Doubly-unintegrated parton distributions

Graeme Watt

Institute for Particle Physics Phenomenology

University of Durham, UK

Outline of talk

- Collinear vs. k_t-factorisation
- 'Last-step' approach to unintegrated parton distributions
- Why *doubly*-unintegrated parton distributions?
- *Applications:*
  - Inclusive jet production at HERA
  - W and Z production at Tevatron
  - Standard Model Higgs production at LHC
- Talk based on:
  - M. A. Kimber, A. D. Martin and M. G. Ryskin (KMR), Phys. Rev. D **63** (2001) 114027 [arXiv:hep-ph/0101348]
  - G. W., A. D. Martin and M. G. Ryskin, Eur. Phys. J. C **31** (2003) 73 [arXiv:hep-ph/0306169]
  - G. W., A. D. Martin and M. G. Ryskin, to appear in Phys. Rev. D [arXiv:hep-ph/0309096]

Collinear factorisation

In DIS:

$$\boxed{\sigma^{\gamma^* p} = \sum_{a=g,q} \int_0^1 \frac{\mathrm{d}x}{x}\, a(x,\mu^2)\; \hat{\sigma}^{\gamma^* a}}$$

- $\sigma^{\gamma^* p}$ is the hadronic cross section
- $a(x,\mu^2) = xg(x,\mu^2)$ or $xq(x,\mu^2)$ are the (integrated) parton distribution functions (PDFs)
  - satisfy DGLAP evolution in the factorisation scale μ^2
    $\iff$ resum $\alpha_S \ln(\mu^2)$ terms
    $\iff$ strongly-ordered transverse momentum (k_t) along evolution chain ($\ldots \ll k_{n-1,t} \ll k_{n,t} \ll \mu$)
- $\hat{\sigma}^{\gamma^* a}$ are the partonic cross sections
  - calculate assuming incoming parton has momentum $k = x\,p$, $k^2 = 0$

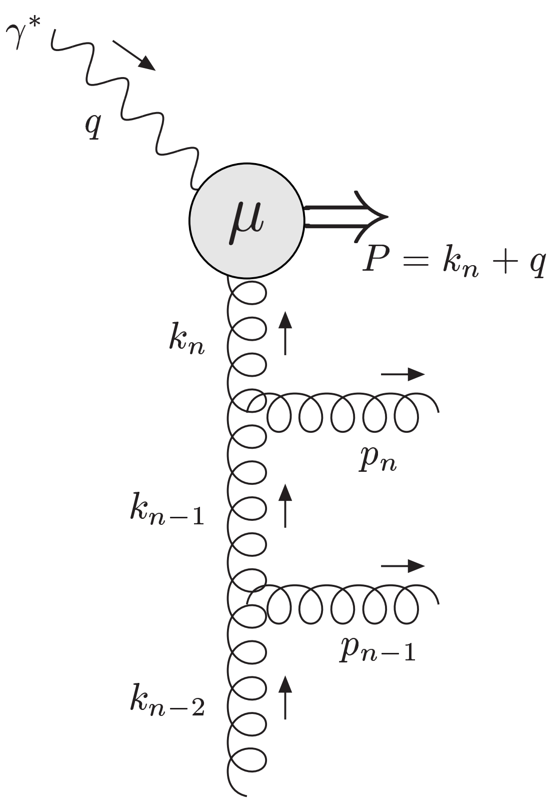
k_t-factorisation (for small-x gluons)

In DIS: $$\boxed{\sigma^{\gamma^* p} = \int_0^1 \frac{\mathrm{d}x}{x} \int_0^\infty \frac{\mathrm{d}k_t^2}{k_t^2}\, f_g(x, k_t^2[, \mu^2])\, \hat{\sigma}^{\gamma^* g}}$$

- $f_g(x, k_t^2[, \mu^2])$ is the *unintegrated* gluon distribution:
  - $f_g(x, k_t^2)$ satisfies BFKL evolution in x
    $\iff$ resum $\alpha_S \ln(1/x)$ terms
    $\iff$ strongly-ordered x $(\ldots \gg x_{n-1} \gg x_n \gg x_B)$
  - $f_g(x, k_t^2, \mu^2)$ satisfies CCFM evolution in μ^2 (or Ξ)
    $\iff$ resum $\alpha_S \ln(1/x)$ and $\alpha_S \ln(1/(1-x))$ terms
    $\iff$ strongly-ordered rapidities
    $(\ldots \ll \xi_{n-1} \ll \xi_n \ll \Xi)$
- $\hat{\sigma}^{\gamma^* g}$ calculated assuming incoming gluon has momentum $k = x\,p + k_\perp$, $k^2 = -k_t^2$

‘Last-step’ approach to uPDFs

- Relax DGLAP strong ordering in last evolution step only: $\ldots \ll k_{n-1,t} \ll k_{n,t} \sim \mu$
- Obtain uPDFs $f_a(x, k_t^2, \mu^2)$ from PDFs $a(x/z, k_t^2)$



- Penultimate parton with $\boxed{k_{n-1} = \frac{x}{z} p}$

  splits to a final parton with

$$\boxed{k_n \equiv k = x\,p - \beta\,q' + k_\perp,}$$

where
$$\beta = \frac{x_B}{x}\frac{z}{(1-z)}\frac{k_t^2}{Q^2}, \quad k^2 = -\frac{k_t^2}{1-z}$$

$$(q' = q + x_B\,p, \quad q^2 = -Q^2,$$

$$p^2 = 0 = q'^2, \quad k_\perp^2 = -k_t^2)$$

Unintegrated from integrated PDFs

- Start from the LO DGLAP equation evaluated at a scale k_t:

$$\frac{\partial\, a(x,k_t^2)}{\partial \log k_t^2} = \frac{\alpha_S(k_t^2)}{2\pi} \sum_{b=g,q} \left[\int_x^1 \mathrm{d}z\, P_{ab}(z)\, b\left(\frac{x}{z}, k_t^2\right) - a(x,k_t^2) \int_0^1 \mathrm{d}\zeta\; \zeta\, P_{ba}(\zeta) \right]$$

- Resum virtual terms into Sudakov form factors:

$$T_a(k_t^2,\mu^2) \equiv \exp\left(-\int_{k_t^2}^{\mu^2} \frac{\mathrm{d}\kappa_t^2}{\kappa_t^2} \frac{\alpha_S(\kappa_t^2)}{2\pi} \sum_{b=g,q} \int_0^1 \mathrm{d}\zeta\; \zeta\, P_{ba}(\zeta) \right)$$

- Then explicit formula for uPDFs is:

$$\begin{aligned} f_a(x,k_t^2,\mu^2) &\equiv \frac{\partial}{\partial \log k_t^2} \left[a(x,k_t^2)\, T_a(k_t^2,\mu^2) \right] \\ &= T_a(k_t^2,\mu^2)\, \frac{\alpha_S(k_t^2)}{2\pi} \sum_{b=g,q} \int_x^1 \mathrm{d}z\, P_{ab}(z)\, b\left(\frac{x}{z}, k_t^2\right) \end{aligned}$$

Impose angular ordering in last step

- After resumming virtual DGLAP terms, need to regulate singularities from soft gluon emission
- Colour coherence
  $\Longrightarrow$ Gluons emitted in the last evolution step should be closer to the proton direction than the subprocess
  $\Longrightarrow$ Rapidity of gluons $>$ rapidity of subprocess (“angular ordering”)
- This leads to the condition:

$$z \frac{k_t}{1-z} < \mu \qquad \Longleftrightarrow \qquad z < \frac{\mu}{\mu + k_t}$$

- Apply only to emitted gluons, not emitted quarks (improvement to the KMR prescription)

Normalisation of the uPDFs

$$\boxed{\int_0^{\mu^2} \frac{\mathrm{d}k_t^2}{k_t^2}\, f_a(x, k_t^2, \mu^2) = a(x, \mu^2)}$$

- Problem: $f_a(x, k_t^2, \mu^2)$ not defined for $k_t < \mu_0 \sim 1$ GeV, since $a(x/z, k_t^2)$ not defined below this scale
- Solution: Know that $f_a \sim k_t^2$ as $k_t^2 \to 0$, due to gauge invariance

  Assume the form: $$\boxed{f_a(x, k_t^2, \mu^2)\big|_{k_t<\mu_0} = \frac{k_t^2}{\mu_0^2}\left[A(x, \mu^2) + \frac{k_t^2}{\mu_0^2}\, B(x, \mu^2)\right]}$$

  Determine coefficients $A(x, \mu^2)$ and $B(x, \mu^2)$ to ensure

  1. Correct normalisation: $\int_0^{\mu_0^2} \frac{\mathrm{d}k_t^2}{k_t^2}\, f_a(x, k_t^2, \mu^2) = a(x, \mu_0^2)\, T_a(\mu_0^2, \mu^2)$,
  2. Continuity of $f_a(x, k_t^2, \mu^2)$ at $k_t = \mu_0$
- Numerical results are insensitive to the precise form used for the $k_t < \mu_0$ contribution

Why *doubly*-unintegrated PDFs?

Answer: To account for the precise kinematics

- Reminder: parton entering subprocess has momentum

$$\boxed{k = x\,p - \beta\,q' + k_\perp, \qquad \beta = \frac{x_B}{x}\,\frac{z}{(1-z)}\,\frac{k_t^2}{Q^2}}$$

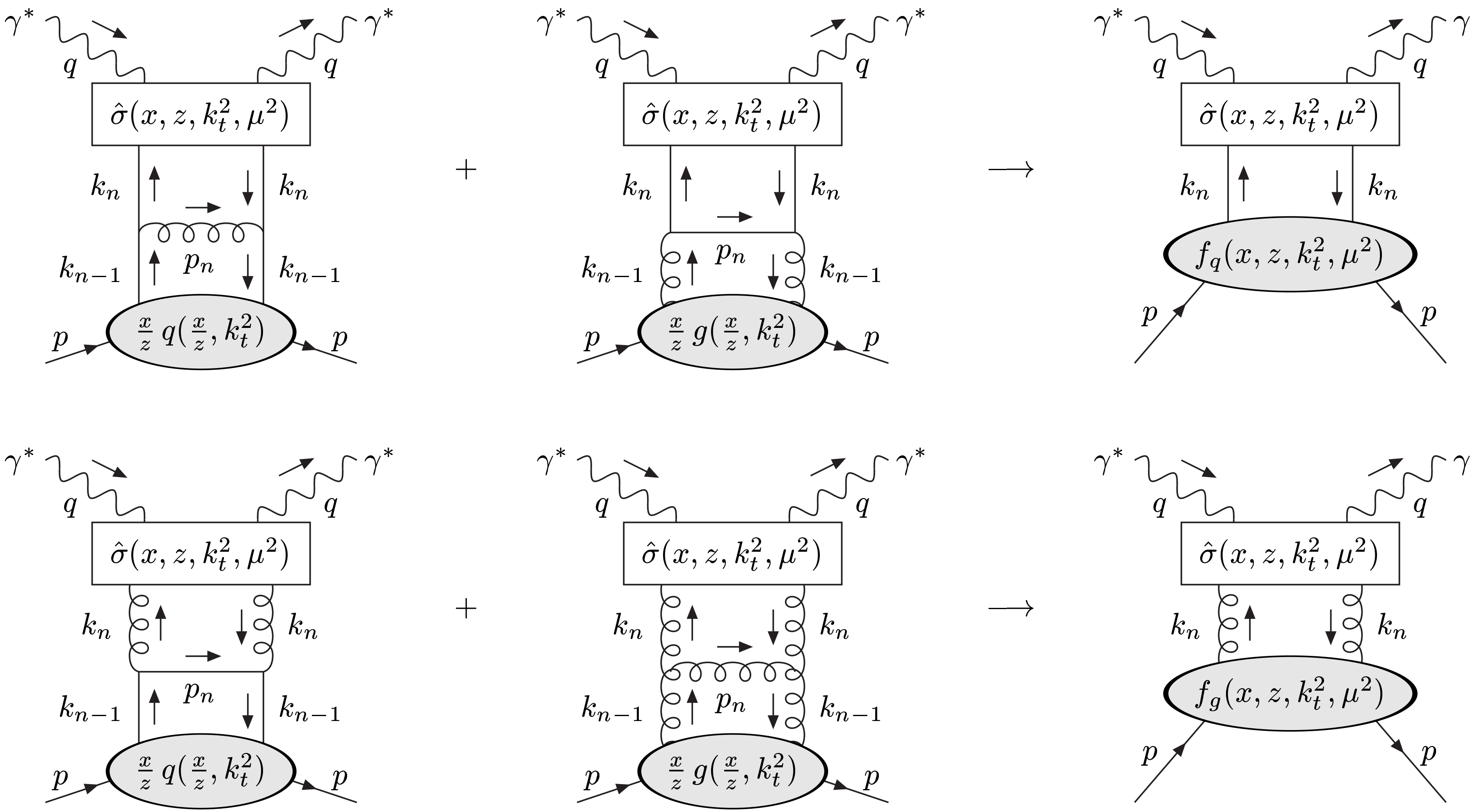
- Collinear factorisation: $z \neq 0, k_t = 0$ ($\Rightarrow \beta = 0$), $\Rightarrow$ PDFs
- k_t-factorisation: $z = 0, k_t \neq 0$ ($\Rightarrow \beta = 0$), $\Rightarrow$ uPDFs
- (z, k_t)-factorisation: $z \neq 0, k_t \neq 0$ ($\Rightarrow \beta \neq 0$), $\Rightarrow$ duPDFs

$$\int_x^1 \mathrm{d}z\, f_a(x, z, k_t^2, \mu^2) = f_a(x, k_t^2, \mu^2)$$

$$\Longrightarrow \boxed{f_a(x, z, k_t^2, \mu^2) = T_a(k_t^2, \mu^2)\,\frac{\alpha_S(k_t^2)}{2\pi}\sum_b P_{ab}(z)\, b\left(\frac{x}{z}, k_t^2\right)}$$

+ angular-ordering constraints

(z, k_t)-factorisation



(z, k_t)-factorisation

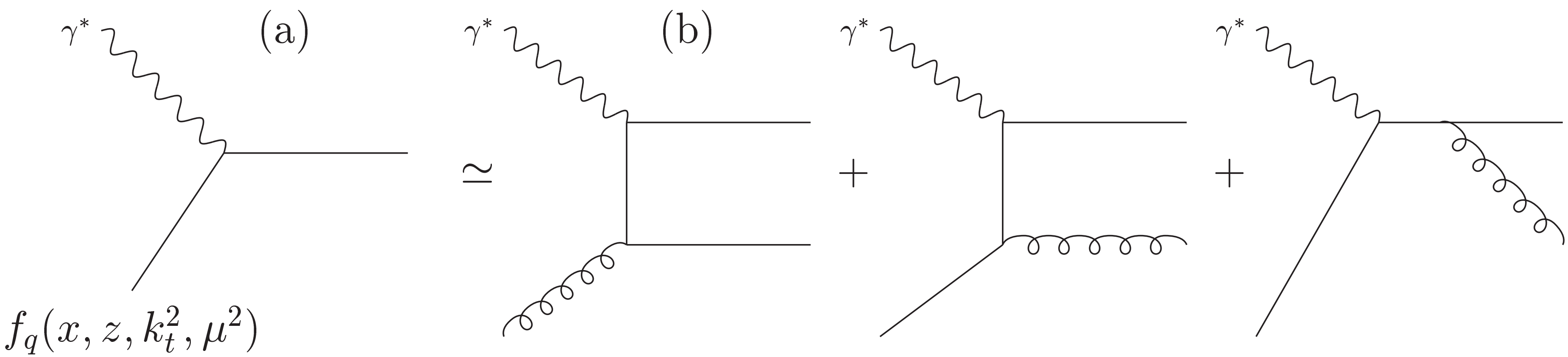
$$\boxed{\sigma^{\gamma^* p} = \sum_a \int_0^1 \frac{\mathrm{d}x}{x} \int_x^1 \mathrm{d}z \int_0^\infty \frac{\mathrm{d}k_t^2}{k_t^2}\, f_a(x, z, k_t^2, \mu^2)\, \hat{\sigma}^{\gamma^* a}}$$

How to calculate $\hat{\sigma} = \int \mathrm{d}\Phi\, |\mathcal{M}|^2 \,/\, F$?

- F is the flux factor: same as in collinear approximation (and in k_t-factorisation)
- $|\mathcal{M}|^2$ is the squared matrix element: last evolution step only factorises (to give LO DGLAP splitting kernels) if evaluated in collinear approximation ($k = x\, p$)
- $\mathrm{d}\Phi$ is the phase space element: evaluate with full kinematics ($k = x\, p - \beta\, q' + k_\perp$)

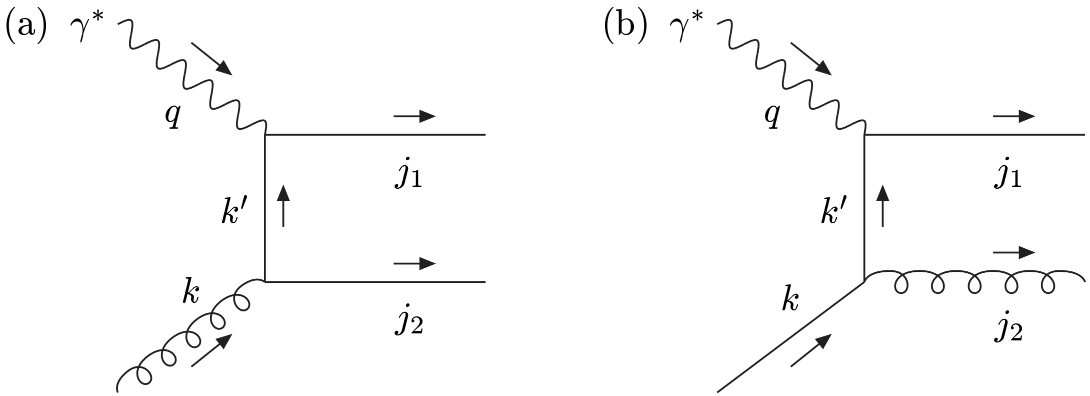
Application: inclusive jets in DIS

- Inclusive jet cross section: count all jets satisfying cuts on transverse energy E_T and rapidity η



- (a) (z, k_t)-factorisation: subprocess $\mathcal{O}(\alpha_S^0)$
  - Count last-step emission $\Rightarrow$ 2 jets with $E_T = k_t$
- (b) Collinear approximation: subprocess $\mathcal{O}(\alpha_S^1)$ (LO QCD)

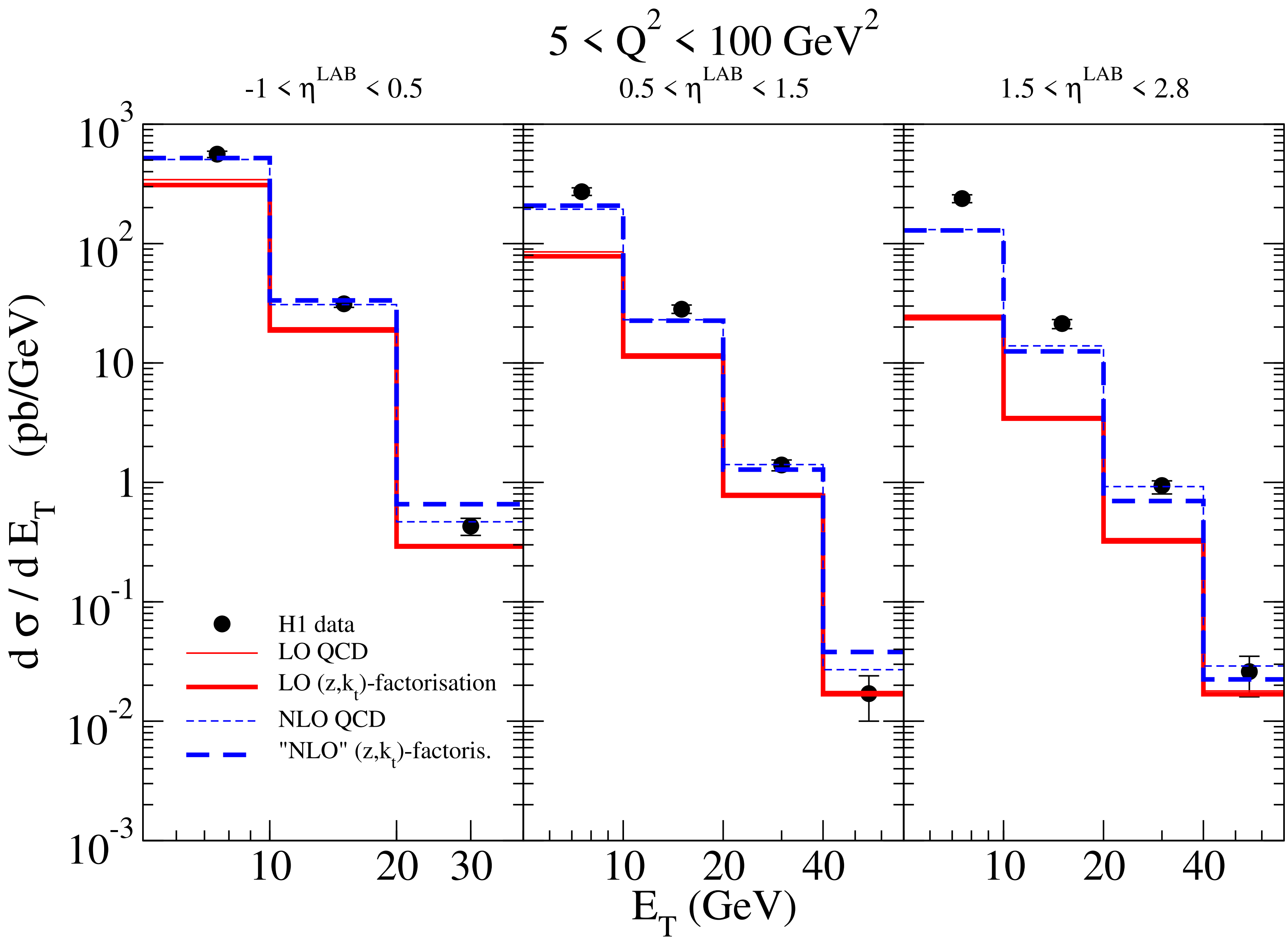
Inclusive jets in DIS at "NLO"



- $k = x\,p - \beta\,q' + k_\perp$
- Use axial gluon gauge to suppress other diagrams

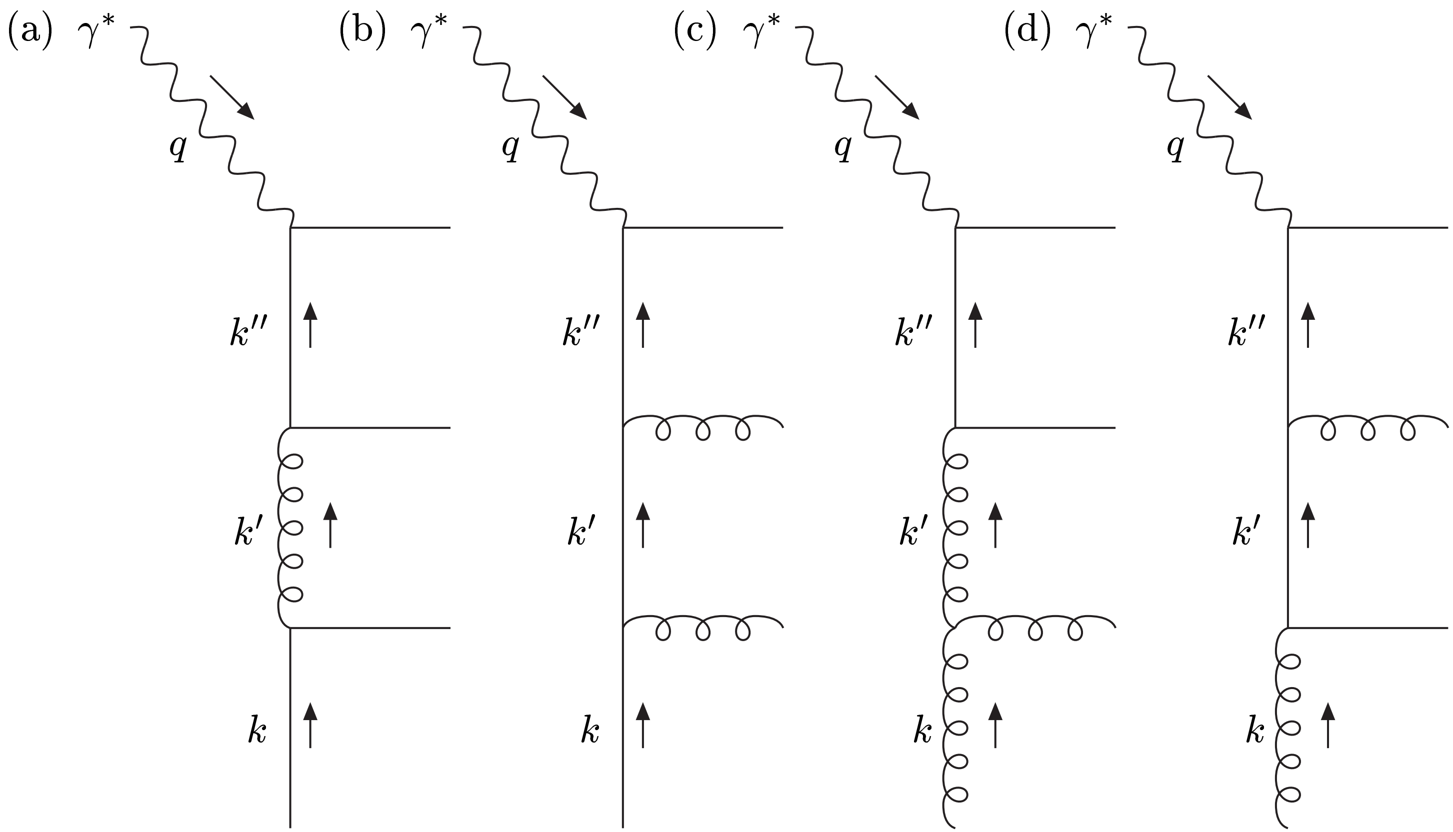
- (a) depends on doubly-unintegrated gluon distribution
- (b) depends on doubly-unintegrated quark distribution
  - Angular ordering regulates soft gluon singularity ($\eta_{j_1} < \eta_{j_2}$)
- 3 outgoing partons $\Rightarrow$ pass through jet algorithm
- Approximation to full $\mathcal{O}(\alpha_S^2)$ NLO QCD calculation

Comparison with H1 data



Jets in DIS: possible extensions

- “NNLO” (z, k_t)-factorisation $\simeq \mathcal{O}(\alpha_S^3)$ NNLO QCD ?



- Define duPDFs of the photon and calculate resolved photon contribution ?

$W, Z,$ Higgs P_T distributions

- Fixed-order cross section: divergent terms $\propto \ln(M_{V,H}/P_T)$ $(V = W, Z)$ appear due to soft and collinear gluon emission
- Need to analytically resum these terms to all orders in α_S (or use a numerical parton shower simulation)

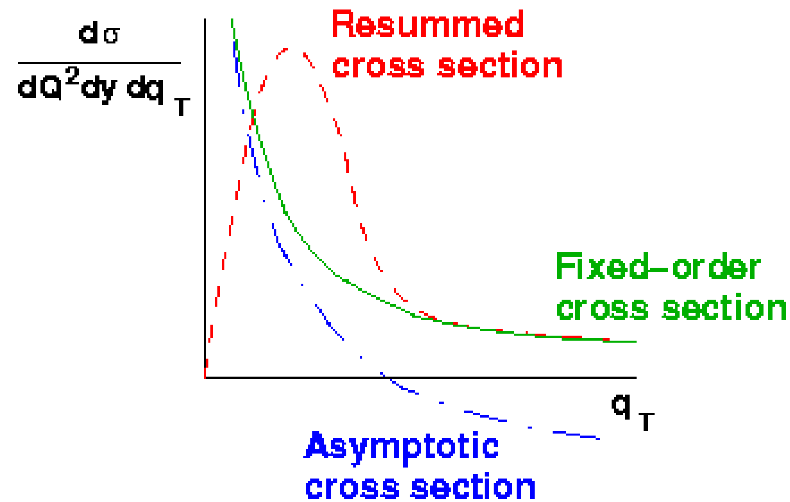


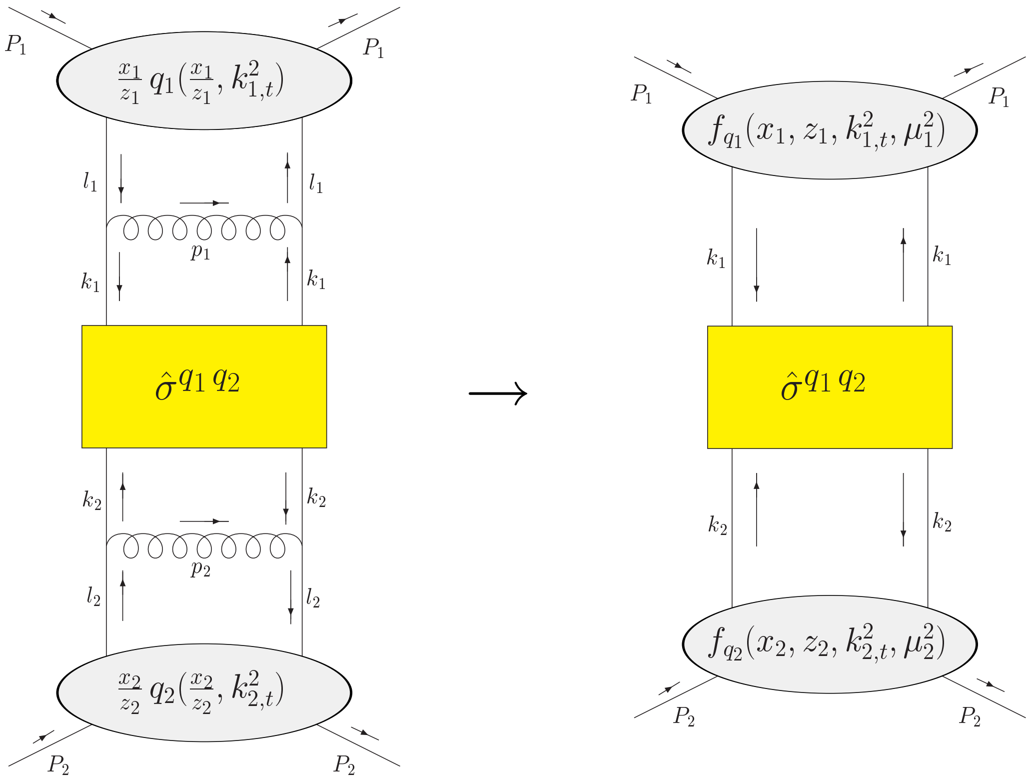
Figure taken from:

`http://hep.pa.msu.edu/wwwlegacy/`

Here, $Q^2 \equiv M^2_{V,H}$, $q_T \equiv P_T$ and y is rapidity

- CCFM equations in "single loop approximation" embody conventional soft gluon resummation formulae (Gawron, Kwieciński, Szczurek)
- Alternative approach: use duPDFs

(z, k_t)-factorisation at pp and $p\bar{p}$ colliders



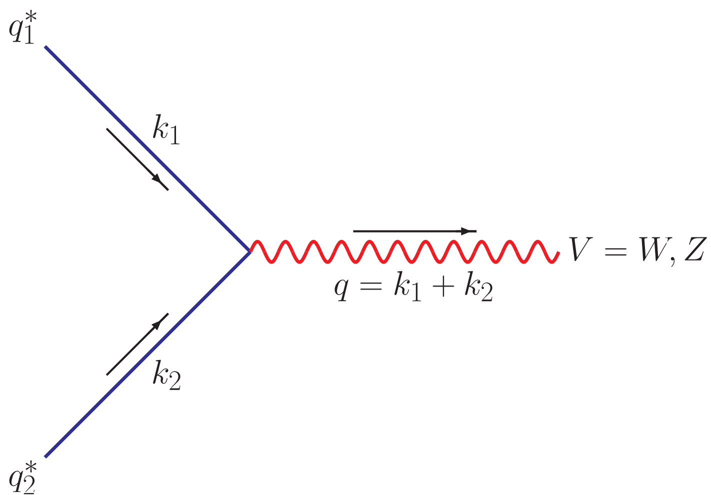
Kinematics:

- $l_i = \frac{x_i}{z_i} P_i$
- $k_i = x_i P_i - \beta_i P_j + k_{i\perp}$
- $\beta_i = \frac{z_i}{x_i (1-z_i)} \frac{k_{i,t}^2}{s}$
- $(i, j) = (1, 2)$ or $(2, 1)$
- $s = (P_1 + P_2)^2 = 2 P_1 \cdot P_2$

$$\sigma = \sum_{a_1, a_2} \int_0^1 \frac{\mathrm{d}x_1}{x_1} \int_0^1 \frac{\mathrm{d}x_2}{x_2} \int_{x_1}^1 \mathrm{d}z_1 \int_{x_2}^1 \mathrm{d}z_2 \int_0^\infty \frac{\mathrm{d}k_{1,t}^2}{k_{1,t}^2} \int_0^\infty \frac{\mathrm{d}k_{2,t}^2}{k_{2,t}^2}$$

$$f_{a_1}(x_1, z_1, k_{1,t}^2, \mu_1^2)\; f_{a_2}(x_2, z_2, k_{2,t}^2, \mu_2^2)\; \hat{\sigma}^{a_1 a_2}$$

Application: W and Z P_T distributions



- Precise kinematics:

  $\Rightarrow \delta(q^2 - M_V^2)$ and $\delta(q_t - P_T)$,

  where $q^2 = (x_1 - \beta_2)(x_2 - \beta_1)\, s - q_t^2$

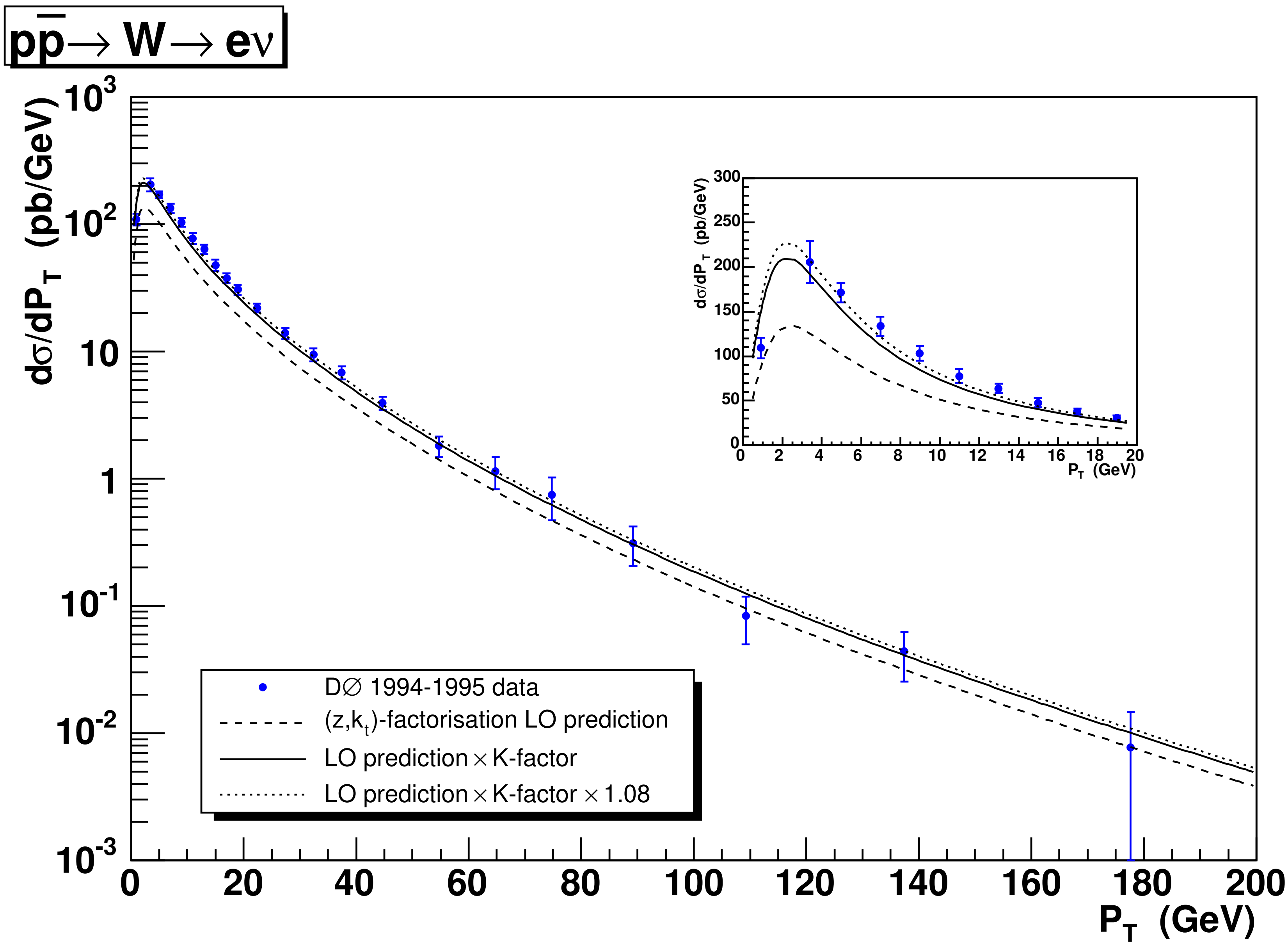
  and $q_t = |\boldsymbol{k_{1,t}} + \boldsymbol{k_{2,t}}|$

- Include non-logarithmic π^2-enhanced loop corrections (Parisi, Curci, Greco):

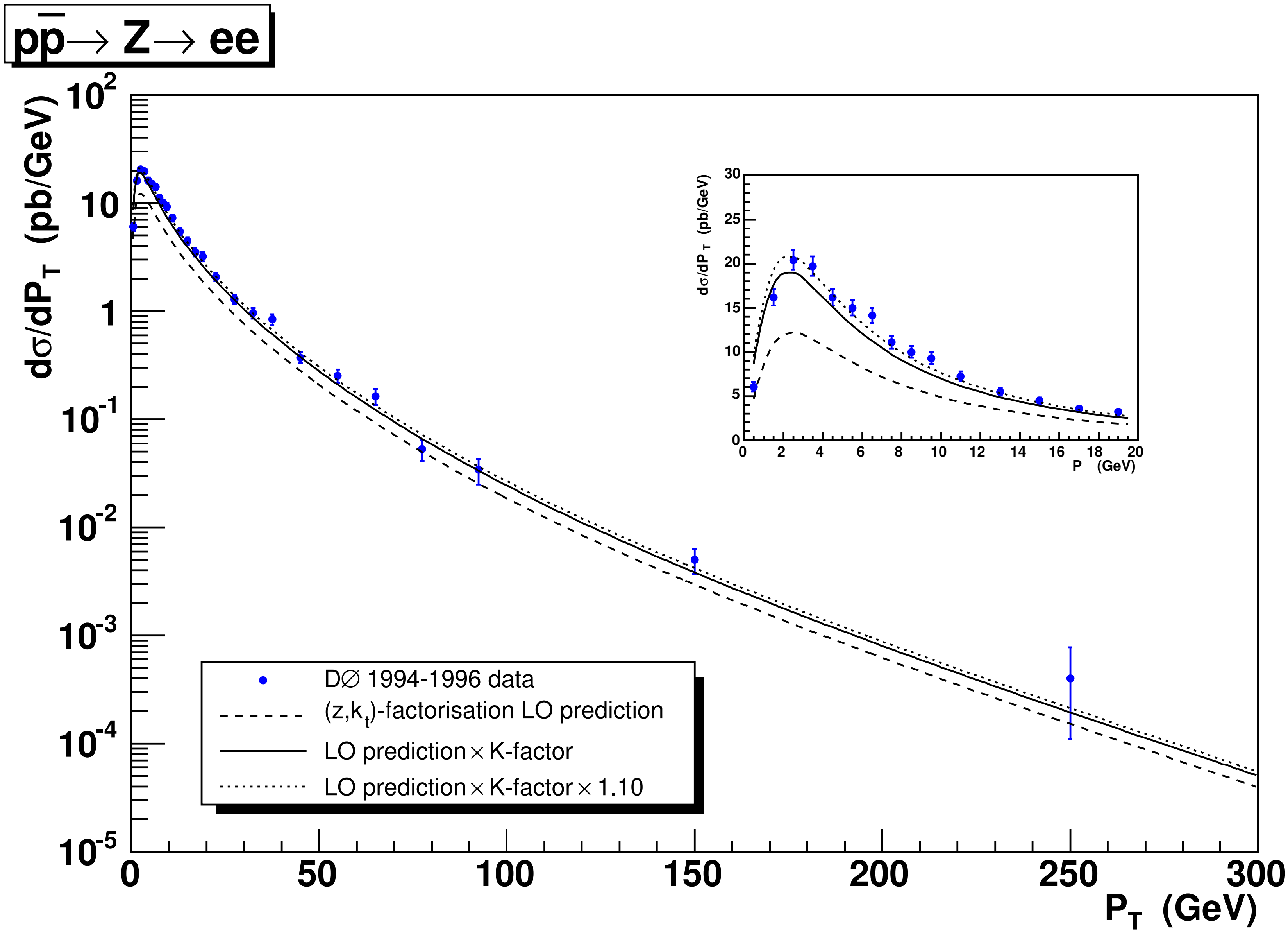
$$K(q_1^* \, q_2^* \to V) \simeq \left| \frac{T_q(k_t^2, -\mu^2)}{T_q(k_t^2, \mu^2)} \right|^2 \simeq \exp\left(C_F \, \frac{\alpha_S(\mu^2)}{2\,\pi} \, \pi^2 \right)$$

  - Scale choice $\mu = P_T^{2/3} \, M_V^{1/3}$ eliminates certain sub-leading logarithms in T_q (Kulesza, Stirling)

$W\ P_T$ distribution at Tevatron Run 1

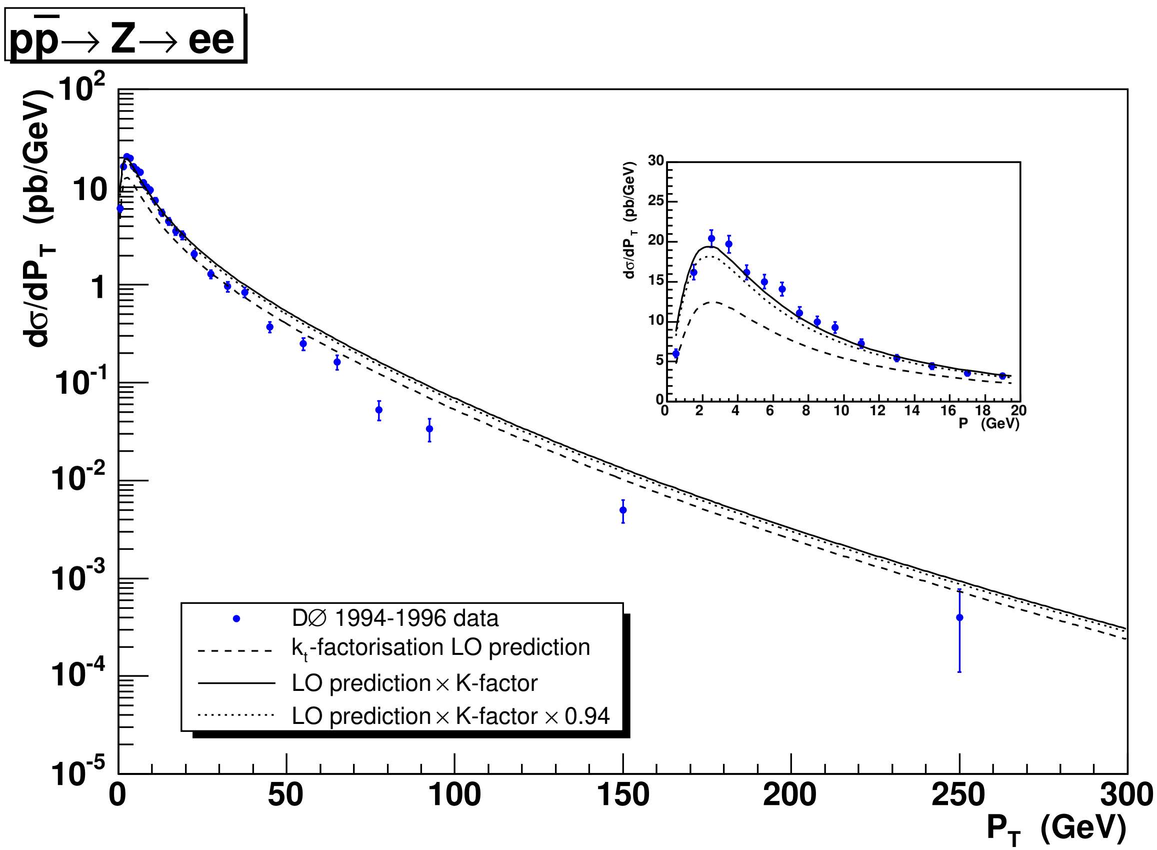


$Z\ P_T$ distribution at Tevatron Run 1



What difference does the extra z make?

- Set $z_i \to 0$ in $\hat{\sigma}$, $k_i = x_i\, P_i + k_{i,\perp} \quad \Rightarrow \quad \approx k_t$-factorisation



- Not much difference at small P_T, but too big at large P_T
- Description of jets in DIS much worse

Proton structure function $F_2(x_B, Q^2)$

Compare predictions using:

- Collinear approximation ($\gamma^* q \to q$):

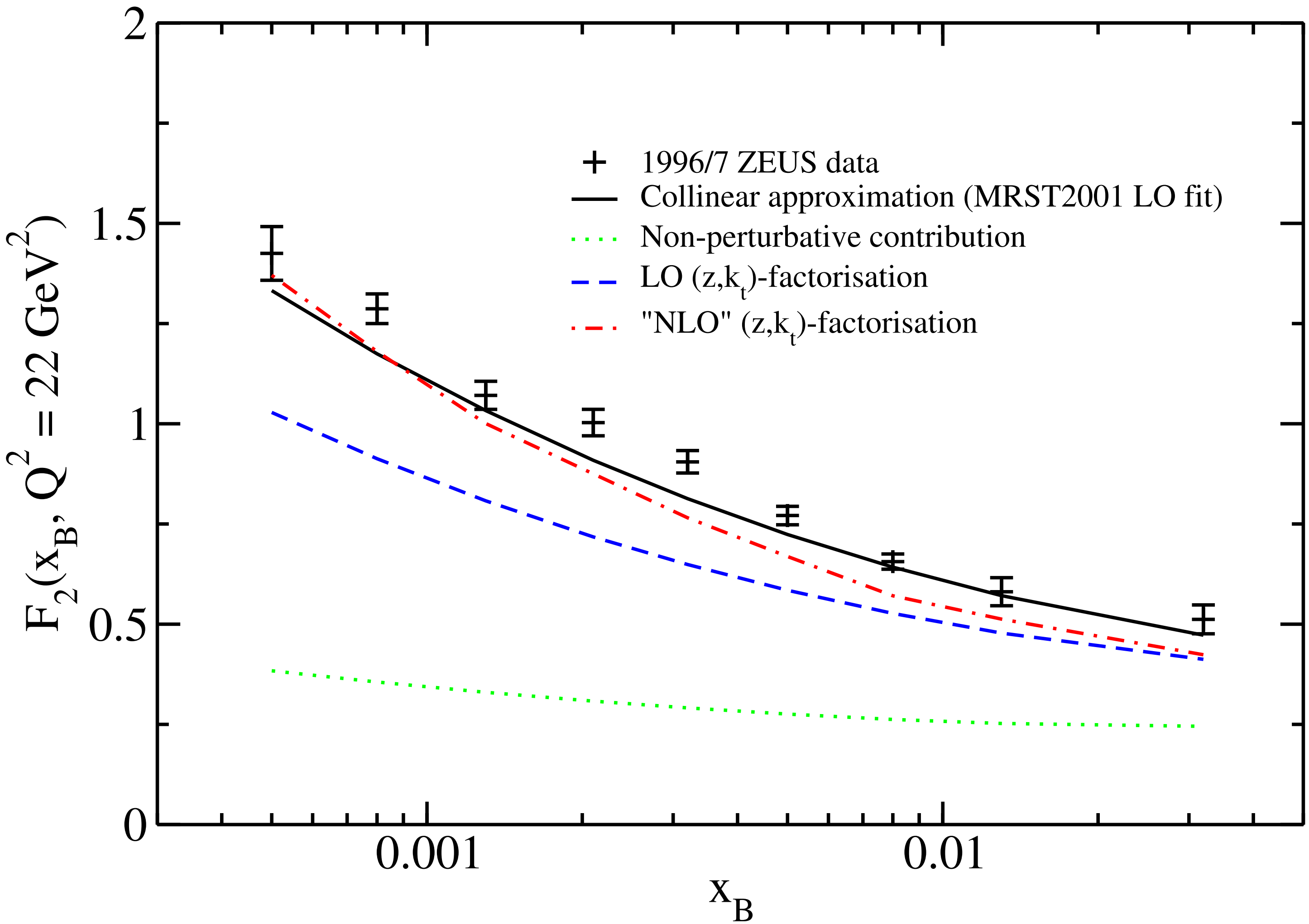
$$F_2(x_B, Q^2) = \sum_q e_q^2\, x_B q(x_B, Q^2)$$

- LO (z, k_t)-factorisation ($\gamma^* q^* \to q$):

$$F_2(x_B, Q^2) = \sum_q e_q^2\, x_B q(x_B, \mu_0^2)\, T_q(\mu_0^2, Q^2)$$
$$+ \int_x^1 \mathrm{d}z \int_{\mu_0^2}^{\infty} \frac{\mathrm{d}k_t^2}{k_t^2}\, \frac{x_B/x}{1 - x_B\beta/x} \sum_q e_q^2 f_q(x, z, k_t^2, \mu^2)$$

- "NLO" (z, k_t)-factorisation ($\gamma^* g^* \to q\bar{q}$ and $\gamma^* q^* \to qg$)

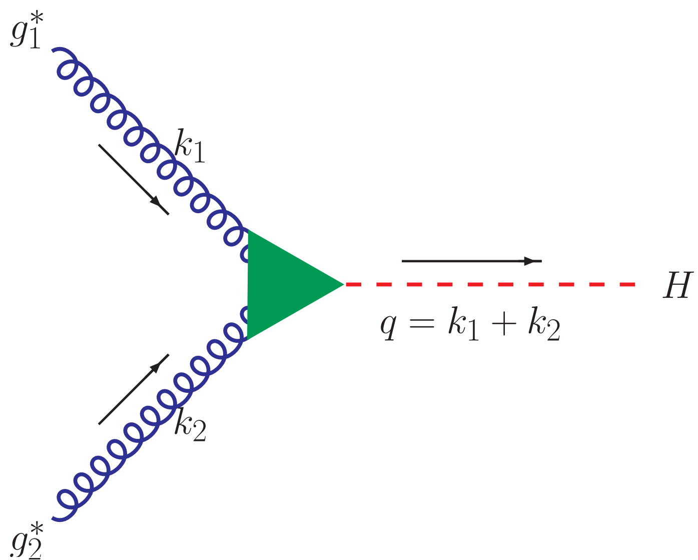
Proton structure function $F_2(x_B, Q^2)$



- For higher accuracy, should refit input integrated PDFs

Application: SM Higgs P_T at LHC

- Dominant production mechanism: gluon-gluon fusion via top quark loop

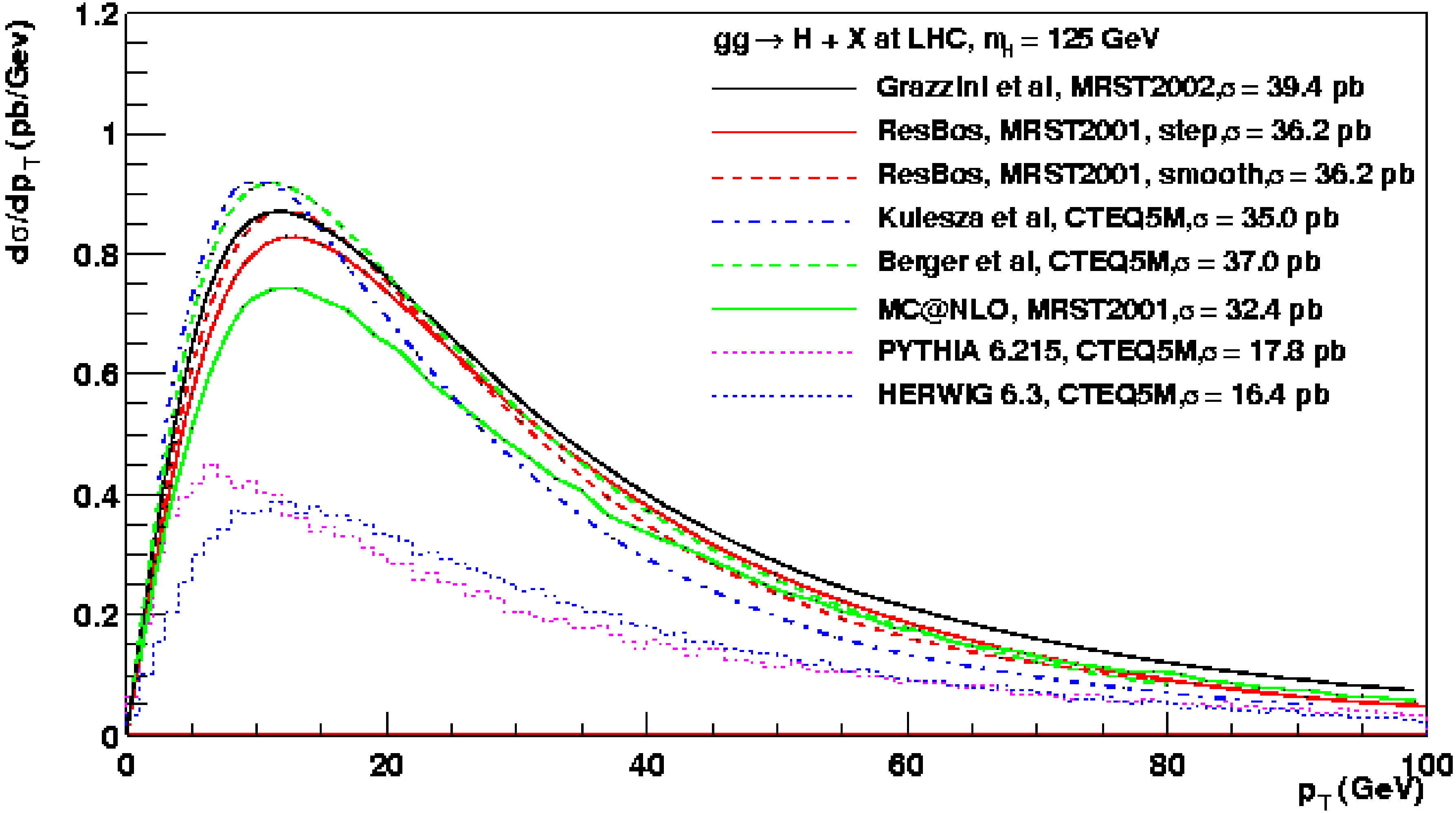


- Triangle represents effective vertex in limit $M_H \ll 2m_t$
- Correction due to top quark mass $\approx \left[1 + \left(\frac{M_H}{2m_t}\right)^2\right]^2$

- Kinematics identical to $q_1^* \, q_2^* \to V$
- K-factor same apart from different colour factor:

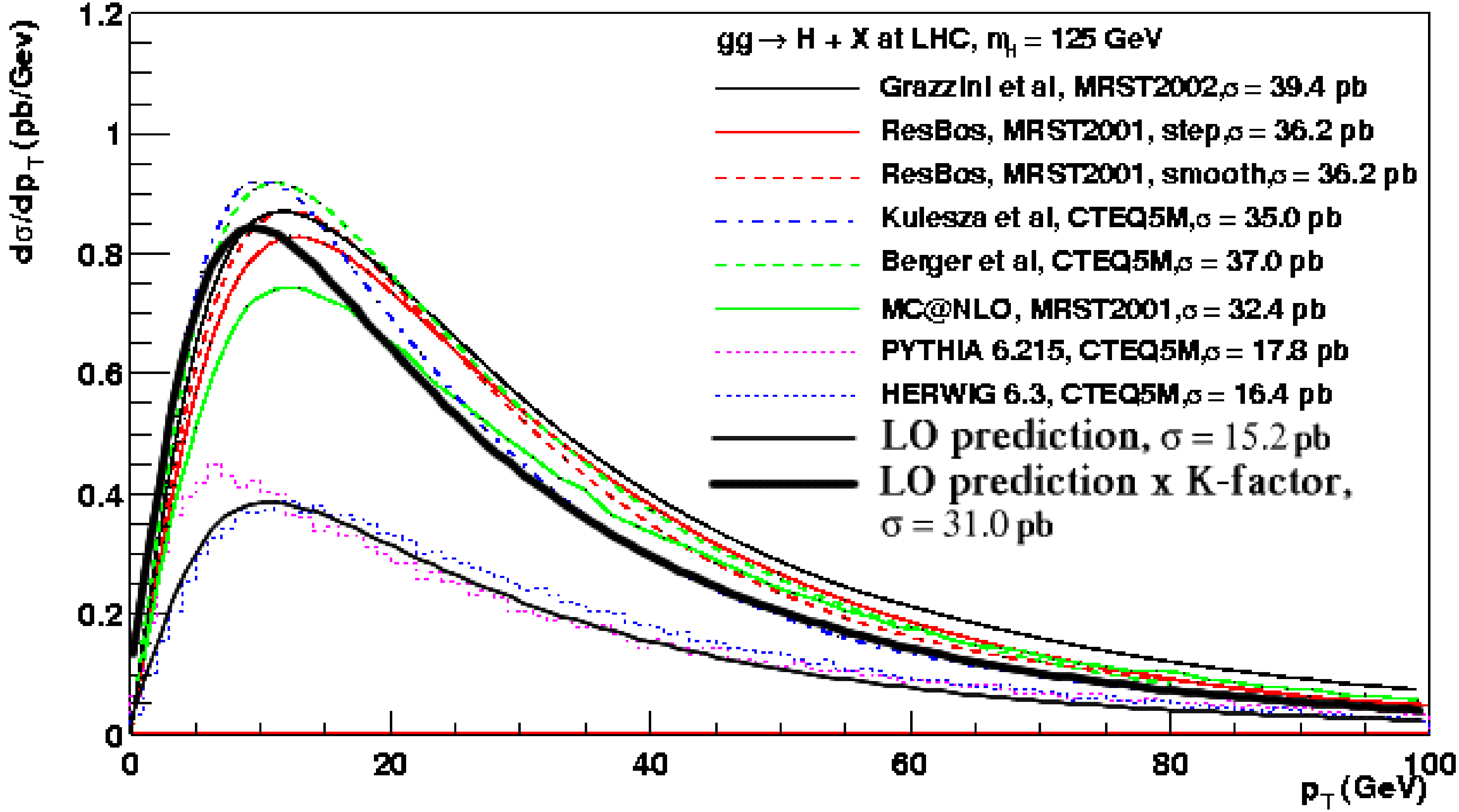
$$C_F = 4/3 \to C_A = 3$$

Higgs P_T distribution at LHC



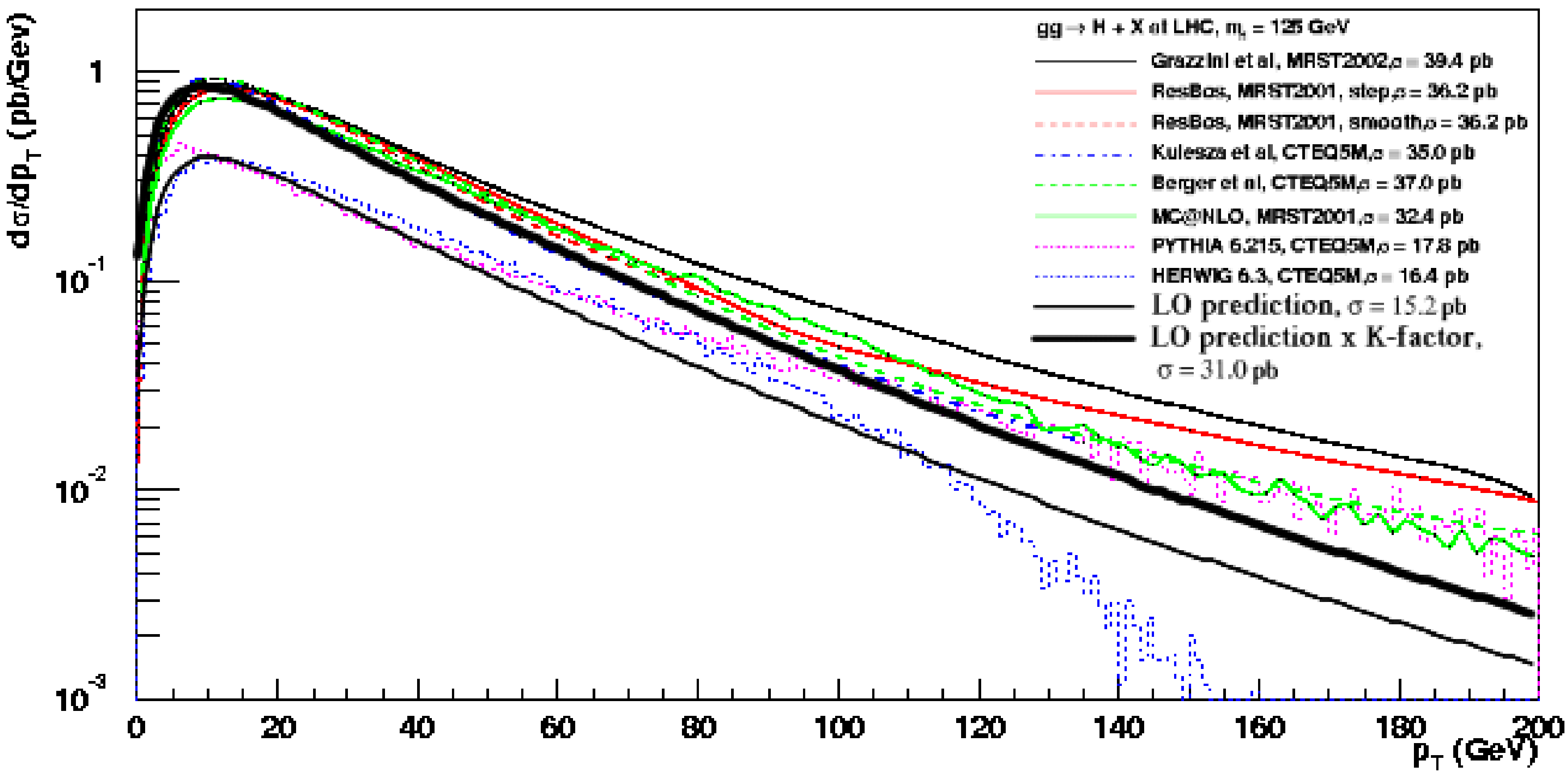
C. Balazs, M. Grazzini, J. Huston, A. Kulesza and I. Puljak, arXiv:hep-ph/0403052.

Higgs P_T distribution at LHC



Good agreement with more sophisticated approaches

Higgs P_T distribution at LHC



Need $g_1^* \, g_2^* \to gH$ and $q_i^* \, g_j^* \to qH$ subprocesses at large P_T

Conclusions

- Refined KMR 'last-step' procedure for determining uPDFS from conventional integrated PDFs
- Both quarks and gluons naturally included, with full LO DGLAP splitting kernels, not just the singular terms
- Only input needed is usual LO DGLAP-evolved integrated PDFs (MRST, CTEQ, . . .)
- To keep the precise kinematics in the subprocess ($z \neq 0$), need *doubly*-unintegrated PDFs and (z, k_t)-factorisation
- Good description of inclusive jets in DIS and P_T distributions of electroweak bosons (W, Z, Higgs)
- Other processes to study: prompt photon, $b\bar{b}$ production (but maybe better to refit input integrated PDFs first)