Diffractive, generalised and unintegrated PDFs

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Introduction

- Most effort has been concentrated on determining "standard" PDFs for use in fixed-order (usually NLO, $\overline{\mathrm{MS}}$) collinear factorisation calculations of inclusive processes.
- (This is the cleanest theoretical situation, but is not necessarily consistent with calculations in MC generators.)
- **This talk:** discuss some "non-standard" PDFs used for specific types of processes: diffractive, generalised and unintegrated.
- (Won't discuss polarised, nuclear, multiparton, . . . , PDFs.)
- See also talk by K. Kutak for a different approach to unintegrated PDFs.

Introduction to diffraction

- Diffractive processes characterised by a Large Rapidity Gap.
- Exchange of vacuum quantum numbers at high energies ($\equiv$ "Pomeron" exchange). What is the Pomeron ($\mathbb{P}$)?

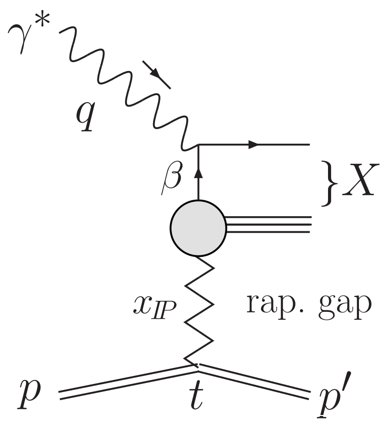
"Soft" processes ($\sim$ 1 fm)

- Use Regge theory.
- $\mathbb{P}$ is a Regge trajectory: $\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}\, t$.
- $\sigma_{\rm tot} \sim s^{\alpha_{\mathbb{P}}(0)-1}$
- $\mathrm{d}\sigma_{\rm el}/\mathrm{d}t \sim s^{2[\alpha_{\mathbb{P}}(t)-1]}$
- Donnachie–Landshoff fit [hep-ph/9209205]: $\alpha_{\mathbb{P}}(0) = 1.08$, $\alpha'_{\mathbb{P}} = 0.25$ GeV^{-2}
- *Effective:* includes multi-$\mathbb{P}$.

"Hard" processes ($\gtrsim$ 1 GeV)

- Use perturbative QCD.
- $\mathbb{P} \sim$ DGLAP/BFKL ladder $\sim$ usual gluon density.
- Appears in cross section for inclusive processes.
- Appears in **amplitude** for diffractive processes. $\Rightarrow$ Appears **squared** in the diffractive cross section.

Inclusive diffractive DIS (DDIS) kinematics



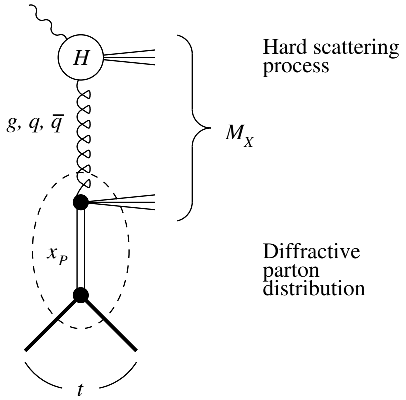
- $q^2 \equiv -Q^2$ (photon virtuality)
- $W^2 \equiv (q+p)^2 \simeq -Q^2 + 2\,p\cdot q$ $\Rightarrow$ $x_{\rm Bj} \equiv \frac{Q^2}{2\,p\cdot q} \simeq \frac{Q^2}{Q^2+W^2}$ (at LO, fraction of proton's momentum carried by struck quark)
- $t \equiv (p-p')^2 \simeq 0$, $(p-p') \simeq x_{\mathbb{P}}\, p$
- $M_X^2 \equiv (q+p-p')^2 \simeq -Q^2 + x_{\mathbb{P}}(Q^2+W^2)$ $\Rightarrow$ $x_{\mathbb{P}} \simeq \frac{Q^2+M_X^2}{Q^2+W^2} \ll 1$ (fraction of proton's momentum carried by Pomeron)
- $\beta \equiv \frac{x_{\rm Bj}}{x_{\mathbb{P}}} \simeq \frac{Q^2}{Q^2+M_X^2}$ (at LO, fraction of Pomeron's momentum carried by struck quark)

Leading-twist collinear factorisation in DDIS

Diffractive structure function (integrated over t):

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \sum_{a=q,g} \beta \int_\beta^1 \frac{\mathrm{d}z}{z}\, C_{2,a}\left(\frac{\beta}{z}\right) f^{\mathrm{D}}_{a/p}(x_{\mathbb{P}}, z, \mu_F^2)$$
$$= \sum_q e_q^2\, \beta f^{\mathrm{D}}_{q/p}(x_{\mathbb{P}}, \beta, \mu_F^2) \quad \text{at LO.}$$

- $C_{2,a}$ are the **same** coefficient functions as in inclusive DIS.
- Diffractive PDFs $f^{\mathrm{D}}_{a/p}$ satisfy DGLAP evolution.
- Proven by J. Collins [hep-ph/9709499] to hold up to power-suppressed corrections.

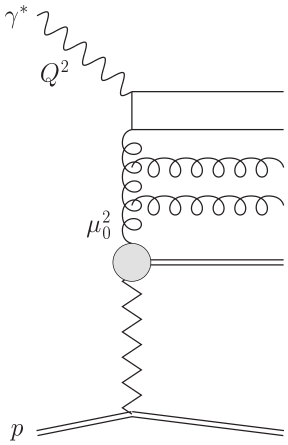


Regge factorisation in DDIS

Ingelman, Schlein [PLB 152 (1985) 256].
Assume Regge factorisation:

$$f^{\mathrm{D}}_{a/p}(x_{\mathbb{P}}, z, \mu_F^2) = f_{\mathbb{P}/p}(x_{\mathbb{P}})\, f_{a/\mathbb{P}}(z, \mu_F^2)$$

- $f_{\mathbb{P}/p}(x_{\mathbb{P}})$ is the Pomeron flux factor. Take from Regge phenomenology.
- $f_{a/\mathbb{P}}(z, \mu_F^2)$ are the Pomeron PDFs DGLAP-evolved from an input scale μ_0^2 up to the factorisation scale μ_F^2 of the hard scattering process.



Paradox: hard scattering, but assumption of soft Pomeron exchange due to DGLAP strong-ordering in virtualities: $\mu_F^2 \gg \ldots \gg \mu^2 \gg \ldots \gg \mu_0^2 \sim 1\ \mathrm{GeV}^2$.

H1 2006/7 analysis of diffractive PDFs

Pomeron flux factor

$$f_{\mathbb{P}/p}(x_{\mathbb{P}}) = \int_{t_{\rm cut}}^{t_{\rm min}} \mathrm{d}t\ \mathrm{e}^{B_{\mathbb{P}}\, t}\ x_{\mathbb{P}}^{1-2\alpha_{\mathbb{P}}(t)},$$

- Fix $\alpha'_{\mathbb{P}} = 0.06^{+0.19}_{-0.06}$ GeV^{-2} and $B_{\mathbb{P}} = 5.5^{+0.7}_{-2.0}$ GeV^{-2} from H1 leading-proton data [hep-ex/0606003].
- Treat $\alpha_{\mathbb{P}}(0)$ as a free parameter.

Input Pomeron PDFs

$$z\, f_{a/\mathbb{P}}(z,\mu_0^2) = A_a\, z^{B_a}\, (1-z)^{C_a}$$

- Evolve from $\mu_0^2 \simeq 2$ GeV^2 with usual NLO DGLAP evolution.
- No momentum sum rule: Pomeron is not a particle.
- $u = d = s = \bar{u} = \bar{d} = \bar{s}$, so only singlet and gluon.

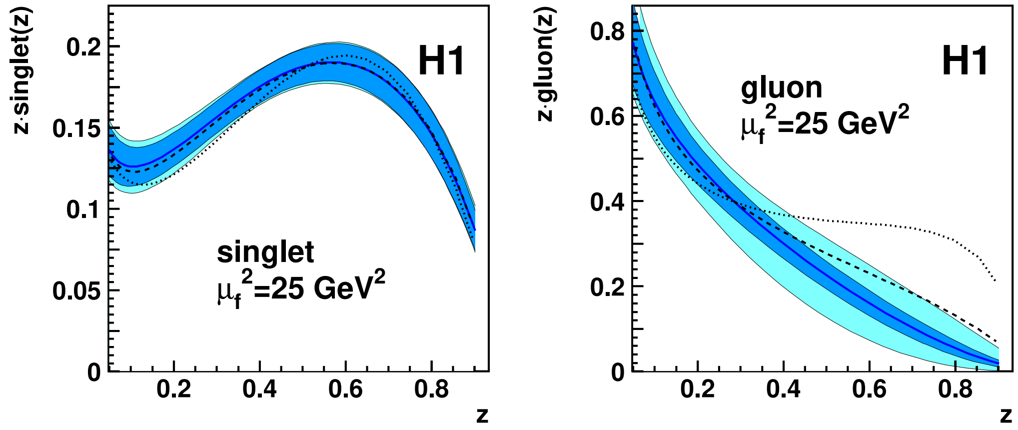
H1 2006/7 extraction of diffractive PDFs

- Fit to inclusive DDIS data points with $M_X \geq 2$ GeV and $Q^2 \geq 8.5$ GeV^2 [hep-ex/0606004].
- Also simultaneous fit to inclusive DDIS and DDIS dijets [0708.3217].

	$\alpha_{\mathbb{P}}(0)$	$z\, f_{g/\mathbb{P}}(z,\mu_0^2)$	$\chi^2_{\rm DDIS}/N_{\rm pts.}$	$\chi^2_{\rm dijet}/N_{\rm pts.}$
H1 2006 Fit A	1.118 ± 0.007	$A_g\,(1-z)^{C_g}$	158/190	—
H1 2006 Fit B	1.111 ± 0.007	A_g	164/190	—
H1 2007 Jets Fit	1.104 ± 0.007	$A_g\, z^{B_g}\,(1-z)^{C_g}$	169/190	27/36

- Comments:
  1. Cut on $Q^2 \geq 8.5$ GeV^2 required to achieve stability.
  2. $\alpha_{\mathbb{P}}(0)$ values larger than values for soft Pomeron. No reason to expect universality of these values.
  3. Fit B would be excluded if $\Delta\chi^2 = 1$ is taken literally.
  4. χ^2 of inclusive DDIS data deteriorates when dijet data are added: suggests some inconsistency within this framework.

H1 2006/7 extraction of diffractive PDFs



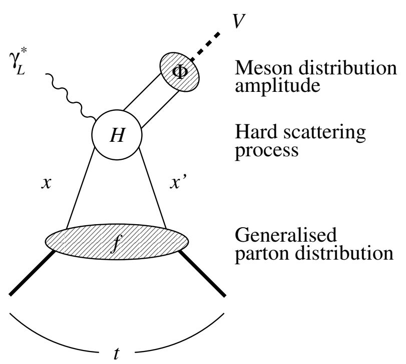
- H1 2007 Jets DPDF
- exp. uncertainty
- exp. + theo. uncertainty
- H1 2006 DPDF fit A
- H1 2006 DPDF fit B

- Gluon determined **indirectly** from scaling violations of inclusive DDIS data (Fit A) larger at high z compared to gluon determined **directly** from dijet data.

Collinear factorisation for exclusive hard processes

Collins, Frankfurt, Strikman [hep-ph/9611433]:

$$\mathcal{A}(\gamma_L^* p \to Vp) = \sum_{i,j} \int_0^1 \mathrm{d}z \int \mathrm{d}x \; \overbrace{f_{i/p}(x,x',t,\mu^2)}^{\text{Generalised PDF}} \; \overbrace{H_{ij}(x,x',z,Q^2,\mu^2)}^{\text{Hard scattering}} \; \overbrace{\Phi_j^V(z,\mu^2)}^{\text{Meson dist. amp.}}$$
$$+ \quad \text{power-suppressed corrections.}$$



- Valid also for large x, not just in the diffractive region $x \ll 1$.
- Generalised (a.k.a. skewed, off-diagonal) PDFs reduce to usual PDFs in limit of $x = x'$ and $t = 0$.
- Fourier transform from $\mathbf{\Delta}$ (where $t = -|\mathbf{\Delta}|^2$) to impact parameter $\mathbf{b}$ $\Rightarrow$ spatial distribution of partons in transverse plane.

Progress in determination of generalised PDFs

- **NLO corrections** for $\gamma_L^* p \to \rho\, p$ are huge at small-x [Diehl, Kugler, 0708.1121]. Small-x resummation needed. Important power corrections due to parton transverse momenta.
- Situation better for $\gamma^* p \to \gamma\, p$ (deeply virtual Compton scattering): known to NNLO [Kumerički, Müller, Passek-Kumerički, hep-ph/0703179].
- **"Global" analysis** of **generalised PDFs** still a long way off. Generally rely on models/approximations to relate generalised PDFs to usual PDFs, e.g. for small $|t|$ and $x' \ll x \ll 1$:

$$f_{g/p}(x, x', t, \mu^2) = \exp(B_D\, t/2)\, R_g(x, \mu^2)\, xg(x, \mu^2),$$

with t-slope B_D taken from HERA data and

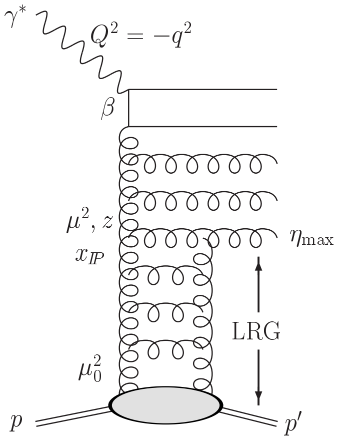
$$R_g(x, \mu^2) = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 4)},$$

assuming $xg(x, \mu^2) \sim x^{-\lambda}$ [Shuvaev *et al.*, hep-ph/9902410].

Perturbative Pomeron contribution to inclusive DDIS

Martin, Ryskin, G.W. [hep-ph/0406224, hep-ph/0504132, hep-ph/0609273]

- Hard Pomeron responsible for exclusive diffractive processes at HERA also contributes to inclusive DDIS.



- Contribution to **diffractive** PDFs is **calculable** in terms of the generalised PDFs:

$$f^{\mathrm{D}}_{a/p}(x_{\mathbb{P}}, z, \mu_F^2) = \int_{\mu_0^2}^{\mu_F^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2)\, f_{a/\mathbb{P}}(z, \mu_F^2; \mu^2)$$

- Perturbative Pomeron flux factor:

$$f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2) = \frac{1}{x_{\mathbb{P}} B_D} \left[R_g \frac{\alpha_S(\mu^2)}{\mu} x_{\mathbb{P}} g(x_{\mathbb{P}}, \mu^2) \right]^2$$

- Pomeron PDFs $f_{a/\mathbb{P}}(z, \mu_F^2; \mu^2)$ DGLAP-evolved up to μ_F^2 from input Pomeron-to-parton splitting functions $P_{a\mathbb{P}}(z) \equiv f_{a/\mathbb{P}}(z, \mu^2; \mu^2)$.

Evolution equation for diffractive PDFs

- Treat contribution from $\mu^2 < \mu_0^2 \sim 1\ \mathrm{GeV}^2$ as in usual Regge factorisation approach (need to fit to data):

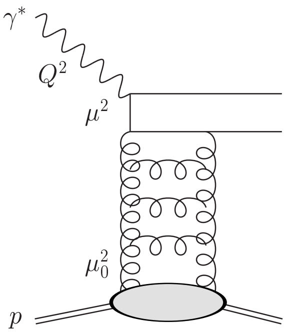
$$\Rightarrow \quad f^{\mathrm{D}}_{a/p}(x_{\mathbb{P}}, z, \mu_F^2) = f_{\mathbb{P}/p}(x_{\mathbb{P}})\, f_{a/\mathbb{P}}(z, \mu_F^2) + \int_{\mu_0^2}^{\mu_F^2} \frac{\mathrm{d}\mu^2}{\mu^2}\, f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2)\, f_{a/\mathbb{P}}(z, \mu_F^2; \mu^2)$$

- Differentiate with respect to $\ln \mu_F^2$:

$$\frac{\partial f^{\mathrm{D}}_{a/p}(x_{\mathbb{P}}, z, \mu^2)}{\partial \ln \mu^2} = \sum_{a'=q,g} P_{aa'} \otimes f^{\mathrm{D}}_{a'/p} + P_{a\mathbb{P}}(z) f_{\mathbb{P}/p}(x_{\mathbb{P}}; \mu^2)$$

- Extra **inhomogeneous term** in the evolution equation due to the perturbative Pomeron-to-parton splitting.
- cf. the evolution equation for the **photon** PDFs also has an inhomogeneous term from the photon-to-parton splitting.

Direct Pomeron contribution to DDIS

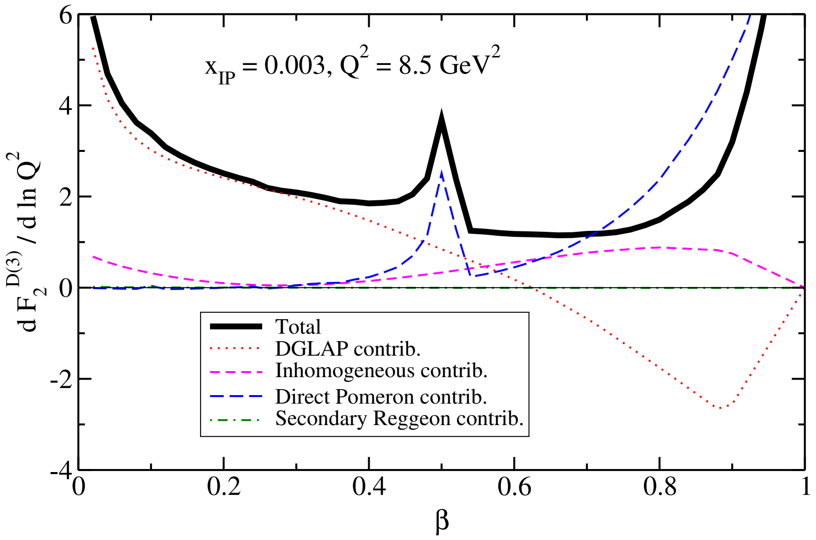


- Perturbative Pomeron can participate **directly** in the hard scattering process.
- "Direct Pomeron" contribution is analogous to the "direct photon" component of the photon structure function.

$$F_2^{\mathrm{D}(3)}(x_{\mathbb{P}}, \beta, Q^2) = \underbrace{\sum_{a=q,g} C_{2,a} \otimes f^{\mathrm{D}}_{a/p}}_{\text{Resolved Pomeron}} + \underbrace{C_{2,\mathbb{P}}}_{\text{Direct Pomeron}}$$

$$\text{where} \quad \frac{\partial f^{\mathrm{D}}_{a/p}(x_{\mathbb{P}}, z, Q^2)}{\partial \ln Q^2} = \underbrace{\sum_{a'=q,g} P_{aa'} \otimes f^{\mathrm{D}}_{a'/p}}_{\text{DGLAP term}} + \underbrace{P_{a\mathbb{P}}(z)\, f_{\mathbb{P}/p}(x_{\mathbb{P}}; Q^2)}_{\text{Inhomogeneous term}}$$

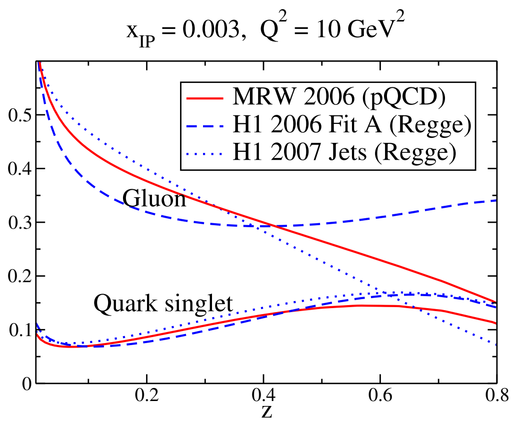
Effect of perturbative Pomeron terms on Q^2 slope



- Peak due to threshold for $\gamma^*\mathbb{P} \to c\bar{c}$ at $\beta = Q^2/(Q^2 + 4m_c^2)$.
- Additional contributions to scaling violations apart from DGLAP contribution, important for $\beta \gtrsim 0.3$.

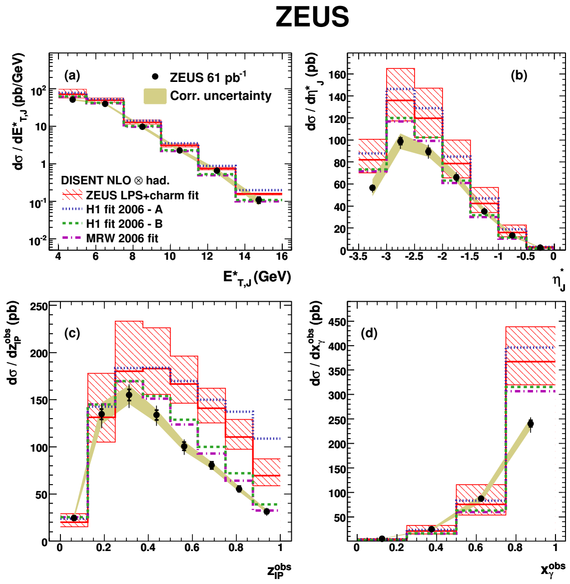
MRW 2006 analysis of diffractive PDFs [hep-ph/0609273]

- Repeat the H1 2006 analysis with the additional LO perturbative Pomeron terms included (using MRST NLO PDFs in the perturbative Pomeron flux factor).



- Smaller gluon at high z compared to H1 2006 Fit A.
- $\Rightarrow$ Tension between inclusive DDIS and dijet data in Regge factorisation approach partly alleviated by inclusion of perturbative Pomeron terms.
- MRW 2006 diffractive PDFs available from `http://durpdg.dur.ac.uk/hepdata/mrw.html`

Comparison to ZEUS data on DDIS dijets [0708.1415]



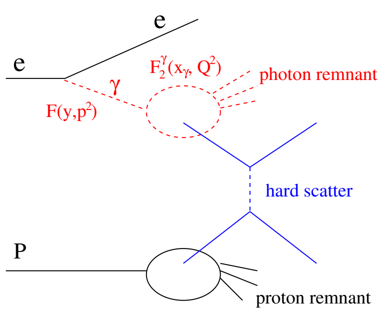
- Only the "direct photon" and "resolved Pomeron" component.
- MRW 2006 diffractive PDFs give the best description of data.

Diffractive PDFs in Monte Carlo event generators

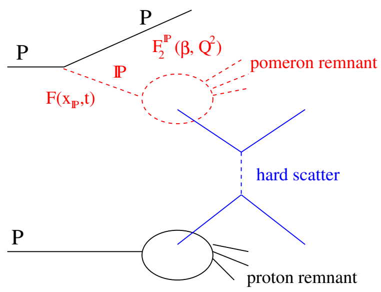
Not used in the "standard" MCs (PYTHIA, HERWIG, SHERPA).

- RAPGAP for *ep* collisions (H. Jung).
- POMWIG (B. Cox and J. Forshaw). Philosophy:

Replace



with



- POMPYT (P. Bruni, A. Edin and G. Ingelman): not current.

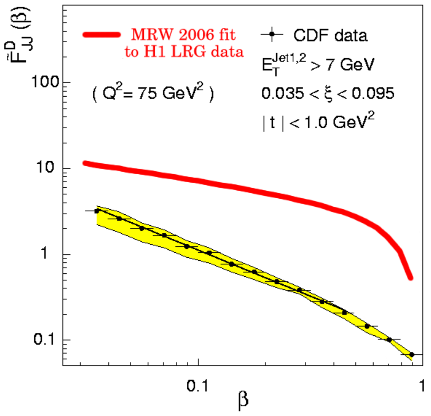
RAPGAP and POMWIG have the recent H1 2006 diffractive PDFs.

Factorisation breaking in diffractive pp and $p\bar{p}$ collisions

- Consider diffractive dijet production at Tevatron. Diffractive structure function of the antiproton:

$$\tilde{F}^{\rm D}_{JJ}(\beta) = \frac{1}{\xi_{\rm max} - \xi_{\rm min}} \int_{\xi_{\rm min}}^{\xi_{\rm max}} {\rm d}\xi \left[\beta g^{\rm D}(\xi,\beta,Q^2) + \frac{4}{9}\beta\Sigma^{\rm D}(\xi,\beta,Q^2)\right],$$

measured by CDF [PRL **84** (2000) 5043].

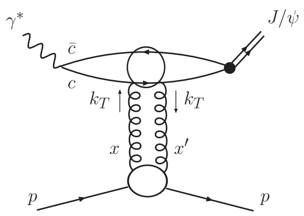


- Factorisation broken in diffractive hadron–hadron collisions by (soft) interaction between spectator partons.
- Suppression factor S^2 calculable from eikonal models with parameters fitted to soft hadron–hadron data.

Unintegrated, generalised PDFs

Martin, Ryskin, Teubner [hep-ph/9912551]:

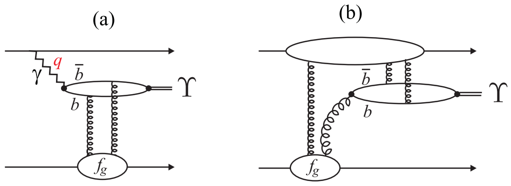
- Exclusive J/ψ production at HERA calculated within k_t-factorisation using a generalised, unintegrated gluon density (T_g = Sudakov form factor):



$$\boxed{f_g(x,x',t,k_t^2,\mu^2) = \exp(B_D\, t/2) R_g \frac{\partial[T_g(k_t^2,\mu^2)xg(x,k_t^2)]}{\partial \ln k_t^2}}$$

Khoze, Martin, Ryskin [0802.0177]:

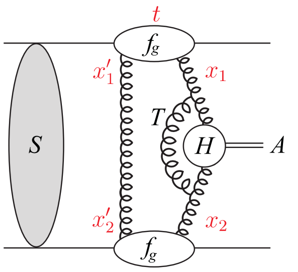
- Probe f_g by measuring exclusive Υ production at LHC via photon/odderon exchange.



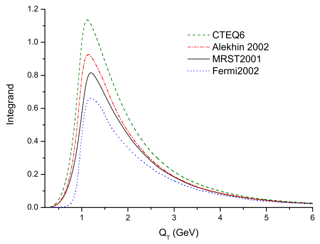
Relevance for central exclusive production at LHC

Khoze, Martin, Ryskin [0802.0177, and references therein]:

$$\sigma(pp \to p+A+p) \sim \frac{S^2}{B_D^2} \left| \frac{\pi}{8} \int \frac{\mathrm{d}Q_T^2}{Q_T^4} f_g(x_1, x_1', Q_T^2, \mu^2)\, f_g(x_2, x_2', Q_T^2, \mu^2) \right|^2 \hat{\sigma}(gg \to A)$$



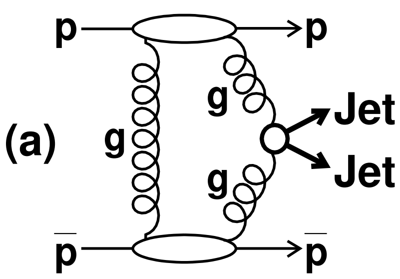
$$\boxed{f_g(x, x', Q_T^2, \mu^2) = R_g \frac{\partial}{\partial \ln Q_T^2} \left[\sqrt{T_g(Q_T^2, \mu^2)}\, xg(x, Q_T^2) \right]}$$

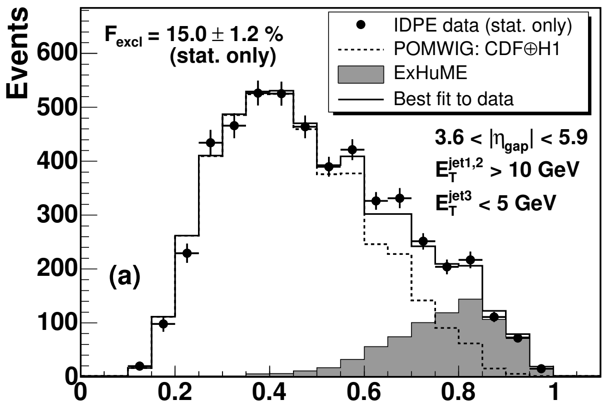


- Sudakov factor, $T_g(Q_T^2, \mu^2)$, suppresses the infrared Q_T region.
- Plot by J. Forshaw [hep-ph/0508274].
- Integrand dominated by $Q_T \sim 1$–2 GeV $\Rightarrow$ pQCD applicable (just).
- Calculations implemented in ExHuME event generator (J. Monk and A. Pilkington).

Exclusive diffractive dijet production at the Tevatron

- Distribution of dijet mass fraction $R_{jj} = M_{jj}/M_X$ measured by CDF [0712.0604].





1. POMWIG = generator for resolved Pomeron contribution.
2. ExHuME = generator for direct Pomeron contribution.

Resolved Pomeron contribution is a background to the exclusive diffractive signal (shown shaded).

Doubly-unintegrated PDFs

G.W., Martin, Ryskin [hep-ph/0306169, hep-ph/0309096]

Type of PDF	PDF integrated over	Incoming momentum
Integrated	k^-, $\mathbf{k_t}$	$k = (k^+, 0, \mathbf{0})$
Unintegrated	k^-	$k = (k^+, 0, \mathbf{k_t})$
Doubly-unintegrated	Nothing!	$k = (k^+, k^-, \mathbf{k_t})$

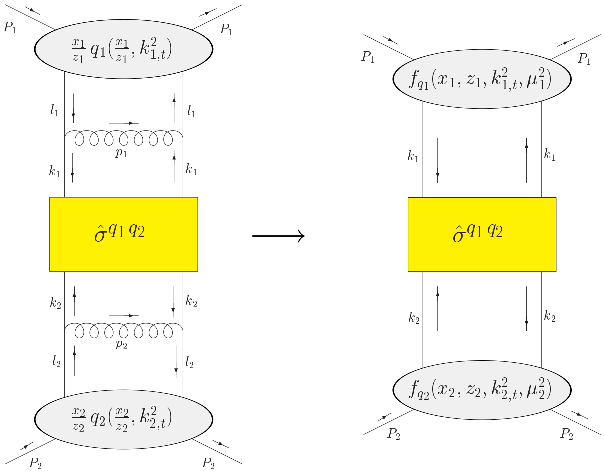
- Doubly-unintegrated PDFs in terms of integrated PDFs from last step of LO DGLAP evolution:

$$
\begin{aligned}
f_a(k, \mu^2) &\equiv f_a(x, z, k_t^2, \mu^2) \\
&= T_a(k_t^2, \mu^2)\, \frac{\alpha_S(k_t^2)}{2\pi} \sum_{b=q,g} P_{ab}(z)\, \frac{x}{z} b\left(\frac{x}{z}, k_t^2\right)
\end{aligned}
$$

$+$ angular-ordering constraints

- Take input integrated PDFs $b(x/z, k_t^2)$ from MRST 2001 LO.

Factorisation with doubly-unintegrated PDFs



$$
\boxed{\sigma = \int \mathrm{d}^4 k_1 \int \mathrm{d}^4 k_2\; f_{q_1}(k_1, \mu_1^2) f_{q_2}(k_2, \mu_2^2)\; \hat{\sigma}^{q_1 q_2}}
$$

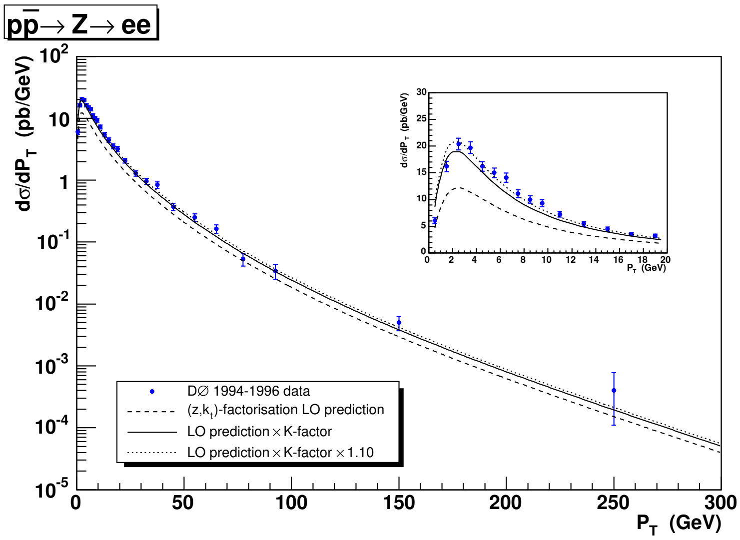
See also: Collins, Zu [hep-ph/0411332]; Collins, Rogers, Staśto [0708.2833]; Höche, Krauss, Teubner [0705.4577].

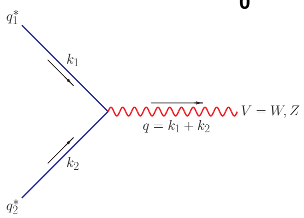
[hep-ph/0309096]

$$
\boxed{\mathrm{d}\hat{\sigma} = \mathrm{d}\Phi\; |\mathcal{M}|^2 / F}
$$

- Evaluate $|\mathcal{M}|^2 / F$ in collinear approximation: $k_i = (k_i^+, 0, \mathbf{0})$.
- Evaluate $\mathrm{d}\Phi$ with full kinematics: $k_i = (k_i^+, k_i^-, \mathbf{k_t})$.
- No small-x approximations are made.

Z P_T distribution at Tevatron Run I

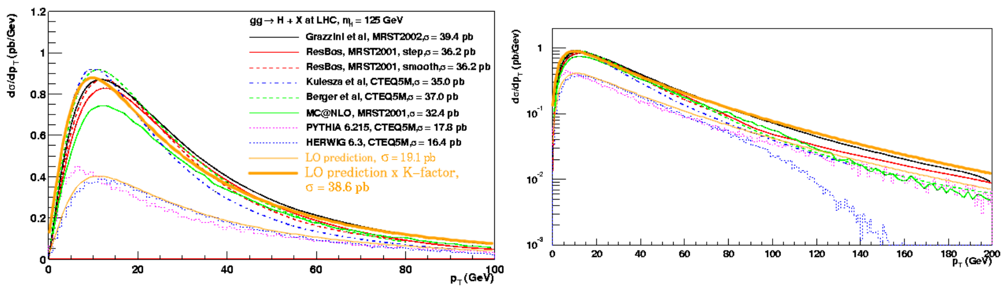


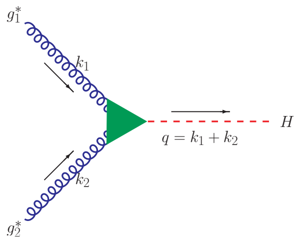


- K-factor $= \exp\left(\pi^2\, C_F\, \alpha_S/2\pi\right)$
- Good description of data over entire P_T range from only LO $q\bar{q} \to Z$ subprocess: explicit matrix-element corrections not needed at large P_T.

Higgs P_T distribution at LHC

C. Balazs, M. Grazzini, J. Huston, A. Kulesza and I. Puljak [hep-ph/0403052]:





- Overlay predictions using doubly-unintegrated PDFs (orange lines).
- K-factor $= \exp\left(\pi^2\, C_A\, \alpha_S/2\pi\right)$
- Only LO $gg \to H$ subprocess: explicit matrix-element corrections not needed at large P_T, cf. HERWIG.
- Code: `http://www.hep.ucl.ac.uk/~watt/`

Diffractive PDFs

- Inclusive diffraction $\Rightarrow$ resolved and direct Pomeron.
- Exclusive diffraction $\Rightarrow$ only direct Pomeron.
- Diffractive PDFs relevant for resolved Pomeron contribution.
- Inhomogeneous evolution analogous to photon PDFs.
- POMWIG event generator for LHC.

Generalised, unintegrated PDFs

- Relevant for exclusive diffractive processes.
- Can be approximately written in terms of usual PDFs.
- ExHuME event generator for LHC.

Doubly-unintegrated PDFs

- Relevant for p_T distributions of final-state particles.
- Get the kinematics right at LO
  $\Rightarrow$ less need for higher-order corrections.