO Pomeron-to-parton splitting functions calculated in Eur. Phys. J. C **44** (2005) 69.
- **Notation:** '$\mathbb{P} = G$' means gluonic Pomeron, '$\mathbb{P} = S$' means sea-quark Pomeron, '$\mathbb{P} = GS$' means interference between these.

$$
\begin{aligned}
z\Sigma^{\mathbb{P}=G}(z, \mu^2; \mu^2) &= P_{q,\mathbb{P}=G}(z) = z^3\,(1-z),\\
zg^{\mathbb{P}=G}(z, \mu^2; \mu^2) &= P_{g,\mathbb{P}=G}(z) = \frac{9}{16}\,(1+z)^2\,(1-z)^2,\\
z\Sigma^{\mathbb{P}=S}(z, \mu^2; \mu^2) &= P_{q,\mathbb{P}=S}(z) = \frac{4}{81}\,z\,(1-z),\\
zg^{\mathbb{P}=S}(z, \mu^2; \mu^2) &= P_{g,\mathbb{P}=S}(z) = \frac{1}{9}\,(1-z)^2,\\
z\Sigma^{\mathbb{P}=GS}(z, \mu^2; \mu^2) &= P_{q,\mathbb{P}=GS}(z) = \frac{2}{9}\,z^2\,(1-z),\\
zg^{\mathbb{P}=GS}(z, \mu^2; \mu^2) &= P_{g,\mathbb{P}=GS}(z) = \frac{1}{4}\,(1+2z)\,(1-z)^2
\end{aligned}
$$

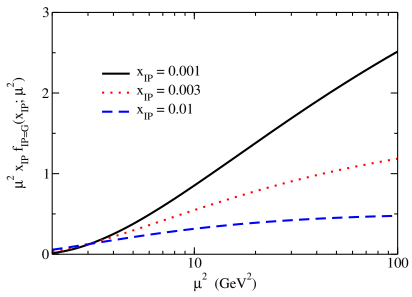
Evolve these input Pomeron PDFs from μ^2 up to Q^2 using NLO DGLAP evolution.

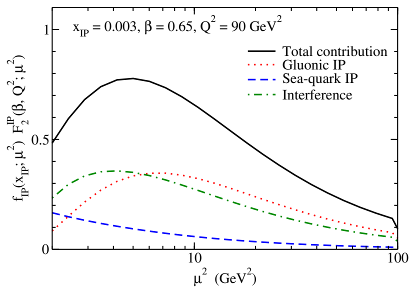
Contribution to $F_2^{\mathrm{D}(3)}$ as a function of μ^2

$$F_2^{\mathrm{D}(3)} = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, F_2^{\mathbb{P}}(\beta,Q^2;\mu^2)$$

$$f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) = \frac{1}{x_{\mathbb{P}} B_D}\left[R_g \frac{\alpha_S(\mu^2)}{\mu}\, x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2)\right]^2$$

- Naïvely, $f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sim 1/\mu^2$, so contributions from large μ^2 are strongly suppressed.
- But $x_{\mathbb{P}} g(x_{\mathbb{P}},\mu^2) \sim (\mu^2)^\gamma$, where γ is the anomalous dimension. In BFKL limit $\gamma \simeq 0.5$, so $f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sim$ constant.
- HERA domain is in an intermediate region: γ is not small, but is less than 0.5.
- Upper plot: $\mu^2 x_{\mathbb{P}} f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)$ is not flat for small $x_{\mathbb{P}}$. Lower plot: integrand as a function of μ^2 (using MRST2004F3 NLO PDFs) $\Rightarrow$ large contribution from large μ^2.
- Recall that fits using 'Regge factorisation' include contributions from $\mu^2 \le Q_0^2$ in the input distributions, but neglect all contributions from $\mu^2 > Q_0^2$.





Inhomogeneous evolution of DPDFs

$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}},$$

$$\text{where } a^{\mathrm{D}}(x_{\mathbb{P}},z,Q^2) = \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, a^{\mathbb{P}}(z,Q^2;\mu^2)$$

$$\begin{aligned}
\Longrightarrow \frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} &= \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \frac{\partial a^{\mathbb{P}}}{\partial \ln Q^2} + f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2)\, a^{\mathbb{P}}(z,Q^2;\mu^2)\Big|_{\mu^2=Q^2} \\
&= \int_{\mu_0^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}}(x_{\mathbb{P}};\mu^2) \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathbb{P}} + f_{\mathbb{P}}(x_{\mathbb{P}};Q^2)\, a^{\mathbb{P}}(z,Q^2;Q^2) \\
&= \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} + \underbrace{f_{\mathbb{P}}(x_{\mathbb{P}};Q^2)\, P_{a\mathbb{P}}(z)}_{\text{Extra inhomogeneous term}}
\end{aligned}$$

Inhomogeneous evolution of DPDFs is **not a new idea**:

> *"We introduce a diffractive dissociation structure function and show that it obeys the DGLAP evolution equation, **but**, with an additional inhomogeneous term." [Levin–Wüsthoff,'94]*

Pomeron structure is analogous to photon structure

Photon structure function

$$F_2^{\gamma}(x_B, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\gamma}}_{\text{Resolved photon}} + \underbrace{C_{2,\gamma}}_{\text{Direct photon}}$$

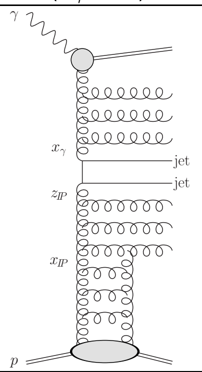
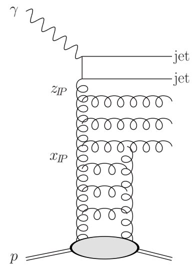
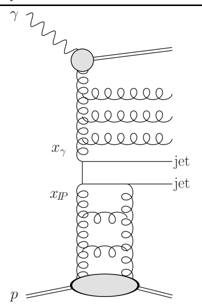
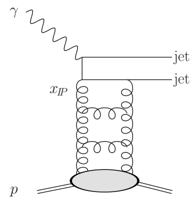
$$\text{where } \frac{\partial a^{\gamma}(x, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\gamma}}_{\text{DGLAP term}} + \underbrace{P_{a\gamma}(x)}_{\text{Inhomogeneous term}}$$

Diffractive structure function

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}}}_{\text{Resolved Pomeron}} + \underbrace{C_{2,\mathbb{P}}}_{\text{Direct Pomeron}}$$

$$\text{where } \frac{\partial a^{\mathrm{D}}(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}}}_{\text{DGLAP term}} + \underbrace{P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)}_{\text{Inhomogeneous term}}$$

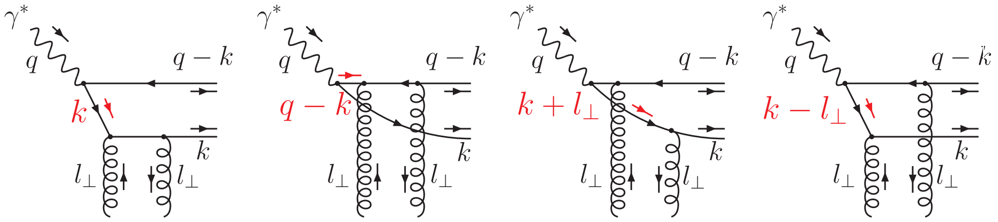
Dijets in diffractive photoproduction

	Resolved photon ($x_\gamma < 1$)	Direct photon ($x_\gamma = 1$)
Resolved Pomeron ($z_{\mathbb{P}} < 1$)		
Direct Pomeron ($z_{\mathbb{P}} = 1$)		

- Direct Pomeron contributions ($z_{\mathbb{P}} = 1$) are neglected in 'Regge factorisation' analyses.

Need for NLO calculations

- NLO analysis of DDIS data is not yet possible.
- Need $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$ at NLO. Should be calculable with usual methods, e.g. LO diagrams are:



  Dimensional regularisation: work in $4-2\varepsilon$ dimensions, collinear singularity appears as $1/\varepsilon$ pole multiplied by $P_{q\mathbb{P}}$, subtract in e.g. $\overline{MS}$ factorisation scheme to leave finite remainder $C_{2,\mathbb{P}}$.
- Here, take NLO $C_{2,a}$ and $P_{aa'}$ $(a, a' = q, g)$, but LO $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$.
  - Work in Fixed Flavour Number Scheme (no charm DPDF), with charm production via NLO $\gamma^* g^{\mathbb{P}} \to c\bar{c}$ [Riemersma *et al.*,'95] and LO $\gamma^*\mathbb{P} \to c\bar{c}$ [Levin–Martin–Ryskin–Teubner,'97].
  - For light quarks, include LO $\gamma_L^*\mathbb{P} \to q\bar{q}$ (higher-twist), with LO $\gamma_T^*\mathbb{P} \to q\bar{q}$ contribution given by $C_{T,\mathbb{P}} = F_{T,q\bar{q}}^{\mathrm{D}(3)} - F_{T,q\bar{q}}^{\mathrm{D}(3)}\Big|_{\mu^2 \ll Q^2}$. This subtraction defines a choice of factorisation scheme.

Analysis of H1 LRG data (`hep-ex/0606004`)

- Take input quark singlet and gluon densities at $Q_0^2 = 2$ GeV^2 in the form:

$$z\Sigma^{\mathrm{D}}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, A_q\, z^{B_q}(1-z)^{C_q},$$
$$zg^{\mathrm{D}}(x_{\mathbb{P}}, z, Q_0^2) = f_{\mathbb{P}}(x_{\mathbb{P}})\, A_g\, z^{B_g}(1-z)^{C_g}.$$

- Take $f_{\mathbb{P}}(x_{\mathbb{P}})$ as in the H1 2006 fit with $\alpha_{\mathbb{P}}(0)$, A_a, B_a, and C_a $(a = q, g)$ as free parameters.
- Treatment of secondary Reggeon as in H1 2006 fit, i.e. using pion PDFs, but using GRV NLO instead of Owens LO. (N.B. No good reason that the $\mathbb{R}$ PDFs should be same as pion PDFs.)
- Fit H1 LRG data binned at fixed $x_{\mathbb{P}}$ values with cut $M_X \geq 2$ GeV. Will study effect of cut $Q^2 \geq Q^2_{\min}$ on fitted data.
- Statistical and systematic experimental errors added in quadrature. (**Caveat:** underestimates numerical values of χ^2, but central DPDFs obtained should be very close to those obtained treating correlated systematic errors separately.)
- Two types of fits:
  - **"Regge"** = 'Regge factorisation' approach (i.e. **no** $C_{2,\mathbb{P}}$ or $P_{a\mathbb{P}}$) $\simeq$ H1 2006 Fit A.
  - **"pQCD"** = 'perturbative QCD' approach **with** LO $C_{2,\mathbb{P}}$ and $P_{a\mathbb{P}}$.
- Use MRST2004F3 NLO PDFs with $\Lambda_{\mathrm{QCD}}^{(n_f=3)} = 407$ MeV.

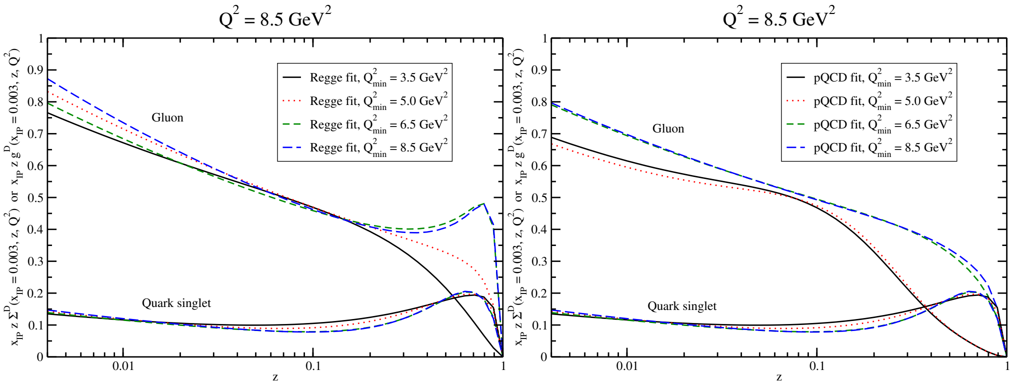
Stability with respect to $Q^2_{\rm min}$ variation

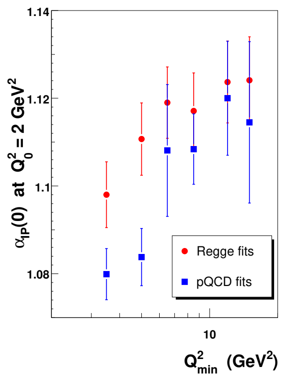
- Stability analysis following MRST [EPJC 35 (2004) 325].

$Q^2_{\rm min}$ (GeV^2)	3.5	5.0	6.5	8.5	12	15
Number of data points	266	239	214	190	164	141
$\chi^2(Q^2 \geq 3.5$ GeV$^2)$	272					
	264					
$\chi^2(Q^2 \geq 5$ GeV$^2)$	233	222				
	227	223				
$\chi^2(Q^2 \geq 6.5$ GeV$^2)$	208	186	174			
	208	201	186			
$\chi^2(Q^2 \geq 8.5$ GeV$^2)$	178	155	144	142		
	182	172	153	150		
$\chi^2(Q^2 \geq 12$ GeV$^2)$	156	136	124	123	122	
	162	153	135	132	131	
$\chi^2(Q^2 \geq 15$ GeV$^2)$	133	111	100	98	97	96
	138	128	109	104	102	101
Stability measure Δ_i^{i+1}	0.41	0.48	0.08	0.04	0.04	
	0.15	0.60	0.13	0.04	0.04	

- Both Regge and pQCD fits stable for $Q^2_{\rm min} \gtrsim 6.5$ GeV^2. To compare directly with H1 2006 fits, take $Q^2_{\rm min} = 8.5$ GeV^2 for default fits (conservative choice).

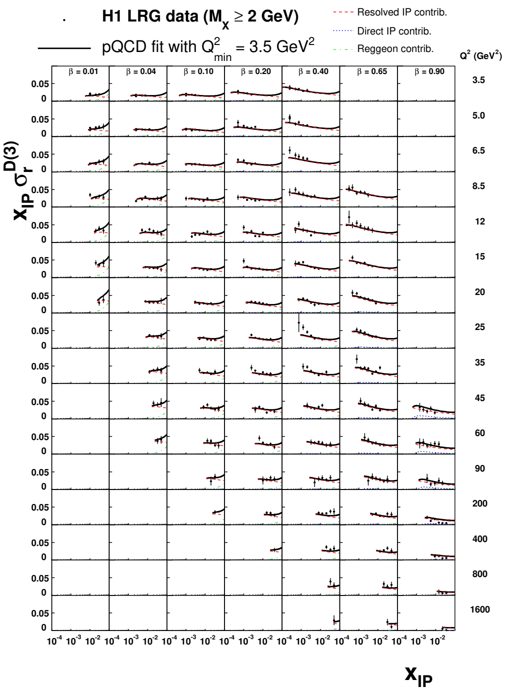
DPDF and $\alpha_{\mathbb{P}}(0)$ dependence on $Q^2_{\rm min}$



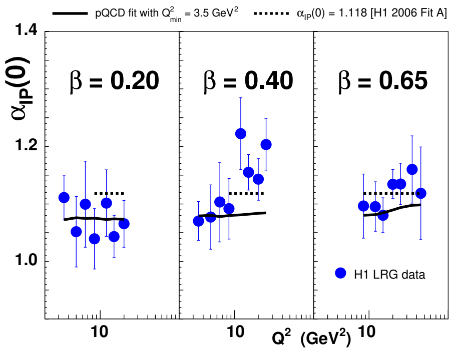


- Again, stability for $Q^2_{\rm min} \gtrsim 6.5$ GeV^2.
- Required $\alpha_{\mathbb{P}}(0)$ at $Q_0^2 = 2$ GeV^2 for pQCD fits is lower than for Regge fits.

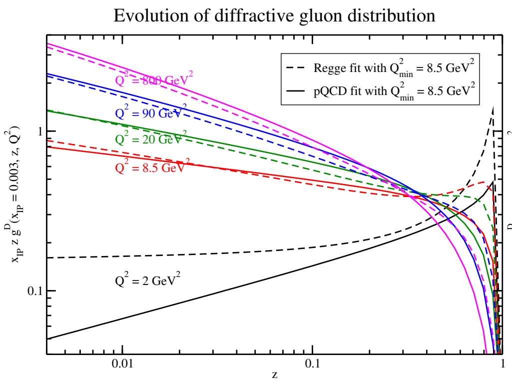
$x_{\mathbb{P}}$ dependence of H1 LRG data

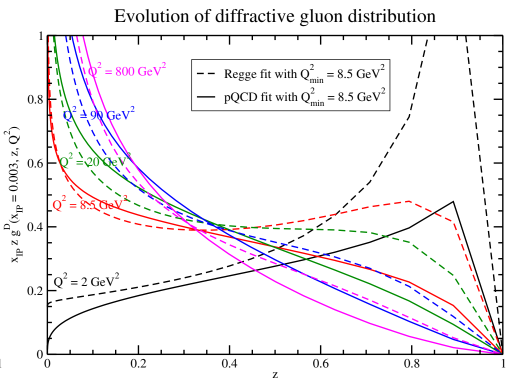


- Fit $\sigma_r^{D(3)} \propto f_{\mathbb{P}}(x_{\mathbb{P}})$ in each (β, Q^2) bin containing four or more data points with $x_{\mathbb{P}} \leq 0.01$, $y \leq 0.45$ and $M_X \geq 2$ GeV.
- For $\beta = 0.40$ and $\beta = 0.65$, clear rise with Q^2 of effective $\alpha_{\mathbb{P}}(0) \Rightarrow$ $x_{\mathbb{P}}$-factorisation broken.
- Inhomogeneous term depends on $x_{\mathbb{P}}$ and therefore $x_{\mathbb{P}}$-factorisation is broken when evolving upwards from Q_0^2 to Q^2 (but seems small effect).



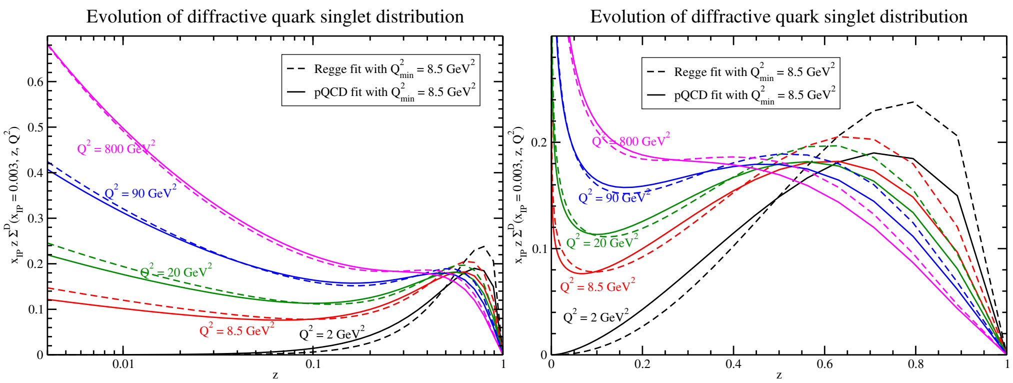
Evolution of diffractive gluon distribution





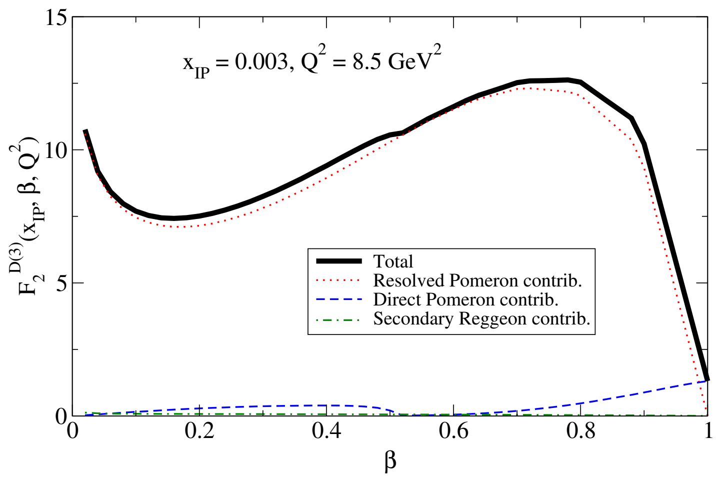
- Extra inhomogeneous term in evolution equation means gluon from pQCD fit needs to be smaller at input scale.

Evolution of diffractive quark singlet distribution



- Quark singlet distribution at input scale is larger at low z in pQCD fit and smaller at large z.

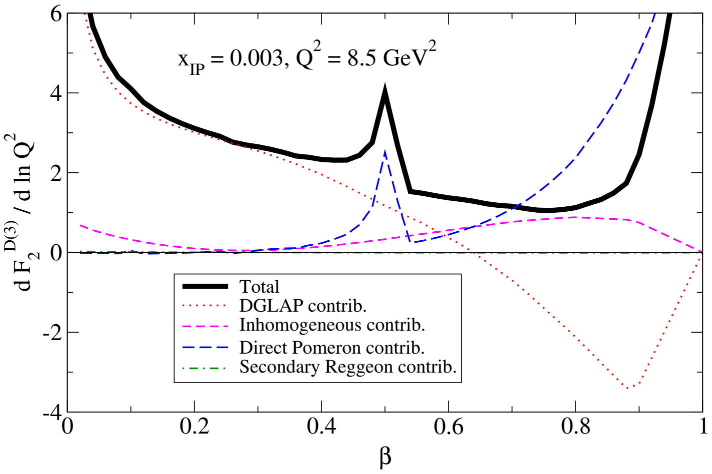
β dependence of $F_2^{D(3)}$



- Direct Pomeron contribution only important for $\beta \gtrsim 0.9$.

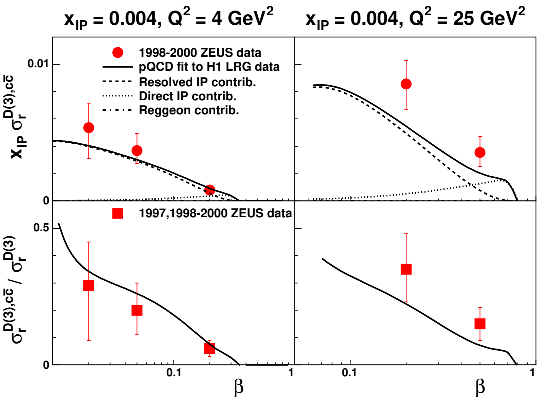
β dependence of $\partial F_2^{\mathrm{D}(3)} / \partial \ln Q^2$

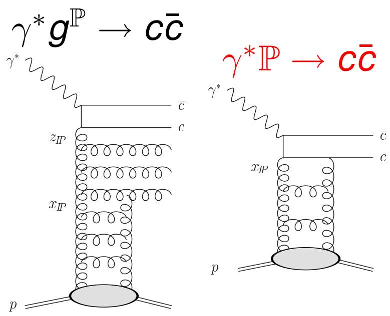
At LO, $\frac{\partial F_2^{\mathrm{D}(3)}}{\partial \ln Q^2} = \sum_q e_q^2 \left(\sum_{a'=q,g} P_{qa'} \otimes a'^{\mathrm{D}} + P_{a\mathbb{P}} f_{\mathbb{P}} \right) + (\text{Direct } \mathbb{P}) + \mathbb{R}.$



- Peak due to threshold for $\gamma^* \mathbb{P} \to c\bar{c}$ at $\beta = Q^2/(Q^2 + 4m_c^2)$.
- Additional contributions to scaling violations apart from DGLAP contribution, important for $\beta \gtrsim 0.3$.

Predictions for diffractive charm production





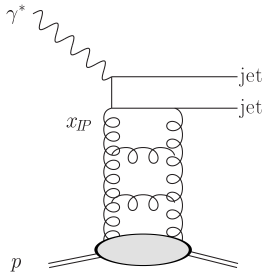
- Direct Pomeron contribution, i.e. $\gamma^* \mathbb{P} \to c\bar{c}$ ($z_{\mathbb{P}} = 1$), is significant at moderate/high β.
- These charm data points are included in the ZEUS LPS fit [ZEUS: Eur. Phys. J. C **38** (2004) 43], but only the $\gamma^* g^{\mathbb{P}} \to c\bar{c}$ contribution was included and not the $\gamma^* \mathbb{P} \to c\bar{c}$ contribution. Therefore, diffractive gluon from ZEUS LPS fit needed to be artificially large to fit the charm data.
- H1 also neglect the $\gamma^* \mathbb{P} \to c\bar{c}$ contribution (see talk by R. Wolf).

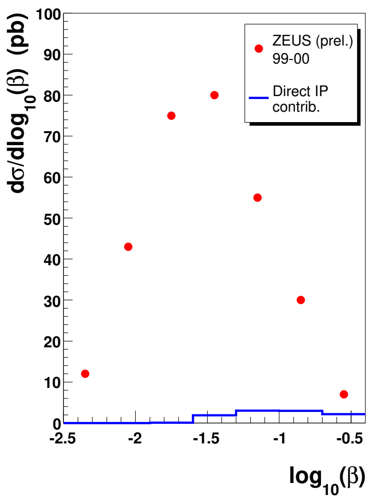
Summary of the Diffractive Working Group at DIS98 (hep-ph/9806485)

> *"From the theoretical point of view one also should take into account that the presently available Monte Carlo models are assuming an **illegitimate Regge factorisation**, in which hard scale dependencies on $x_{\mathbb{P}}$ and β as found in theoretical QCD analyses, and which characterise the final state, are neglected. For instance, one treats the charm production as **entirely due to the familiar photon–gluon fusion, neglecting the direct charm–anticharm excitation** which some theorists claim to be substantial. In this approximation, in order to reproduce the diffractive charm signal one needs a hard glue in the Pomeron fits. Therefore the conclusions drawn from these Monte Carlo studies as to the physical picture underlying the diffractive final states should be handled with care."*

No progress in theory used by H1/ZEUS in 8 years?

Direct Pomeron contribution to dijet production

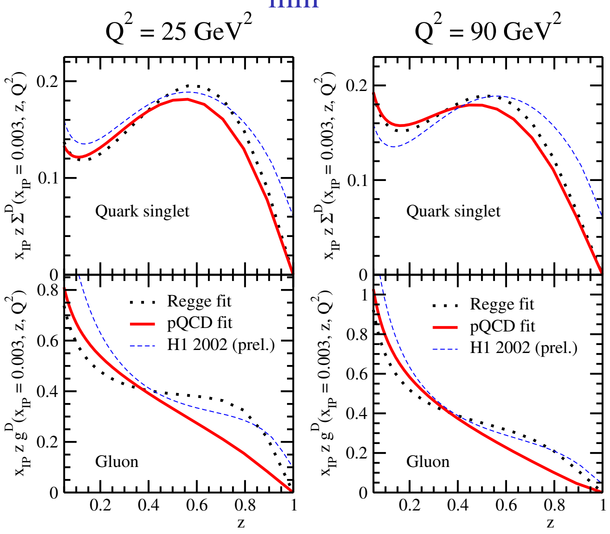


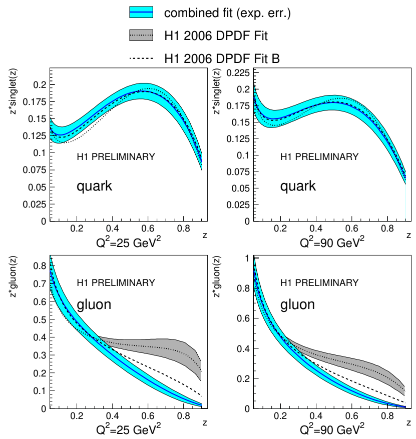


- Direct Pomeron contribution ($z_{\mathbb{P}} = 1$) [EPJC **44** (2005) 69] calculated with ZEUS (prel.) kinematic cuts (see talk by A. Bonato): 31% of data in largest β bin.
- Alternative calculations for exclusive dijets by Braun and Ivanov [PRD **72** (2005) 034016].
- H1 combined fit is to dijet data with $z_{\mathbb{P}} < 0.9$ integrated over β. Therefore, can neglect direct Pomeron contribution and include only the resolved Pomeron contribution using NLOJET++.
- Aside: inconsistency in heavy quark treatment. H1 2006 fit is done in FFNS with **massive** heavy quark contributions, but jet coefficient functions used in programs like NLOJET++ and DISENT[a] are computed for **massless** partons.

[a] Note that DISENT is known to have a small bug at the 1–2% level [Z. Trócsányi, hep-ph/0512004].

DPDFs with $Q^2_{\text{min}} = 8.5$ GeV^2 compared to H1 DPDFs



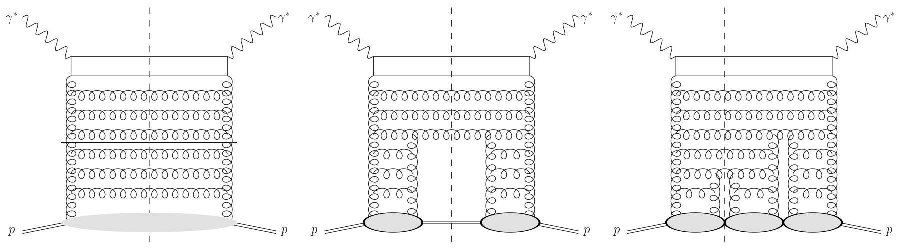


- Regge fit $\simeq$ H1 2006 Fit A. pQCD fit closer to H1 combined fit than H1 2006 Fit A **without** including jet data $\Rightarrow$ will describe dijet data better than H1 2006 Fit A.
- H1 combined fit determines gluon **directly** from dijet data, whereas fits only to inclusive DDIS data determine gluon only **indirectly** so more sensitive to details of evolution, i.e. better test of theory used. Including dijet data in the fit is **not** necessarily a good thing if the theory is unreliable.
- H1 χ^2 for 190 inclusive DDIS points is 158 (H1 Fit A), 164 (H1 Fit B), 169 (H1 combined fit), so some tension between inclusive DDIS and jet data which is alleviated by inclusion of inhomogeneous term in evolution equation.

Further corrections to DPDF evolution

- NNLO parton-to-parton splitting functions (known).
- NLO Pomeron-to-parton splitting functions (unknown).
- Absorptive corrections. Schematically,

$$\frac{\partial g^{\text{D}}}{\partial \ln Q^2} = P_{gg} \otimes g^D + P_{g\mathbb{P}} \otimes g^2 - 4 P_{g\mathbb{P}} \otimes g g^D + \ldots .$$



Possible that further corrections will stabilise the results of the fit with respect to the Q^2_{min} cut.

Conclusions

- Collinear factorisation holds, but we need to account for the direct Pomeron coupling:

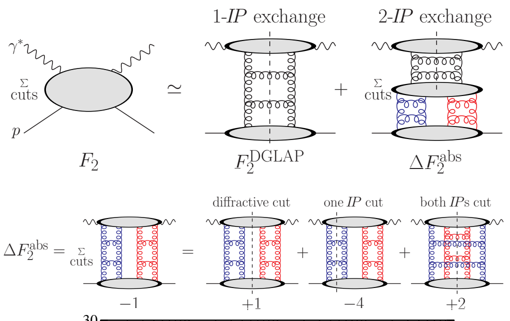
$$F_2^{\mathrm{D}(3)} = \sum_{a=q,g} C_{2,a} \otimes a^{\mathrm{D}} \;+\; C_{2,\mathbb{P}}$$

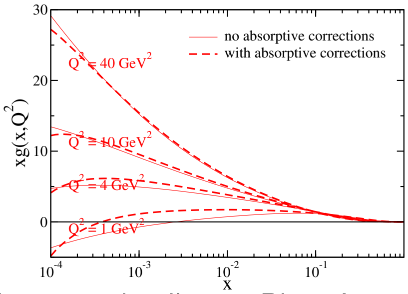
$$\frac{\partial a^{\mathrm{D}}}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a'^{\mathrm{D}} \;+\; P_{a\mathbb{P}}(z)\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2)$$

  Direct coupling and inhomogeneous evolution analogous to the photon case. Direct Pomeron contribution should also be included when calculating jet or heavy quark production.
- New analyses from H1 are a dramatic improvement on previous attempts, but **still** do not include the direct Pomeron contributions.
- Evidence of instability in the fits for $Q^2_{\rm min} \lesssim 6.5$ GeV^2: further theoretical corrections such as NLO $P_{a\mathbb{P}}$ or absorptive corrections may help.
- Claims about factorisation breaking based on previous diffractive PDFs will need to be re-examined. Need to have good understanding of $\gamma^* p$ (HERA) before extending, in turn, to γp (HERA), $p\bar{p}$ (Tevatron) and pp (LHC). Recent H1 and ZEUS data are a large step towards this goal.

Appendix: Non-linear evolution of inclusive PDFs

$$\boxed{\frac{\partial a(x,Q^2)}{\partial \ln Q^2} = \sum_{a'=q,g} P_{aa'} \otimes a' \;-\; \int_x^1 \mathrm{d}x_{\mathbb{P}}\; P_{a\mathbb{P}}(x/x_{\mathbb{P}})\, f_{\mathbb{P}}(x_{\mathbb{P}}; Q^2).}$$





- Interesting application of DDIS formalism to calculate shadowing corrections to inclusive DIS via Abramovsky–Gribov–Kancheli (AGK) cutting rules.
- Inhomogeneous evolution of DPDFs $\Rightarrow$ non-linear evolution of inclusive PDFs.
- More precise version of Gribov–Levin–Ryskin–Mueller–Qiu (GLRMQ) equation derived.
- Fit HERA F_2 data similar to MRST2001 NLO fit. Small-x gluon enhanced at low scales.

For more details see Phys. Lett. B **627** (2005) 97 (`hep-ph/0508093`).