Parton distribution functions and α_S

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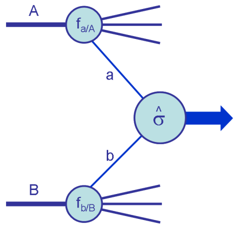
In collaboration with A.D. **M**artin, W.J. **S**tirling and R.S. **T**horne
[arXiv:0901.0002 (“**MSTW 2008**”, 154 pages) and paper in preparation]
MSTW = **MRST** − R.G. Roberts (now retired) + G.**W**. (since 2006)

`http://projects.hepforge.org/mstwpdf/`

Fixed-order collinear factorization at hadron colliders

$$\boxed{\sigma_{AB} = \sum_{a,b=q,g} f_{a/A}(x_a,\mu_F^2) \otimes f_{b/B}(x_b,\mu_F^2) \otimes \hat{\sigma}_{ab}}$$



- Expand $\hat{\sigma}_{ab}$, $P_{aa'}$ and β as perturbative series in α_S ($\overline{\mathrm{MS}}$ scheme):

$$\hat{\sigma}_{ab} = {\alpha_S}^r \left[\hat{\sigma}_{ab}^{\rm LO} + \alpha_S\,\hat{\sigma}_{ab}^{\rm NLO} + {\alpha_S}^2\,\hat{\sigma}_{ab}^{\rm NNLO}\ldots\right] \quad (r \geq 0)$$

$$\frac{\partial f_{a/A}}{\partial \ln Q^2} = \alpha_S \sum_{a'=q,g} \left[P_{aa'}^{\rm LO} + \alpha_S\,P_{aa'}^{\rm NLO} + {\alpha_S}^2\,P_{aa'}^{\rm NNLO}\ldots\right] \otimes f_{a'/A}$$

$$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\rm LO}\,{\alpha_S}^2 - \beta^{\rm NLO}\,{\alpha_S}^3 - \beta^{\rm NNLO}\,{\alpha_S}^4 - \ldots$$

- Need to extract input values $f_{a/A}(x,Q_0^2)$ and $\alpha_S(M_Z^2)$ from data.

Data sets fitted in MSTW 2008 NLO analysis [arXiv:0901.0002]

Data set	χ^2 / $N_{\rm pts.}$
H1 MB 99 e^+p NC	9 / 8
H1 MB 97 e^+p NC	42 / 64
H1 low Q^2 96–97 e^+p NC	44 / 80
H1 high Q^2 98–99 e^-p NC	122 / 126
H1 high Q^2 99–00 e^+p NC	131 / 147
ZEUS SVX 95 e^+p NC	35 / 30
ZEUS 96–97 e^+p NC	86 / 144
ZEUS 98–99 e^-p NC	54 / 92
ZEUS 99–00 e^+p NC	63 / 90
H1 99–00 e^+p CC	29 / 28
ZEUS 99–00 e^+p CC	38 / 30
H1/ZEUS $e^\pm p$ $F_2^{\rm charm}$	107 / 83
H1 99–00 e^+p incl. jets	19 / 24
ZEUS 96–97 e^+p incl. jets	30 / 30
ZEUS 98–00 $e^\pm p$ incl. jets	17 / 30
DØ II $p\bar{p}$ incl. jets	114 / 110
CDF II $p\bar{p}$ incl. jets	56 / 76
CDF II $W \to l\nu$ asym.	29 / 22
DØ II $W \to l\nu$ asym.	25 / 10
DØ II Z rap.	19 / 28
CDF II Z rap.	49 / 29

Data set	χ^2 / $N_{\rm pts.}$
BCDMS μp F_2	182 / 163
BCDMS μd F_2	190 / 151
NMC μp F_2	121 / 123
NMC μd F_2	102 / 123
NMC $\mu n/\mu p$	130 / 148
E665 μp F_2	57 / 53
E665 μd F_2	53 / 53
SLAC ep F_2	30 / 37
SLAC ed F_2	30 / 38
NMC/BCDMS/SLAC F_L	38 / 31
E866/NuSea pp DY	228 / 184
E866/NuSea pd/pp DY	14 / 15
NuTeV νN F_2	49 / 53
CHORUS νN F_2	26 / 42
NuTeV νN xF_3	40 / 45
CHORUS νN xF_3	31 / 33
CCFR $\nu N \to \mu\mu X$	66 / 86
NuTeV $\nu N \to \mu\mu X$	39 / 40
All data sets	**2543 / 2699**

- Red = New w.r.t. MRST 2006 fit.

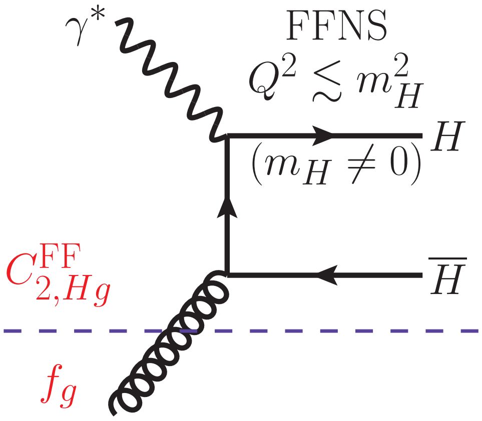
Input parametrization in MSTW 2008 NLO/NNLO fit

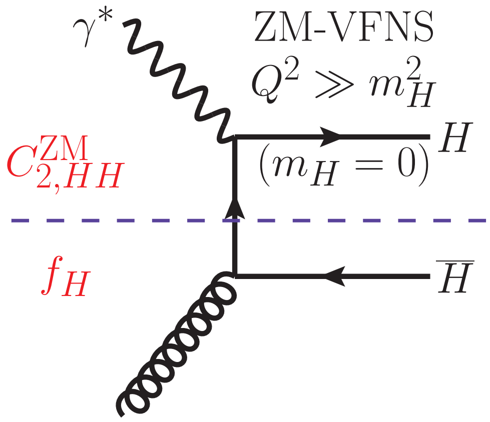
At input scale $Q_0^2 = 1$ GeV^2 (notation: $f_{a/p} \equiv a$):

$$
\begin{aligned}
xu_v &= A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u\, x)\\
xd_v &= A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d\, x)\\
xS &= A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)\\
x(\bar{d}-\bar{u}) &= A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2)\\
xg &= A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}\\
x(s+\bar{s}) &= A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\sqrt{x}+\gamma_S\, x)\\
x(s-\bar{s}) &= A_-\, x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0)
\end{aligned}
$$

- A_u, A_d, A_g and x_0 are determined from sum rules.
- 20 parameters allowed to go free for error propagation, cf. 15 for MRST error PDF sets (1 more for g, 4 more for $s, \bar{s}$).

Heavy quark contribution to DIS structure function F_2





Fixed flavor number scheme

- No heavy quark PDF.
- Includes $\mathcal{O}(m_H^2/Q^2)$ terms.
- No resummation of $\alpha_S \ln(Q^2/m_H^2)$ terms.

Zero-mass variable flavor number scheme

- Use heavy quark PDF.
- Mass dependence neglected.
- Resums $\alpha_S \ln(Q^2/m_H^2)$ terms similar to light quarks.

General-mass variable flavor number scheme (GM-VFNS)

Recent review by R. Thorne and W.-K. Tung [arXiv:0809.0714]

- Interpolate between two well-defined regions: FFNS for $Q^2 \leq m_H^2$, ZM-VFNS for $Q^2 \gg m_H^2$.
- Define by demanding equivalence of the $n_f = n$ (FFNS) and $n_f = n+1$ (VFNS) flavor descriptions above transition point:

$$\begin{aligned} F_i(x,Q^2) &= \sum_k C_{i,k}^{\mathrm{FF},n}(Q^2/m_H^2) \otimes f_k^n(Q^2) \\ &= \sum_j C_{i,j}^{\mathrm{VF},n+1}(Q^2/m_H^2) \otimes f_j^{n+1}(Q^2) \\ &\equiv \sum_{j,k} C_{i,j}^{\mathrm{VF},n+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^n(Q^2) \end{aligned}$$

$$\Rightarrow \quad C_{i,k}^{\mathrm{FF},n}(Q^2/m_H^2) = \sum_j C_{i,j}^{\mathrm{VF},n+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2)$$

- But $C_{i,j}^{\mathrm{VF},n_f,(m)}$ only uniquely defined in massless limit $\Rightarrow$ ambiguous up to $\mathcal{O}(m_H^2/Q^2)$ terms (can redistribute between orders).

Choice of GM-VFNS by MRST/MSTW

- MRST 1998–2004 used the Thorne–Roberts (TR) scheme [hep-ph/9709442]: demand $\partial F_2^H/\partial \ln Q^2$ continuous.
- PDFs are discontinuous at NNLO in a VFNS [Buza *et al.* '96]:

$$f_{j/p}^{n_f+1}(x, m_H^2) = f_{j/p}^{n_f}(x, m_H^2) + \alpha_S^2 \sum_k A_{jk}(x) \otimes f_{k/p}^{n_f}(x, m_H^2),$$

  but neglected in MRST 2001–2004 NNLO analyses.
- Structure functions at NNLO are then discontinuous in ZM-VFNS, but should be continuous in GM-VFNS. Original TR scheme technically difficult to implement at NNLO.
- Instead, R. Thorne [hep-ph/0601245] redefined simpler GM-VFNS (denoted TR′) for use up to NNLO. Adopted elements of "ACOT(χ)" ($\to$ talk by F. Olness):

$$C_{2,HH}^{\mathrm{VF},n_f,(m)}(z, Q^2/m_H^2) = C_{2,HH}^{\mathrm{ZM},n_f,(m)}(z/x_{\max})$$

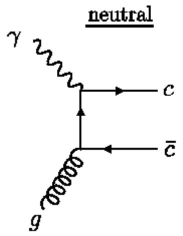
Ordering of the perturbative expansion for F_2^H (TR, TR′)

n-flavor, $Q^2 < m_H^2$	
LO :	$\frac{\alpha_S}{4\pi} C_{2,Hg}^{\mathrm{FF},n,(1)} \otimes g^n$
NLO :	$\left(\frac{\alpha_S}{4\pi}\right)^2 \left(C_{2,Hg}^{\mathrm{FF},n,(2)} \otimes g^n + C_{2,Hq}^{\mathrm{FF},n,(2)} \otimes \Sigma^n\right)$
NNLO :	$\left(\frac{\alpha_S}{4\pi}\right)^3 \sum_j C_{2,Hj}^{\mathrm{FF},n,(3)} \otimes f_j^n$

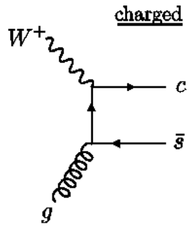
$(n+1)$-flavor, $Q^2 > m_H^2$	
LO :	$C_{2,HH}^{\mathrm{VF},n+1,(0)} \otimes (H + \overline{H})$
NLO :	$\frac{\alpha_S}{4\pi} \left(C_{2,HH}^{\mathrm{VF},n+1,(1)} \otimes (H + \overline{H}) + C_{2,Hg}^{\mathrm{VF},n+1,(1)} \otimes g^{n+1}\right)$
NNLO :	$\left(\frac{\alpha_S}{4\pi}\right)^2 \sum_j C_{2,Hj}^{\mathrm{VF},n+1,(2)} \otimes f_j^{n+1}$

- Maintain continuity at $Q^2 = m_H^2$ by freezing term with highest power of α_S for $Q^2 < m_H^2$ when moving above m_H^2.
- ACOT-type schemes instead use same order of α_S for n- and $(n+1)$-flavors, e.g. $F_2^H = 0$ at LO for $Q^2 < m_H^2$.

Modeling of higher-order massive DIS coefficient functions

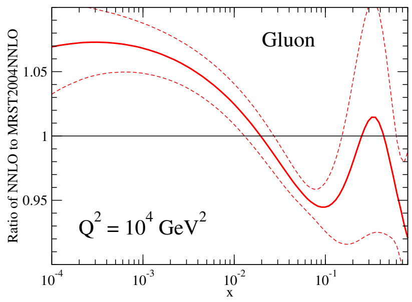


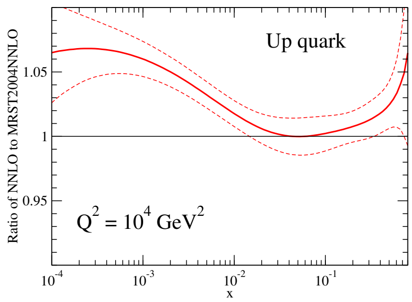
- Massive $\mathcal{O}(\alpha_S^3)$ NC coefficient functions unknown, but needed for GM-VFNS at NNLO.
- Model [Thorne '06] using known leading threshold logarithms [Laenen, Moch '99] and leading $\ln(1/x)$ terms [Catani, Ciafaloni, Hautmann '91]. Variation in free parameters does not lead to a large change.
- Massive $\mathcal{O}(\alpha_S^2)$ CC coefficient functions unknown, but needed for GM-VFNS at NLO (important for $s, \bar{s}$ determination from CCFR/NuTeV $\nu N \to \mu^+\mu^- X$ data).
- Model by modifying $\mathcal{O}(\alpha_S^2)$ NC contributions for different threshold behaviour. More sophisticated modeling in MSTW 2008. No attempt made to model $\mathcal{O}(\alpha_S^3)$ CC contribution needed at NNLO.



Impact of consistent GM-VFNS at NNLO

Ratio 2006 to 2004:





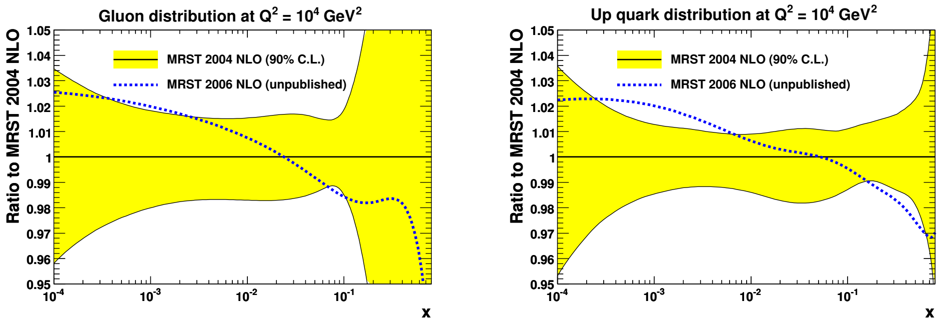
MRST 2006 NNLO [arXiv:0706.0459]

- First implementation of TR′ scheme.
- Increase in low-x PDFs when discontinuities included.
- $\sigma_{W,Z}$ at the LHC sensitive to light sea-quark PDFs at $x \sim 0.006$ (for $y = 0$) $\Rightarrow$ $\sigma_{W,Z}$ increase by 6%. A **correction**, not an *uncertainty*.

- From CTEQ6.1 NLO (ZM-VFNS) to CTEQ6.5 NLO (GM-VFNS): 8% increase in $\sigma_{W,Z}$ at LHC. A **correction**, not an *uncertainty*.

Scheme dependence of GM-VFNS at NLO

- Change TR $\longrightarrow$ TR$'$ allows study of scheme dependence at NLO.
- MRST 2004 (TR) and MRST 2006 (TR$'$) fits use same data.



- Nearly 3% increase in $\sigma_{W,Z}$ at the LHC at NLO. Genuine theory uncertainty: should decrease going to higher orders.
- An **uncertainty**, not a *correction* at NLO.

Path to NNLO evolution in global PDF analysis

MRST 2001/2002 : approximate NNLO splitting [van Neerven, Vogt '00].

MRST 2004 : exact NNLO splitting [Moch, Vermaseren, Vogt '04].

MRST 2006 : added discontinuities at $Q^2 = m_H^2$ [Buza *et al.* '96]:

$$\boxed{f_{j/p}^{n_f+1}(x, m_H^2) = f_{j/p}^{n_f}(x, m_H^2) + \alpha_S^2 \sum_k A_{jk}(x) \otimes f_{k/p}^{n_f}(x, m_H^2),}$$

with TR$'$ GM-VFNS for structure functions [Thorne '06].

MSTW 2008 : minor refinements to NNLO evolution code:

- added perturbative NNLO generation of $q \neq \bar{q}$ (very small).
- improved definition of α_S (MRST form unconventional).
- evolution checked against public PEGASUS [Vogt '04] and HOPPET [Salam, Rojo '08] codes for fitted input PDFs.

Treatment of jet data in MRST/MSTW analyses

MRST 2001–2006

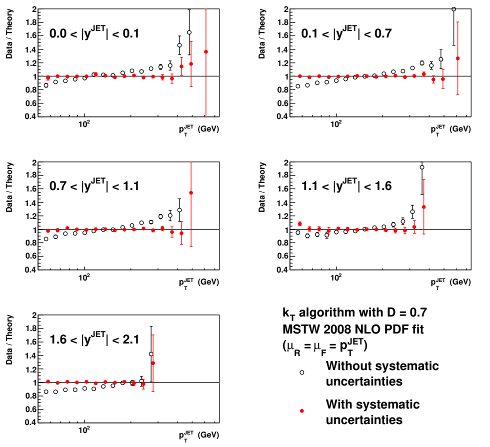
- Fit six "pseudogluon" points at $Q^2 = 2000$ GeV^2 inferred from **Tevatron Run I** inclusive jet data.
- Comparison to actual jet data, calculated at LO with a K-factor, only made *after* the fit.

MSTW 2008

- Fit to **Tevatron Run II** and HERA DIS inclusive jet data.
- Complete treatment of correlated systematic errors.
- Use FASTNLO code [Kluge, Rabbertz, Wobisch '06] to calculate NLO cross sections exactly during the fit.
- Full NNLO $\hat{\sigma}_{ab}$ not yet known: include 2-loop threshold corrections [Kidonakis, Owens '00] for Tevatron jet data at NNLO and exclude HERA DIS jet data.

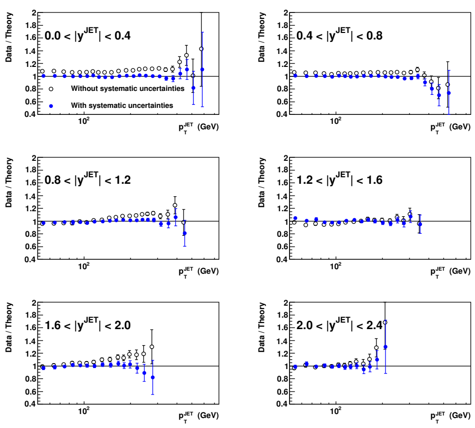
Description of Tevatron Run II inclusive jet data

CDF Run II inclusive jet data, $\chi^2 = 56$ for 76 pts.



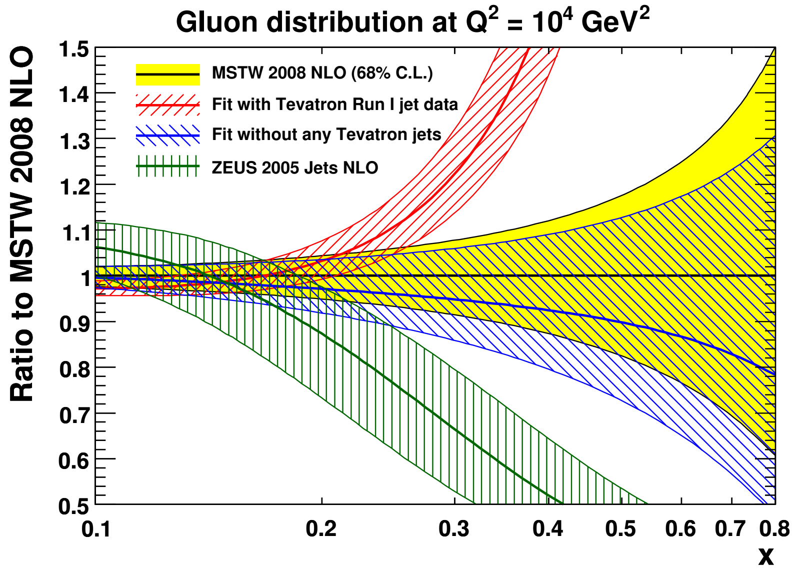
[Data: hep-ex/0701051]

DØ Run II inclusive jet data (cone, R = 0.7)
MSTW 2008 NLO PDF fit ($\mu_R = \mu_F = p_T^{JET}$), $\chi^2 = 114$ for 110 pts.



[Data: arXiv:0802.2400]

Impact of Run II jet data on high-x gluon distribution



- Run II jet data prefer smaller gluon distribution at high x.

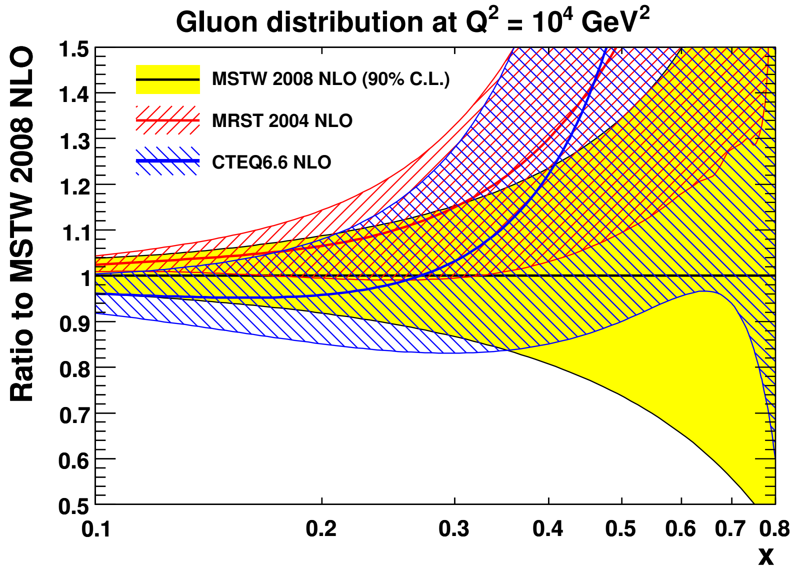
Tension between Run I and Run II inclusive jet data

Highlighted numbers indicate χ^2 values for data sets explicitly included in various NLO global fits:

CDFI (33 pts.)	DØI (90 pts.)	CDFII(k_T) (76 pts.)	DØII (110 pts.)	$\Delta\chi^2_{\text{non-jet}}$ (2513 pts.)	$\alpha_S(M_Z^2)$
53	119	64	117	0	0.1197
51	**48**	132	180	9	0.1214
56	110	**56**	**114**	2	0.1202
53	**85**	**68**	**117**	1	0.1204

- Fit to Run I jets $\Rightarrow$ description of Run II jets bad.
- Fit to Run II jets $\Rightarrow$ description of Run I jets bad.
- Fit neither $\Rightarrow$ similar description as fitting Run II only.
- **Summary**: Some inconsistency between Run I and Run II jets. Run II jets slightly more consistent with rest of data.

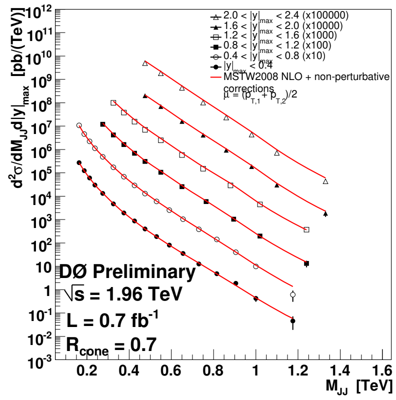
New high-x gluon distribution compared to previous sets

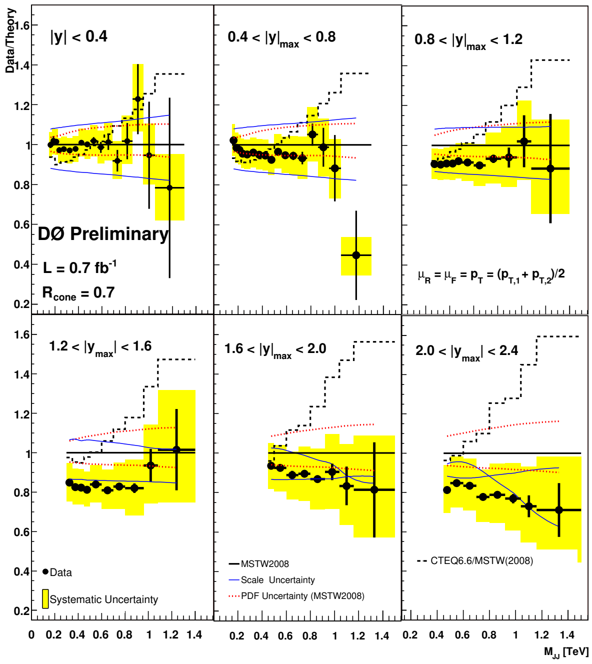


- Smaller high-x gluon than previous MRST and CTEQ fits.

Description of DØ dijet mass spectrum (April 29, 2009)

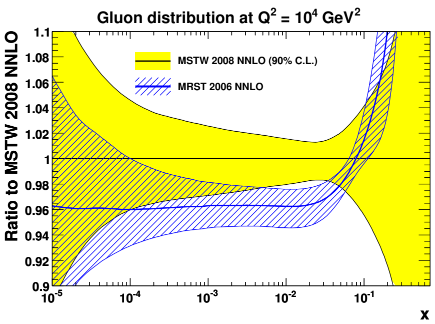
[DØ Note 5919-CONF]



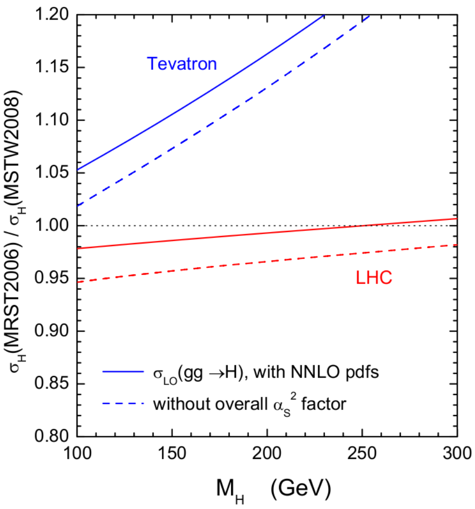


- Data favor **less** gluon at high x (MSTW 2008 over CTEQ6.6).

Implications of new PDFs for Higgs cross sections



- NNLO trend similar to NLO: smaller 2008 gluon at high x, larger 2008 gluon at low x (momentum sum rule).
- $\alpha_S(M_Z^2) = 0.1191$ (2006) $\rightarrow 0.1171$ (MSTW 2008)



- Higgs cross sections smaller at Tevatron with 2008 PDFs ($\rightarrow$ talk by D. de Florian).

Uncertainties in global PDF analysis

"Theoretical" errors

- *Examples:* input parameterisation form, neglected higher-order and higher-twist QCD corrections, electroweak corrections, choice of cuts, nuclear corrections, heavy flavor treatment.
- Difficult to quantify *a priori*. Often correction only known after an improved treatment/calculation is available.

"Experimental" errors

- If all the above sources of "theoretical" errors are fixed, how do we propagate the experimental uncertainties on the fitted data points through to the PDF uncertainties?
- Generally use the **Hessian** method [Pumplin *et al.* '01]: diagonalize covariance matrix from the fit and produce $\pm$ **eigenvector PDF sets** displaced from best-fit PDF set.

Criteria for choice of tolerance $T = \sqrt{\Delta\chi^2_{\rm global}}$

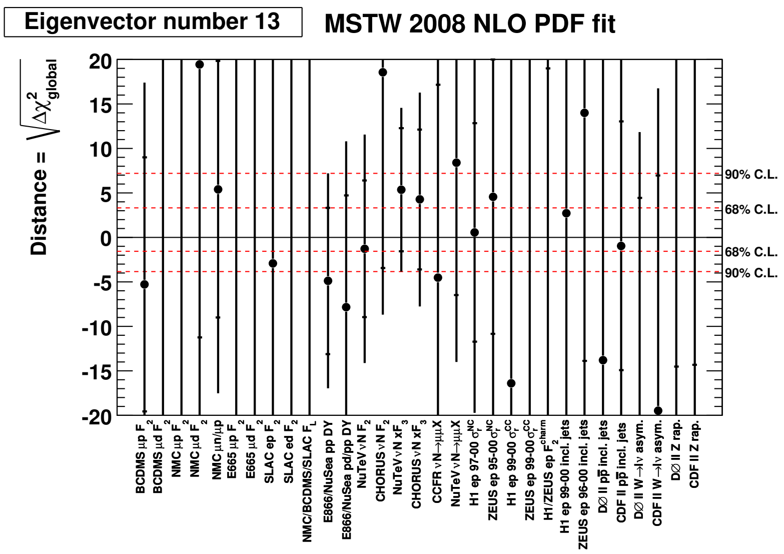
Parameter-fitting criterion

- $T^2 = 1$ for 68% (1-σ) C.L., $T^2 = 2.71$ for 90% C.L.
- **In practice:** minor inconsistencies between fitted data sets, and unknown experimental and theoretical uncertainties, so not appropriate for global PDF analysis.

Hypothesis-testing criterion (proposed by CTEQ)

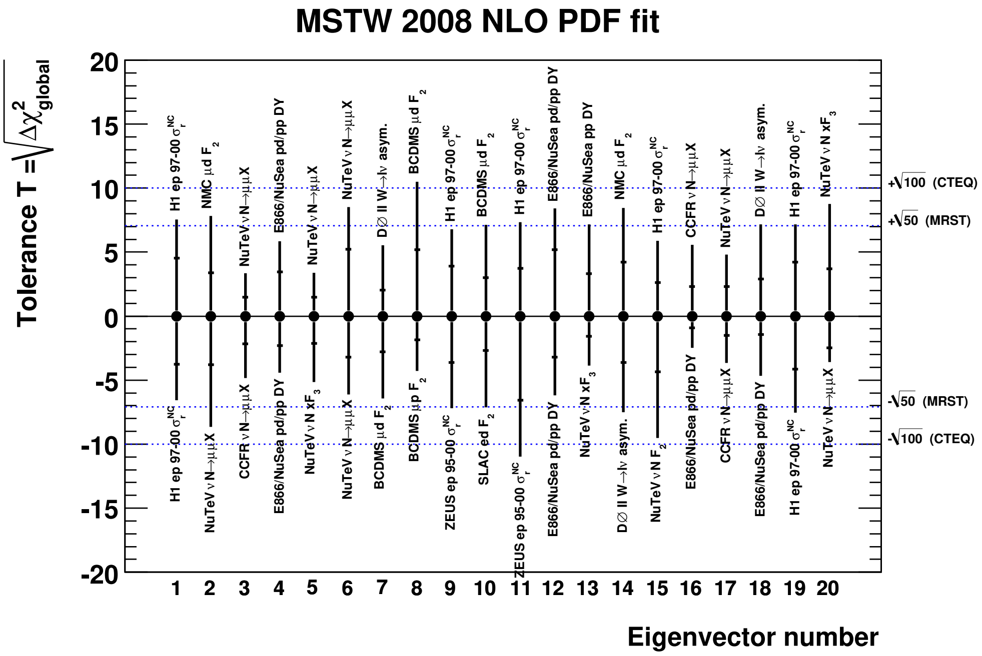
- Much weaker: treat PDF sets obtained from eigenvectors of covariance matrix as alternative hypotheses.
- Determine T^2 from the criterion that each data set should be described within its 90% C.L. limit. Very roughly, a "good" fit has $\chi^2 \simeq N_{\rm pts.} \pm \sqrt{2N_{\rm pts.}}$ for each data set.
- **CTEQ:** $T^2 = 100$ for 90% C.L. limit, **MRST:** $T^2 = 50$.

Determination of tolerance for eigenvector number 13

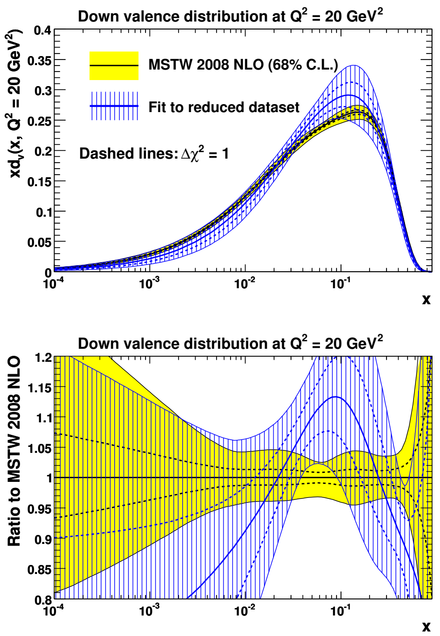


- Tolerance in "+" direction provided by E866/NuSea *pp* DY data.
- Tolerance in "−" direction provided by NuTeV νN xF_3 data.

Dynamic tolerance: different for each eigenvector



Test of dynamic tolerance: fit to reduced dataset



- Fit to reduced dataset comprising 589 DIS data points, cf. 2699 data points in global fit.
- Errors given by $T^2 = 1$ don't overlap $\Rightarrow$ inconsistent data sets included in global fit.
- **Dynamic tolerance** $T^2 > 1$ accommodates mildly inconsistent data sets.

Uncertainties on α_S in global PDF analysis

[MSTW, in preparation]

- PDFs (including uncertainty sets) are determined for a fixed value of α_S $\Rightarrow$ use **same** value in cross section calculations.
- In MRST/MSTW analyses, α_S is fitted, then **fixed** at the best-fit value for final error propagation.

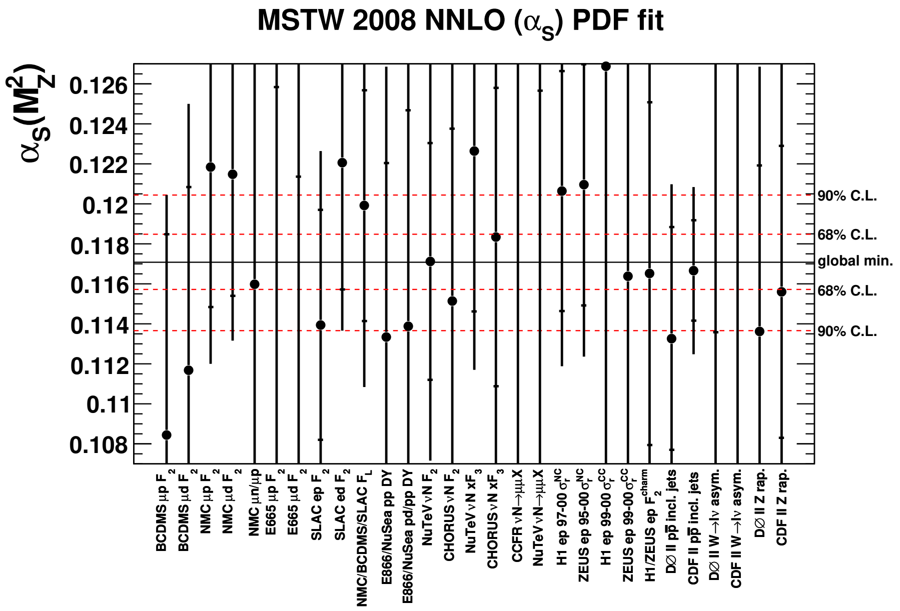
Problems:

1. What is the (experimental) error on α_S?
2. How to include this error in cross section calculations?

Solutions:

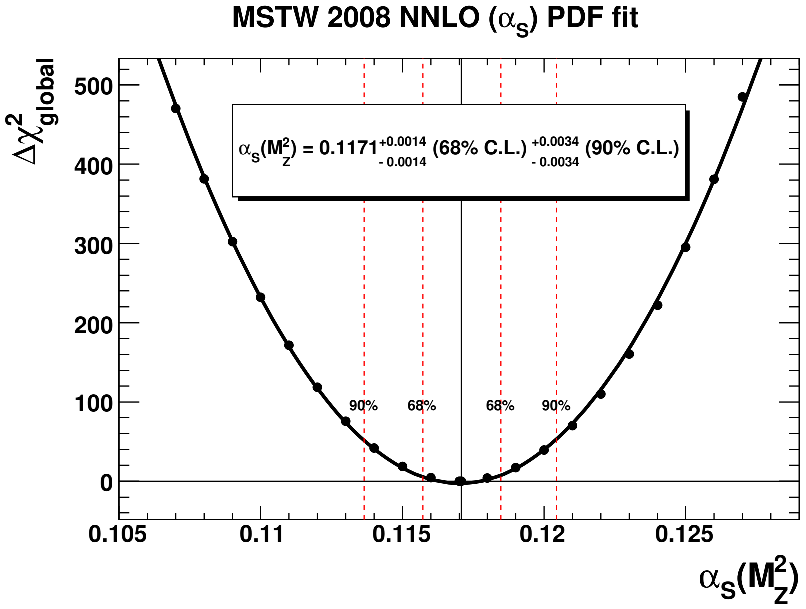
1. Apply same method used to determine the tolerance for each eigenvector to determine the experimental error on $\alpha_S(M_Z^2)$.
2. Then generate best-fit and eigenvector PDF sets for different fixed α_S values for use in calculations of cross sections.

Ranges of $\alpha_S(M_Z^2)$ for which data sets are well described



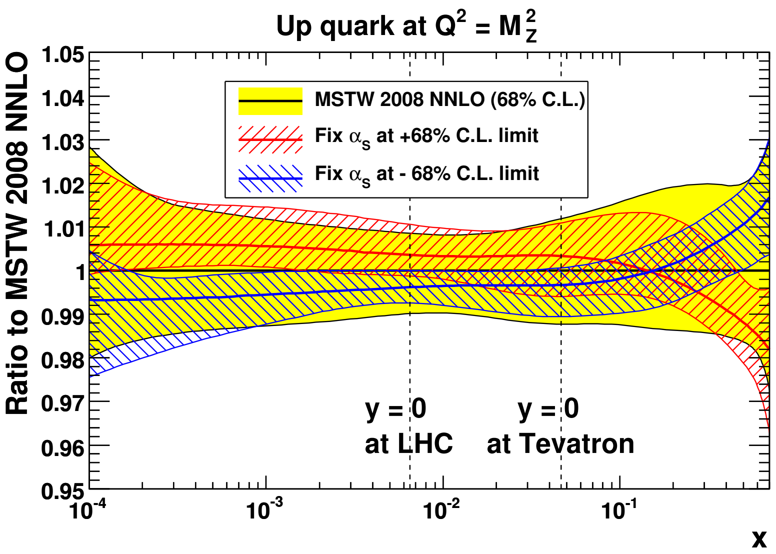
- **Upper** limit on $\alpha_S(M_Z^2)$ provided by BCDMS $F_2^{\mu p}$ data.
- **Lower** limit on $\alpha_S(M_Z^2)$ provided by SLAC F_2^{ed} data.

$\Delta\chi^2_{\mathrm{global}}$ as a function of $\alpha_S(M_Z^2)$ for the NNLO global fit



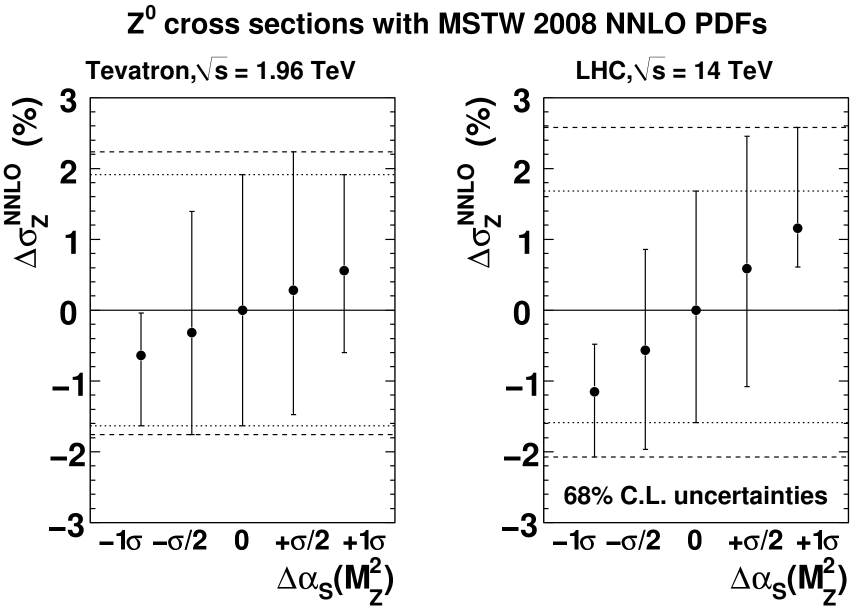
- Additional theory uncertainty ($\sim |\mathrm{NNLO} - \mathrm{NLO}| = \pm 0.003$).
- cf. PDG world average value of $\alpha_S(M_Z^2) = 0.1176 \pm 0.002$.

Dependence of up quark distribution on $\alpha_S(M_Z^2)$



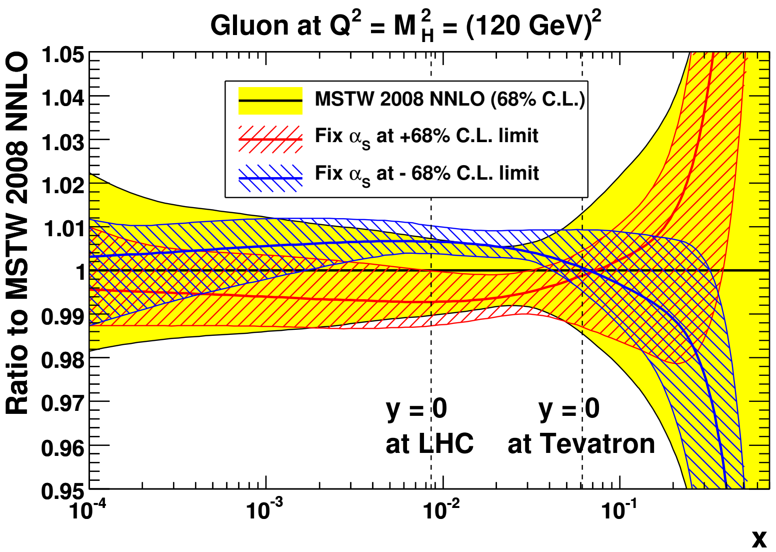
- **Correlation** at low x, **anticorrelation** at high x.
- PDF uncertainties smaller when α_S shifted to limits.

Impact of α_S on Z total cross section at NNLO



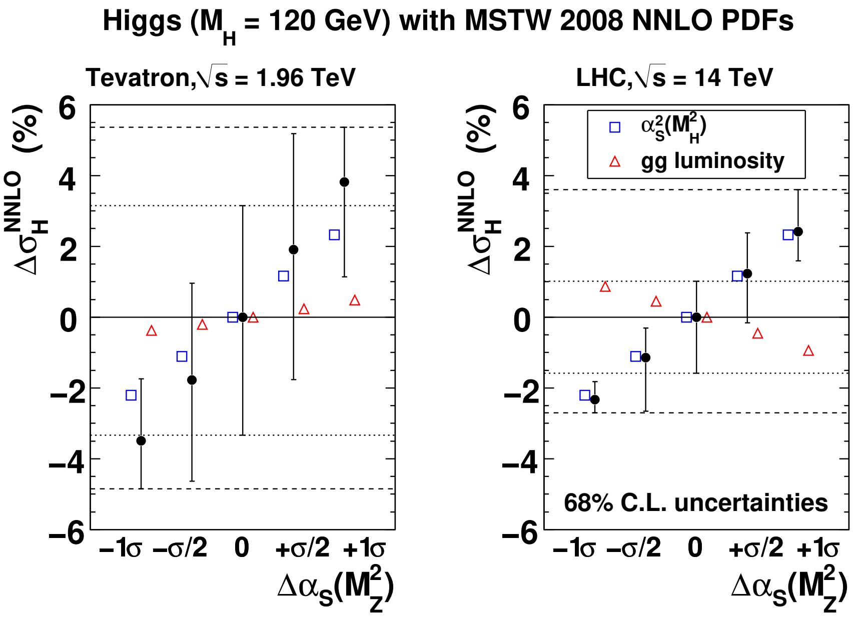
- Two effects: (i) correlation of quark distributions with α_S at LHC (less correlation at Tevatron), (ii) higher-order corrections.

Dependence of gluon distribution on $\alpha_S(M_Z^2)$



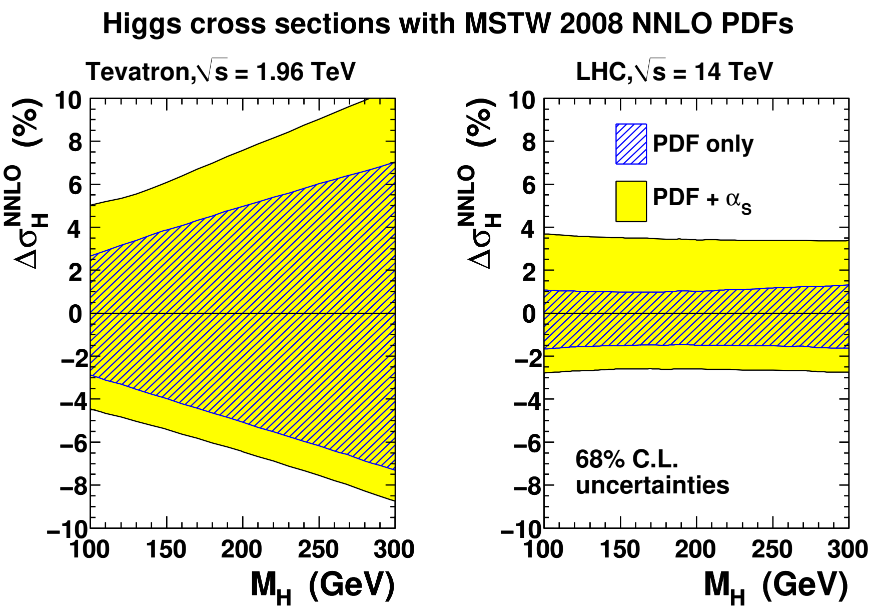
- **Anticorrelation** at low x: HERA $\partial F_2/\partial \ln Q^2 \sim \alpha_S\, g$.
- **Correlation** at high x to maintain momentum sum rule.

Impact of α_S on SM Higgs total cross section at NNLO



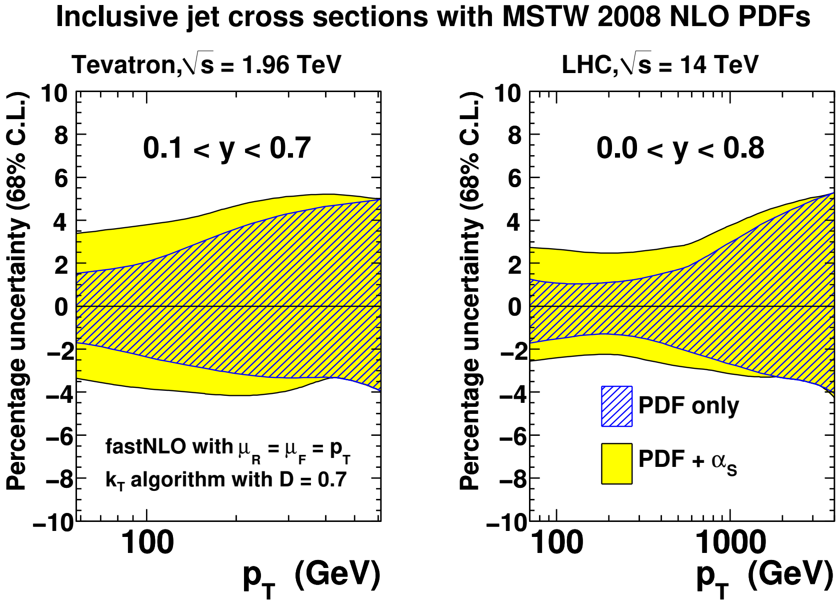
- **Anticorrelation** of gg luminosity with α_S at LHC cancels out (almost exactly) **correlation** due to higher-order corrections.

Impact of α_S on SM Higgs uncertainty versus M_H



- Enhanced "PDF+α_S" uncertainty compared to "PDF only" uncertainty, particularly at the LHC.

Impact of α_S on inclusive jet uncertainty versus p_T

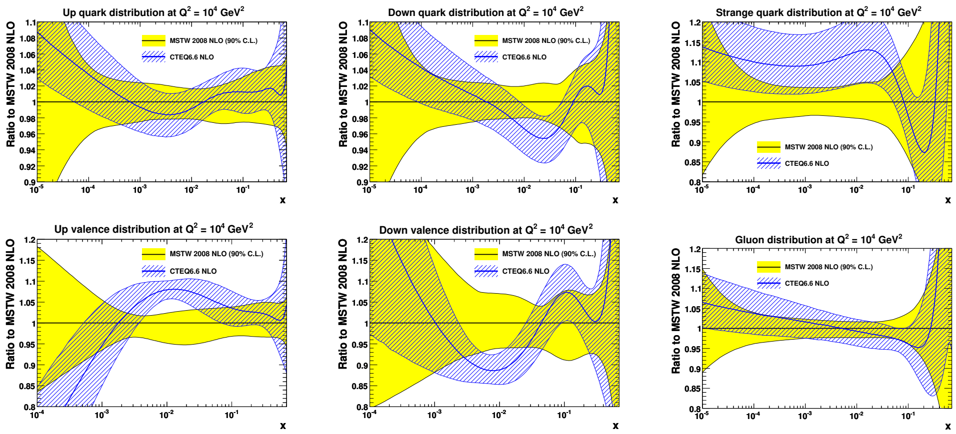


- Mostly gluon-initiated at low $p_T \Rightarrow$ correlated with α_S.
- Mostly quark-initiated at high $p_T \Rightarrow$ anticorrelated with α_S.

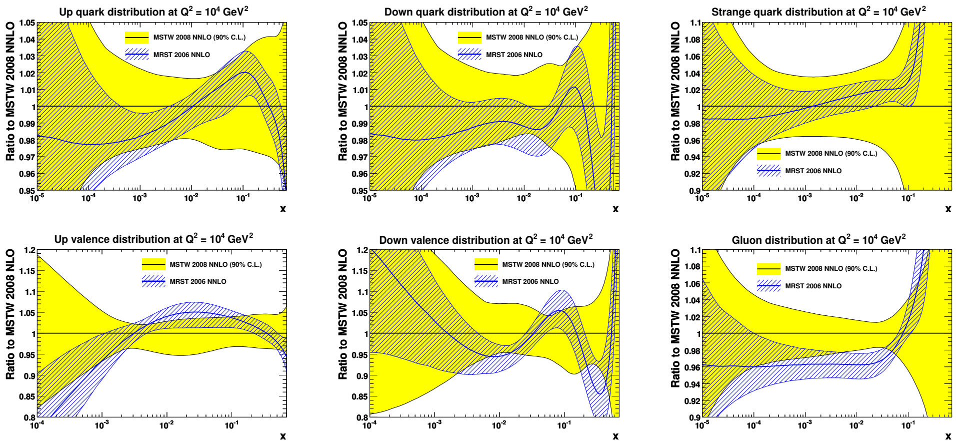
Summary

- **MSTW 2008** (LO, NLO, NNLO) PDF fits [`arXiv:0901.0002`] are the most comprehensive to date: supersede MRST sets.
- Pre-2006 MRST *NNLO* PDF sets should be considered obsolete due to incomplete heavy flavor treatment.
- *Almost* all necessary NNLO processes are now known. *Exceptions:* inclusive jets, massive $\mathcal{O}(\alpha_S^3)$ NC and $\mathcal{O}(\alpha_S^2)$ CC DIS.
- **Tevatron Run II jets** prefer smaller high-x gluon than Run I: impact on Higgs cross sections at Tevatron.
- **Improved** "dynamic tolerance" controlling propagation of experimental errors through to PDF uncertainties.
- Now possible to consistently calculate combined "PDF+α_S" uncertainty on cross sections: additional sets public soon.

Comparison to CTEQ6.6 NLO



Comparison to MRST 2006 NNLO



Alternative approach: NNPDF Collaboration

NNPDF Collaboration: R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, A. Piccione, J. Rojo, M. Ubiali

MSTW approach [arXiv:0901.0002]

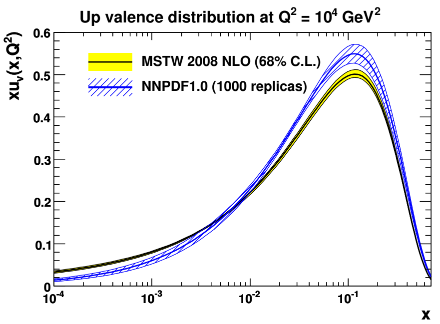
Parametrization	$xf_{a/p} \sim A_a x^{\Delta_a}(1-x)^{\eta_a}(1+\epsilon_a\sqrt{x}+\gamma_a x)$
Minimisation	Non-linear least-squares (Marquardt method)
Error propagation	Hessian method with dynamical tolerance
Application	Use best-fit and 40 eigenvector PDF sets

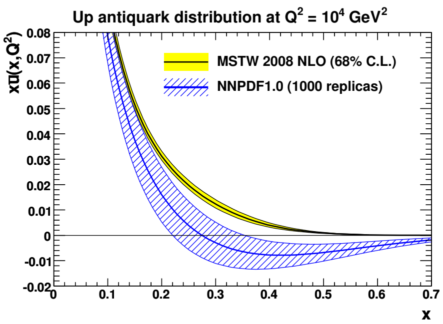
NNPDF approach [arXiv:0808.1231]

Parametrization	Neural network (37 free parameters per PDF)
Minimisation	Genetic algorithm (stop before overlearning)
Error propagation	Generate $N_{\rm rep} \sim \mathcal{O}(1000)$ MC data replicas
Application	Calculate average and s.d. over $N_{\rm rep}$ PDF sets

First results from NNPDF1.0 [arXiv:0808.1231]

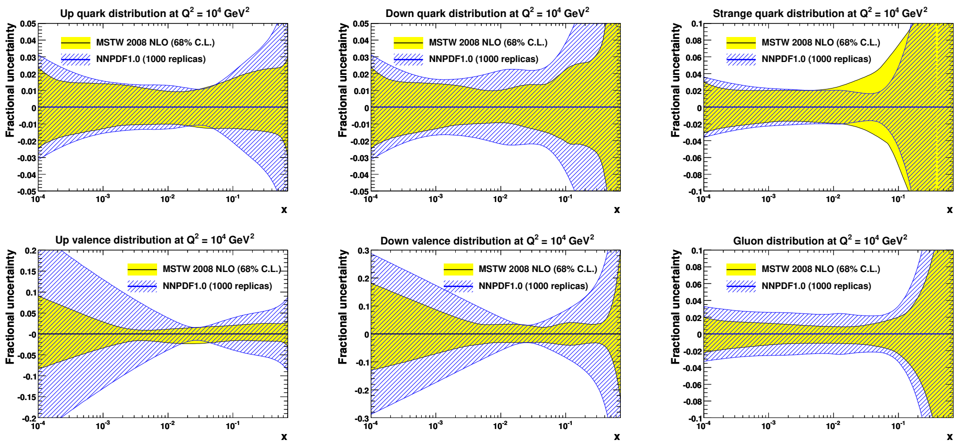
- Fit restricted set of only DIS structure function data (SLAC, BCDMS, NMC, H1, ZEUS, CHORUS).
- Inadequate treatment of heavy quarks (ZM-VFNS).





- Up valence: relative data set normalizations fitted by MSTW.
- Up antiquark: NNPDF1.0 negative by $\sim$ 2-σ, no Drell–Yan.

Uncertainties for MSTW 2008 and NNPDF1.0



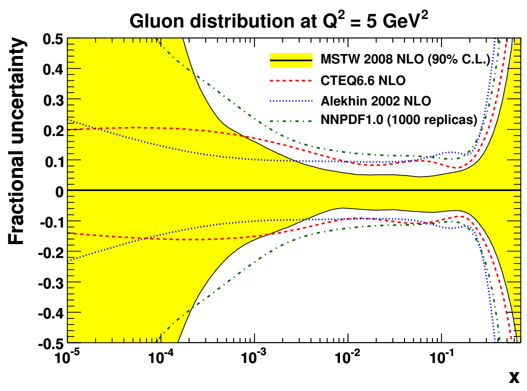
- NNPDF1.0 has fixed $s = \bar{s} = (\bar{u} + \bar{d})/4$ at $Q_0^2 = 2$ GeV^2.
- NNPDF1.1 [arXiv:0811.2288]: free strangeness but no νN dimuon data to constrain, so huge PDF uncertainties.
- **Conclusion:** NNPDF approach looks promising, but PDFs not yet directly comparable to those from standard approach.

Parametrization dependence of low-x gluon

- PDFs lose probabilistic interpretation beyond LO.
- Negative small-x gluon distribution preferred at low scales.
- MRST/MSTW parametrize as:

$$xg(x, Q_0^2) = xg_1(x, Q_0^2) + xg_2(x, Q_0^2) \sim A_g\, x^{\delta_g} + A_{g'}\, x^{\delta_{g'}}$$

$$\Rightarrow \Delta g(x, Q_0^2) \sim \pm g_1(x, Q_0^2)\, \Delta\delta_g \ln(1/x) \pm g_2(x, Q_0^2)\, \Delta\delta_{g'} \ln(1/x)$$



- Other groups (CTEQ, Alekhin) parametrize with a valence-like $xg(x, Q_0^2) \sim x^{\delta_g}$: less freedom at small x.