Proposal for an improved "M_X method"

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Abstract

The "M_X method" is shown to be justified by the Regge theory of DDIS only in the limit of $Q^2 \ll M_X^2$. A more general method is proposed, where the subtraction of non-diffractive events is made by looking at either the $(M_X^2 + Q^2)$ distribution, or better, the $x_{\mathbb{P}}$ distribution. It is possible that a more precise subtraction of non-diffractive events may explain the apparent discrepancy in the Q^2 dependence of DDIS data extracted using the "M_X method" compared to DDIS data extracted by looking for a large rapidity gap.

Recall that the present "M_X method" [1] subtracts the non-diffractive contribution in each (W, Q^2) bin according to

$$\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln M_X^2} = D + \underbrace{c\exp(b\ln M_X^2)}_{\text{non-diffractive}}, \tag{1}$$

where $\ln M_X^2 \in [\ln W^2 - 5.6, \ln W^2 - 2.2]$, and the parameters D, c and b are fitted to data. This formula is motivated by the Regge theory of DDIS (see Chapter 10.5 of [2]) in certain kinematic limits. Here, we examine the approximations made in the derivation of (1), and propose a more precise version of this formula.[1]

For $W^2 \gg M_X^2$ and $Q^2 \gg t$, DDIS can be described by triple Regge diagrams. There are two contributions: the triple Pomeron ($\mathbb{PPP}$) diagram and the Pomeron–Pomeron–Reggeon ($\mathbb{PPR}$) diagram. The DDIS cross section is then given by

$$\begin{aligned}\frac{\mathrm{d}^2\sigma^{\mathrm{D}}_{\gamma^* p}}{\mathrm{d}M_X^2\mathrm{d}t} = {} & A_{\mathbb{PPP}}(t)(W^2)^{2\alpha_{\mathbb{P}}(t)-2}(M_X^2+Q^2)^{\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{P}}(t)} \\ & + A_{\mathbb{PPR}}(t)(W^2)^{2\alpha_{\mathbb{P}}(t)-2}(M_X^2+Q^2)^{\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{P}}(t)};\end{aligned} \tag{2}$$

see (10.21) of [2]. The non-diffractive cross section is given similarly, as a sum of the contributions from the $\mathbb{RRP}$ diagram and the $\mathbb{RRR}$ diagram, by

$$\begin{aligned}\frac{\mathrm{d}^2\sigma^{\mathrm{ND}}_{\gamma^* p}}{\mathrm{d}M_X^2\mathrm{d}t} = {} & A_{\mathbb{RRP}}(t)(W^2)^{2\alpha_{\mathbb{R}}(t)-2}(M_X^2+Q^2)^{\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{R}}(t)} \\ & + A_{\mathbb{RRR}}(t)(W^2)^{2\alpha_{\mathbb{R}}(t)-2}(M_X^2+Q^2)^{\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{R}}(t)};\end{aligned} \tag{3}$$

[1] Although we use the language of Regge theory, the results should also be true in pQCD, after making the approximation that the Pomeron and secondary Reggeons, which in pQCD are branch cuts in the complex angular momentum plane, can be replaced by 'effective' poles.

see (10.37) of [2]. The $\ln(M_X^2+Q^2)$ distribution will therefore be given by a sum of four terms:

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\gamma^*p}}{\mathrm{d}\ln(M_X^2+Q^2)} =& A_{\mathbb{PPP}}(\bar{t})(W^2)^{2\alpha_{\mathbb{P}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{P}}(\bar{t})}\\ &+A_{\mathbb{PPR}}(\bar{t})(W^2)^{2\alpha_{\mathbb{P}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{P}}(\bar{t})}\\ &+A_{\mathbb{RRP}}(\bar{t})(W^2)^{2\alpha_{\mathbb{R}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{R}}(\bar{t})}\\ &+A_{\mathbb{RRR}}(\bar{t})(W^2)^{2\alpha_{\mathbb{R}}(\bar{t})-2}(M_X^2+Q^2)^{1+\alpha_{\mathbb{R}}(0)-2\alpha_{\mathbb{R}}(\bar{t})}, \end{aligned}\tag{4}$$

where $\bar{t}$ is some average value of t. The "M_X method" is valid[2] when $Q^2 \ll M_X^2$, meaning that $\beta = Q^2/(Q^2+M_X^2) \ll 1$. In this limit, the $\mathbb{PPP}$ and $\mathbb{RRP}$ diagrams dominate over the $\mathbb{PPR}$ and $\mathbb{RRR}$ diagrams, and so (4) reduces to

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\gamma^*p}}{\mathrm{d}\ln M_X^2} &= c_{\mathbb{P}}(M_X^2)^{-b_{\mathbb{P}}} + c_{\mathbb{R}}(M_X^2)^{b_{\mathbb{R}}}\\ &= c_{\mathbb{P}}\exp(-b_{\mathbb{P}}\ln M_X^2) + c_{\mathbb{R}}\exp(b_{\mathbb{R}}\ln M_X^2),\end{aligned}\tag{5}$$

where the powers $-b_{\mathbb{P}} = 1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{P}}(\bar{t}) \leq 0$ and $b_{\mathbb{R}} = 1+\alpha_{\mathbb{P}}(0)-2\alpha_{\mathbb{R}}(\bar{t}) \geq 0$, and the parameters $c_{\mathbb{P}}$ and $c_{\mathbb{R}}$ depend on W. The "M_X method" [1] further assumes that $\alpha_{\mathbb{P}}(0) \approx \alpha_{\mathbb{P}}(\bar{t}) \approx 1$, so that $b_{\mathbb{P}} \approx 0$, then (5) reduces to (1).

A more precise subtraction of the non-diffractive contribution, valid not only when $Q^2 \ll M_X^2$, may thus be obtained by looking at the $\ln(M_X^2+Q^2)$ distribution:

$$\boxed{\frac{\mathrm{d}\sigma_{\gamma^*p}}{\mathrm{d}\ln(M_X^2+Q^2)} = \underbrace{c_{\mathbb{P}}(M_X^2+Q^2)^{-b_{\mathbb{P}}}}_{\text{diffractive}} + \underbrace{c_{\mathbb{R}}(M_X^2+Q^2)^{b_{\mathbb{R}}}}_{\text{non-diffractive}},}\tag{6}$$

where the first term of (6) interpolates between the $\mathbb{PPP}$ and $\mathbb{PPR}$ contributions to (4) and the second term of (6) interpolates between the $\mathbb{RRP}$ and $\mathbb{RRR}$ contributions to (4). The four parameters $c_{\{\mathbb{P},\mathbb{R}\}} \geq 0$ and $b_{\{\mathbb{P},\mathbb{R}\}} \geq 0$ should be fitted separately in each (W, Q^2) bin. Note that the effective Pomeron intercept in DDIS, $\alpha_{\mathbb{P}}(0)$, lies significantly above 1 [1], so $b_{\mathbb{P}}$ should not be set to zero.

However, although (6) is an improvement on (1), it uses a combination of two powers of $(M_X^2+Q^2)$ to approximate the combination of four powers of $(M_X^2+Q^2)$ given in (4). A much more direct method of subtracting the non-diffractive contribution to the inclusive cross section is to look at the $x_{\mathbb{P}}$ distribution. In terms of the variables $x_{\mathbb{P}} = (Q^2+M_X^2)/(Q^2+W^2)$ and $\beta = Q^2/(Q^2+M_X^2)$, the diffractive (2) and non-diffractive (3) cross sections can be written as

$$\frac{\mathrm{d}^2\sigma^{\mathrm{D}}_{\gamma^*p}}{\mathrm{d}x_{\mathbb{P}}\mathrm{d}t} = \frac{|g_{\mathbb{P}}(t)|^2}{16\pi^2}x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)}\left[\mathcal{A}_{\mathbb{PPP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{PPR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right],\tag{7}$$

$$\frac{\mathrm{d}^2\sigma^{\mathrm{ND}}_{\gamma^*p}}{\mathrm{d}x_{\mathbb{P}}\mathrm{d}t} = \frac{|g_{\mathbb{R}}(t)|^2}{16\pi^2}x_{\mathbb{P}}^{1-2\alpha_{\mathbb{R}}(t)}\left[\mathcal{A}_{\mathbb{RRP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{RRR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right];\tag{8}$$

see Chapter 10.5 of [2]. The $\ln x_{\mathbb{P}}$ distribution is therefore given by

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\gamma^*p}}{\mathrm{d}\ln x_{\mathbb{P}}} =& \frac{|g_{\mathbb{P}}(\bar{t})|^2}{16\pi^2}x_{\mathbb{P}}^{2-2\alpha_{\mathbb{P}}(\bar{t})}\left[\mathcal{A}_{\mathbb{PPP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{PPR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right]\\ &+\frac{|g_{\mathbb{R}}(\bar{t})|^2}{16\pi^2}x_{\mathbb{P}}^{2-2\alpha_{\mathbb{R}}(\bar{t})}\left[\mathcal{A}_{\mathbb{RRP}}(Q^2)\beta^{1-\alpha_{\mathbb{P}}(0)} + \mathcal{A}_{\mathbb{RRR}}(Q^2)\beta^{1-\alpha_{\mathbb{R}}(0)}\right],\end{aligned}\tag{9}$$

[2]Note that the references [60–62] to the triple Regge formalism, given in [1], all concern hadron–hadron collisions, where the masses of the incoming particles are neglected. Therefore, the same formalism does not directly apply to DIS, where there is a finite photon virtuality.

where again $\bar{t}$ is some average value of t. Therefore, the non-diffractive contribution may be more directly subtracted by looking at the $\ln x_{\mathbb{P}}$ distribution:

$$\boxed{\frac{\mathrm{d}\sigma_{\gamma^* p}}{\mathrm{d}\ln x_{\mathbb{P}}} = \underbrace{c_{\mathbb{P}}(x_{\mathbb{P}})^{-b_{\mathbb{P}}}}_{\text{diffractive}} + \underbrace{c_{\mathbb{R}}(x_{\mathbb{P}})^{b_{\mathbb{R}}}}_{\text{non-diffractive}}\ .} \tag{10}$$

In the most general case, the four parameters $c_{\{\mathbb{P},\mathbb{R}\}} \geq 0$ and $b_{\{\mathbb{P},\mathbb{R}\}} \geq 0$ should be fitted separately in each (β, Q^2) bin, in some limited range of $x_{\mathbb{P}}$ where the inclusive cross section is observed to go from slightly decreasing to strongly increasing with increasing $x_{\mathbb{P}}$. If the data are not sufficient to constrain the powers $b_{\{\mathbb{P},\mathbb{R}\}}$ in each bin, they could be taken to be independent of β, or even independent of both β and Q^2. (The latter assumption is the "Regge factorisation" ansatz made in some DGLAP fits of diffractive structure functions.) The non-diffractive term in (10) should be negligible for $x_{\mathbb{P}} \lesssim 0.01$. Since the "$M_X$ method" (1) is only valid for $Q^2 \ll M_X^2$, the use of the more precise "$(M_X^2 + Q^2)$ method" (6) or the "$x_{\mathbb{P}}$ method" (10) may explain the apparent discrepancy in the Q^2 dependence of DDIS data [1] compared to DDIS data extracted by looking for a large rapidity gap.

References

[1] S. Chekanov *et al.* [ZEUS Collaboration], Nucl. Phys. B **713** (2005) 3.

[2] V. Barone and E. Predazzi, *High-energy particle diffraction*, Heidelberg, Germany: Springer-Verlag (2002).