Parton Distributions

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UCL HEP Group Meeting

18th December 2006

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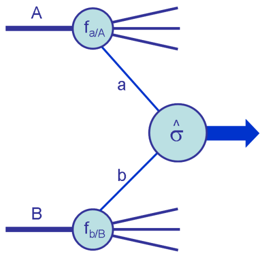
Fixed-order collinear factorisation at hadron colliders

- The "standard" perturbative QCD formalism: work at Leading-Order (LO), Next-to-Leading Order (NLO), etc.
- **Hadronic** cross sections given by convolution of **partonic** cross sections with Parton Distribution Functions (PDFs):



$$\boxed{\sigma_{AB} = \sum_{a,b=q,g} \left[\hat{\sigma}_{ab}^{\rm LO} + \alpha_S(Q)\hat{\sigma}_{ab}^{\rm NLO} + \ldots\right] \otimes f_{a/A}(x_a, Q) \otimes f_{b/B}(x_b, Q).}$$

PDF evolution: $$\frac{\partial f_a}{\partial \ln Q^2} = \sum_{a'=q,g} \left[P_{aa'}^{\rm LO} + \alpha_S P_{aa'}^{\rm NLO} + \ldots\right] \otimes f_{a'}.$$

α_S evolution: $$\frac{\partial \alpha_S}{\partial \ln Q^2} = -\beta^{\rm LO}\alpha_S^2 - \beta^{\rm NLO}\alpha_S^3 - \ldots.$$

Scale dependence of PDFs and α_S is calculable, but need to extract input values $f_a(x, Q_0)$ and $\alpha_S(M_Z)$ from experimental data.

Paradigm for PDF determination by "global analysis"

1. **Parameterise** the x-dependence for each flavour $a = q, g$ at the input scale $Q_0 \sim 1$ GeV in some flexible form, e.g.

$$xf_a(x, Q_0) = A_a x^{\Delta_a}(1-x)^{\eta_a}(1 + \epsilon_a\sqrt{x} + \gamma_a x),$$

   subject to number- and momentum-sum rule constraints.
2. **Evolve** the PDFs to higher scales $Q > Q_0$ using the (DGLAP) evolution equations.
3. **Convolute** the evolved PDFs with the partonic cross sections to calculate hadronic cross sections corresponding to a wide variety of deep-inelastic scattering (and related) data.
4. **Vary** the input parameters $\{A_a, \Delta_a, \eta_a, \epsilon_a, \gamma_a, \ldots\}$ to minimise

$$\chi^2 = \sum_{i=1}^{N_{\rm pts.}} \left(\frac{{\rm Data}_i - {\rm Theory}_i}{{\rm Error}_i}\right)^2.$$

Determination of parton distributions by global analysis

- An "industry" for more than 20 years. Regular updates as new data and theory become available.
- Major group: "**MRST**" = Martin–Roberts–Stirling–Thorne (+ G.W. since April 2006). Other groups: CTEQ, ZEUS, ...
- Why?
  1. **A Fundamental Measurement**: a challenge to understand using methods of nonperturbative QCD.
  2. **A Necessary Evil**: essential input to perturbative calculations of signal and background at hadron colliders.

  [J. Pumplin, talk at DIS 2005, `hep-ph/0507093`]
- As more data becomes available, can relax simplifying assumptions made about form of input parameterisations.
- In this talk, concentrate on:
  1. Strangeness in the proton.
  2. Inclusion of jet data in PDF fits.

Strangeness in the proton

- Protons have **no** valence strange quarks, i.e.

$$\int_0^1 \mathrm{d}x \; [s(x,Q) - \bar{s}(x,Q)] = 0.$$

- But this does **not** necessarily mean that $s(x,Q) = \bar{s}(x,Q) \; \forall x$.
- **Assumption** in all recent PDF fits is that

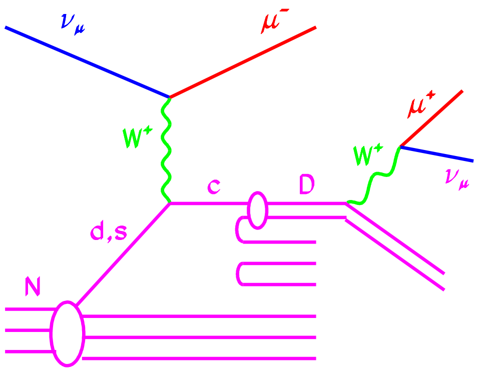
$$s(x,Q_0) = \bar{s}(x,Q_0) = \frac{\kappa}{2}\left[\bar{u}(x,Q_0) + \bar{d}(x,Q_0)\right],$$

with $\kappa \approx 0.5$, justified by a simplified analysis of dimuon data by the CCFR experiment [hep-ex/9406007].

- Updated CCFR and NuTeV dimuon cross sections now available $\Rightarrow$ include in global fit to constrain s and $\bar{s}$ directly.

CCFR/NuTeV dimuon cross sections

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}(\nu_\mu N \to \mu^+\mu^- X) = B_c \, \mathcal{N} \, \mathcal{A} \, \frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}(\nu_\mu N \to \mu^- c \, X),$$



where

B_c = semileptonic branching ratio,
$\mathcal{N}$ = nuclear correction,
$\mathcal{A}$ = acceptance correction.

Isoscalar target: $N = (p+n)/2$.
Isospin symmetry: $f_{d/n} = f_{u/p} \equiv u$.

Cross section for charm production at LO [$\xi \equiv x(1 + m_c^2/Q^2)$]:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y}(\nu_\mu N \to \mu^- c \, X) \propto |V_{cs}|^2 \xi s(\xi, Q) + \frac{1}{2}|V_{cd}|^2 \left[\xi d(\xi,Q) + \xi u(\xi,Q)\right]$$

$\Rightarrow$ ν_μ and $\bar{\nu}_\mu$ cross sections constrain s and $\bar{s}$, respectively.

Motivation: the NuTeV $sin^2\theta_W$ "anomaly"

- NuTeV extraction [hep-ex/0110059] of $\sin^2\theta_W$ from
$$R^- \equiv \frac{\sigma(\nu_\mu N \to \nu_\mu X) - \sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \to \mu^- X) - \sigma(\bar{\nu}_\mu N \to \mu^+ X)} \approx \frac{1}{2} - \sin^2\theta_W$$
was $\sim 3\sigma$ above the standard model prediction.
- New Physics? Or neglected PDF-related uncertainties, e.g.
  1. **Isospin violation?** ($u^p \neq d^n$, $d^p \neq u^n$)
  $$R^- \approx \frac{1}{2} - \sin^2\theta_W + (1 - \frac{7}{3}\sin^2\theta_W)\frac{\int_0^1 \mathrm{d}x\, x\,[(u_v^p - d_v^n) - (d_v^p - u_v^n)]}{2\int_0^1 \mathrm{d}x\, x(u_v + d_v)}$$
     - Generated by QED corrections to parton evolution.
     - Found by MRST [hep-ph/0411040] to remove a little more than 1σ of the total discrepancy in the NuTeV $\sin^2\theta_W$.
  2. **Strange sea asymmetry?** ($s \neq \bar{s}$)
  $$R^- \approx \frac{1}{2} - \sin^2\theta_W - (1 - \frac{7}{3}\sin^2\theta_W)\frac{\int_0^1 \mathrm{d}x\, x(s - \bar{s})}{\int_0^1 \mathrm{d}x\, x(u_v + d_v)}$$

Determination of s and $\bar{s}$ from dimuon data

Parameterise at input scale of $Q_0 = 1$ GeV in the form:

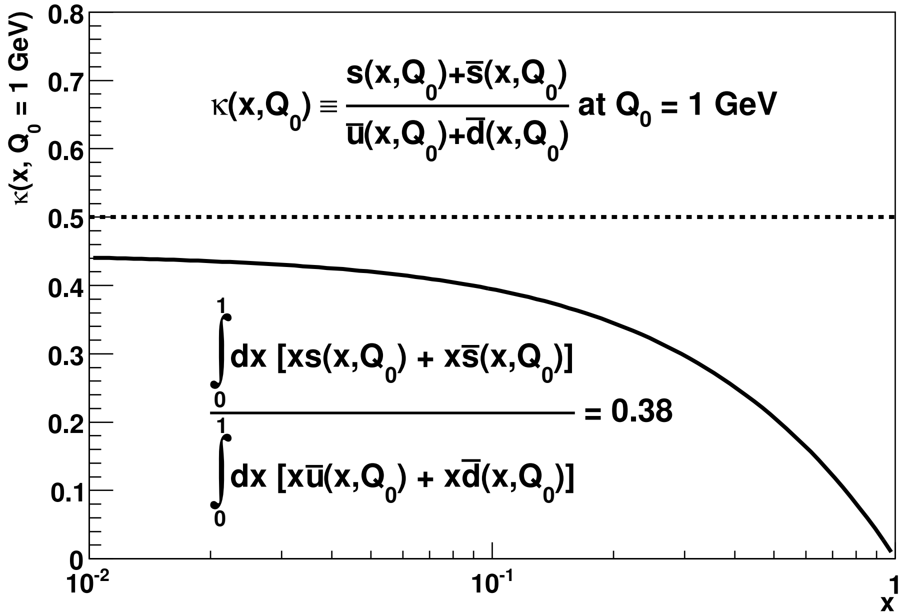
$$s^+(x, Q_0) \equiv s(x, Q_0) + \bar{s}(x, Q_0) = A_+(1-x)^{\eta_+}\, S(x, Q_0),$$
$$s^-(x, Q_0) \equiv s(x, Q_0) - \bar{s}(x, Q_0) = A_- x^{\Delta_- - 1}(1-x)^{\eta_-}(1 - x/x_0),$$

where $S(x, Q_0)$ is the total sea quark density, and x_0 is determined by conservation of strangeness: $\int_0^1 \mathrm{d}x\, s^-(x, Q_0) = 0$.

Study in context of MRST NLO global fit:

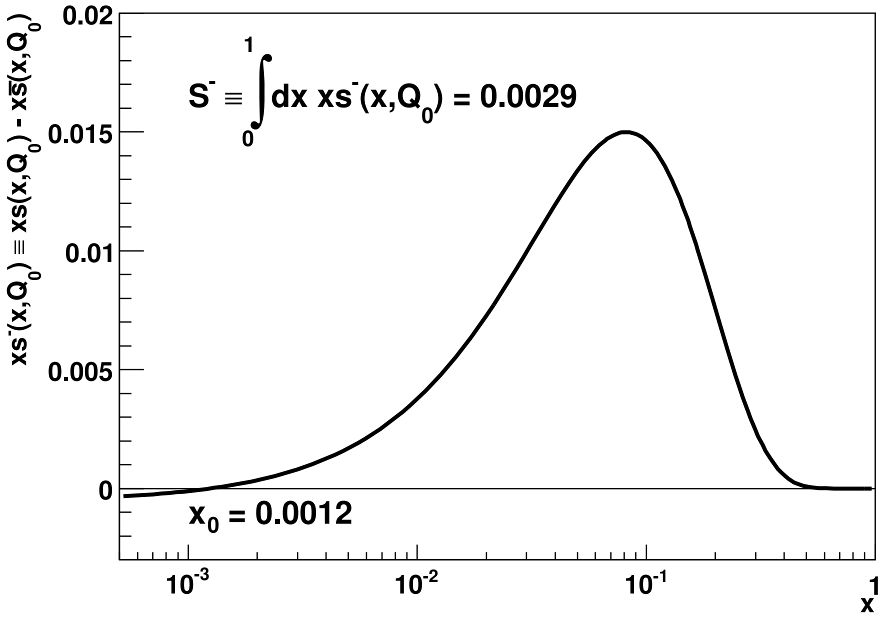
	$\chi^2_{\mathrm{CCFR}\mu\mu}$ (86 pts.)	$\chi^2_{\mathrm{NuTeV}\mu\mu}$ (84 pts.)	χ^2_{global} (2606 pts.)
$s = \bar{s} = (\bar{u} + \bar{d})/4$	68	66	2647
s^+ free, $s^- = 0$	63	54	2617
s^+ free, s^- free	64	40	2606

Ratio of strange sea to non-strange sea: $(s+\bar{s})/(\bar{u}+\bar{d})$



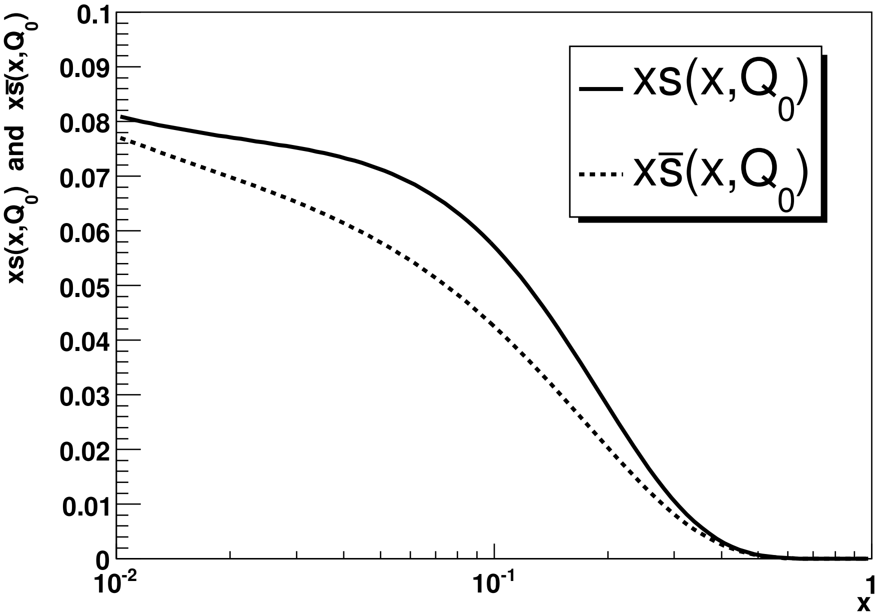
Suppression of strange sea at large x relative to non-strange sea.

Strange sea asymmetry: $xs - x\bar{s}$



cf. $S^- \sim 0.0068$ required to bring NuTeV $\sin^2\theta_W$ to world average.

Strange sea asymmetry: xs and $x\bar{s}$



s and $\bar{s}$ constrained by dimuon data for $0.01 \lesssim x \lesssim 0.2$.

Inclusion of jet data in PDF fits

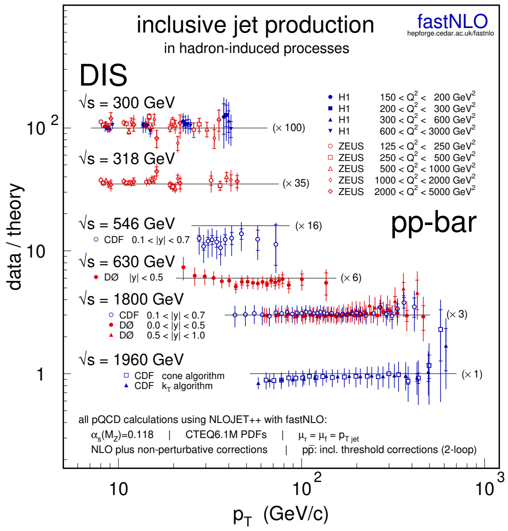
- **Tevatron jet data** are important in constraining the high-x gluon distribution. At NLO,

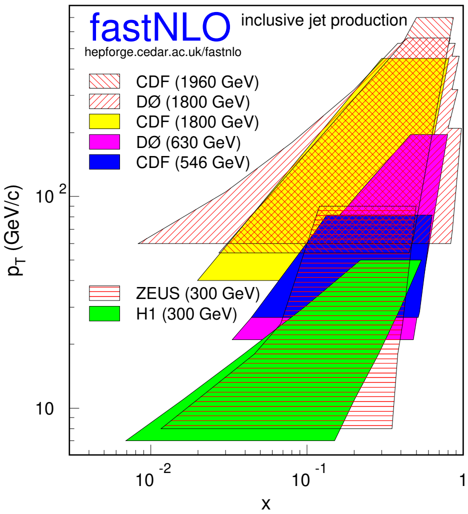
$$\sigma_{p\bar{p}} = \alpha_S^2(Q) \sum_{a,b=q,g} \left[\hat{\sigma}_{ab}^{\mathrm{LO}} + \alpha_S(Q)\hat{\sigma}_{ab}^{\mathrm{NLO}}\right] \otimes f_{a/p}(x_a, Q) \otimes f_{b/\bar{p}}(x_b, Q).$$

- Need multidimensional (Monte Carlo) phase space integration with cuts on E_T and η etc. $\Rightarrow$ takes hours/days of CPU time, so impractical to include in a PDF fit.
- **Old (approximate) solution:** calculate "K-factors" $(\sigma_{p\bar{p}}^{\mathrm{NLO}}/\sigma_{p\bar{p}}^{\mathrm{LO}})$ for a given set of PDFs. Then infer pseudogluon and Λ_{QCD} "data" points from $\sigma_{p\bar{p}}^{\mathrm{LO}}$.
- **New (rigorous) solution:** interpolate PDFs and α_S so that they can be factorised from $\hat{\sigma}_{ab}$. Replaces convolution with multiplication. Phase space integration only needs to be done once, with result stored in a grid for later use during a PDF fit.

“fastNLO”: Fast pQCD Calculations for PDF Fits

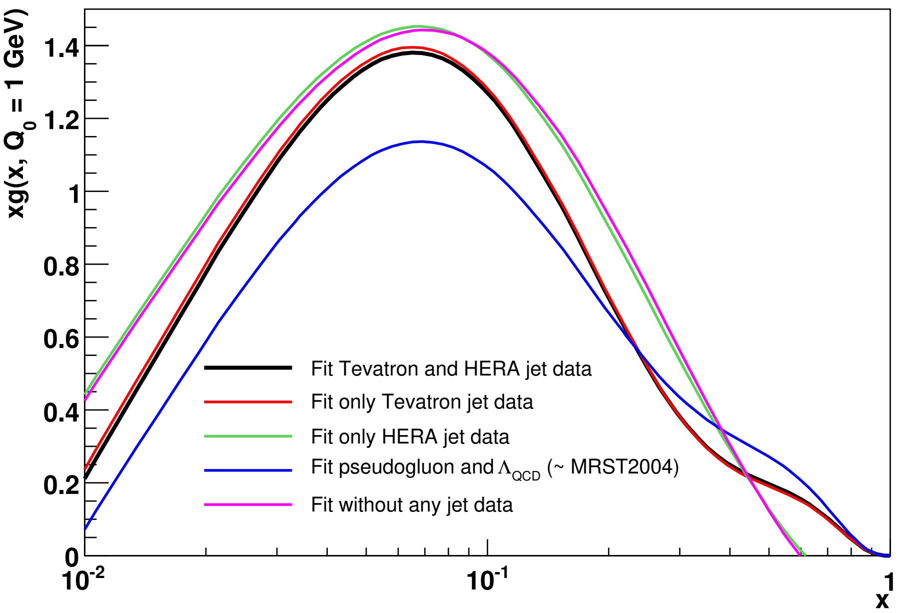
Grids corresponding to Tevatron and HERA kinematic cuts provided by “**fastNLO**” project.[1]



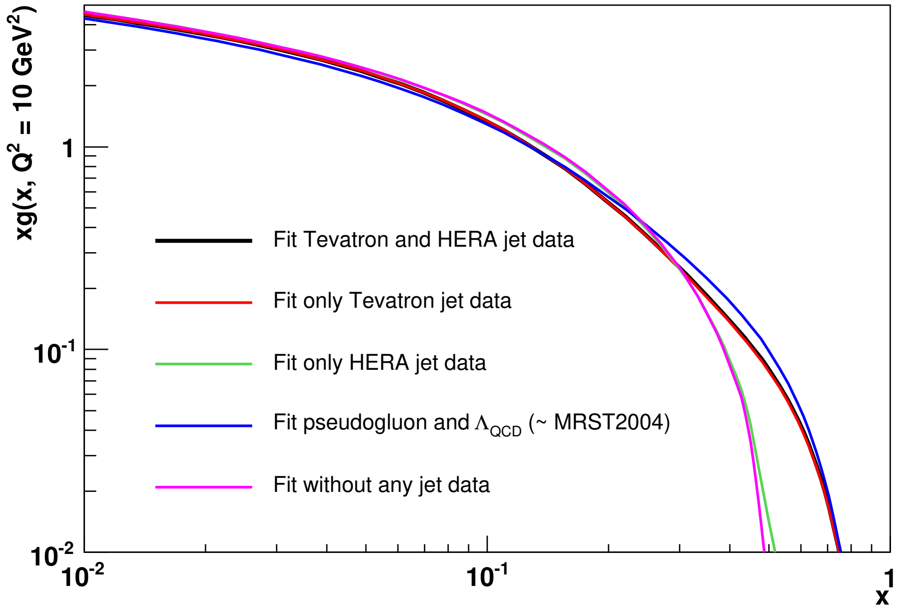


[1] T. Kluge, K. Rabbertz, M. Wobisch, `hep-ph/0609285`.

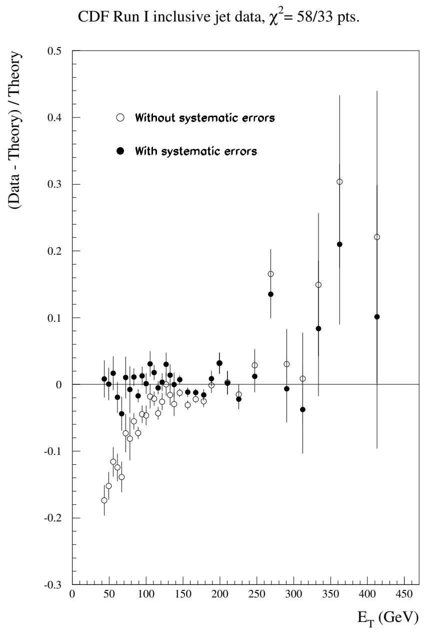
Impact of jet data on gluon distribution at 1 GeV^2

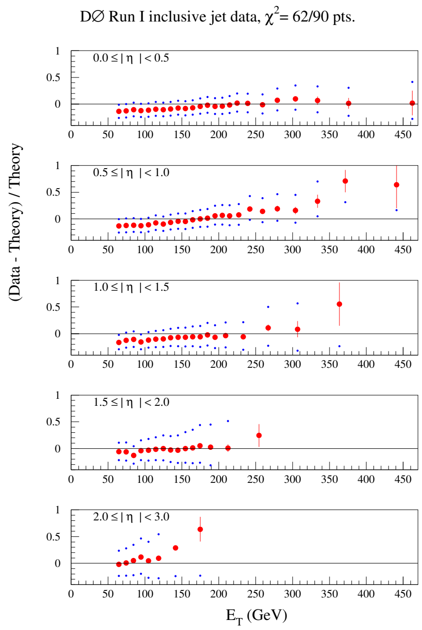


Impact of jet data on gluon distribution at 10 GeV^2



Description of Tevatron Run I jet data using fastNLO





Caveat: hadronisation corrections ($\sim$ 2–6%) not yet applied to theory predictions.

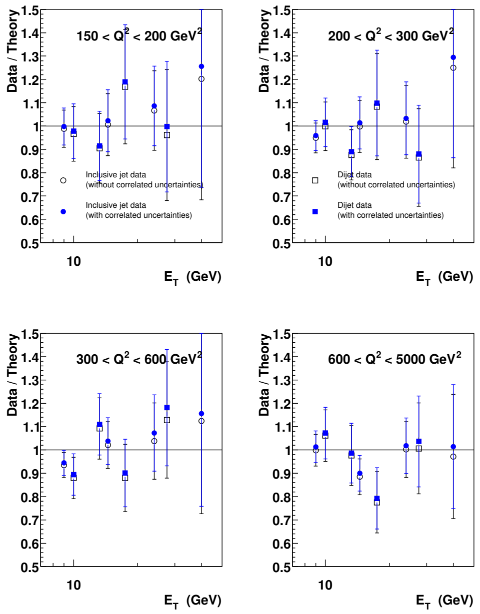
Description of H1 jet data using fastNLO

Use modified χ^2 to account for correlated systematic errors:

$$\chi^2 = \sum_{i=1}^{N_{\rm pts.}} \left(\frac{D_i - T_i - \sum_{k=1}^{N_{\rm corr.}} r_k \sigma_{k,i}^{\rm corr.}}{\sigma_i^{\rm uncorr.}} \right)^2 + \sum_{k=1}^{N_{\rm corr.}} r_k^2 .$$

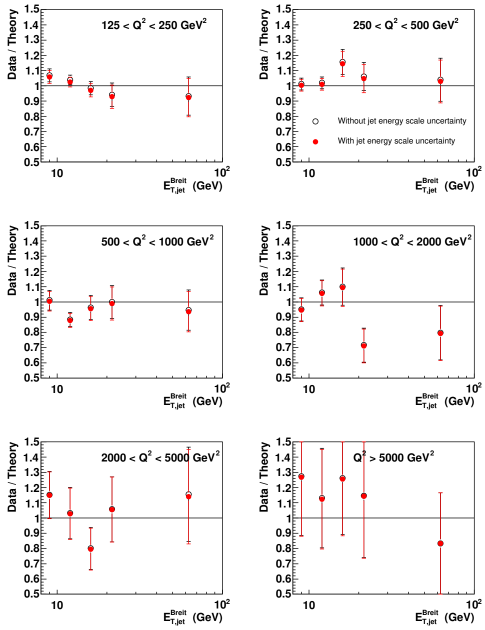
Effect of correlated systematic errors much more important for Tevatron jet data than for HERA jet data.

H1 95-97 incl. jet and dijet data, χ^2 = 14/32 pts.

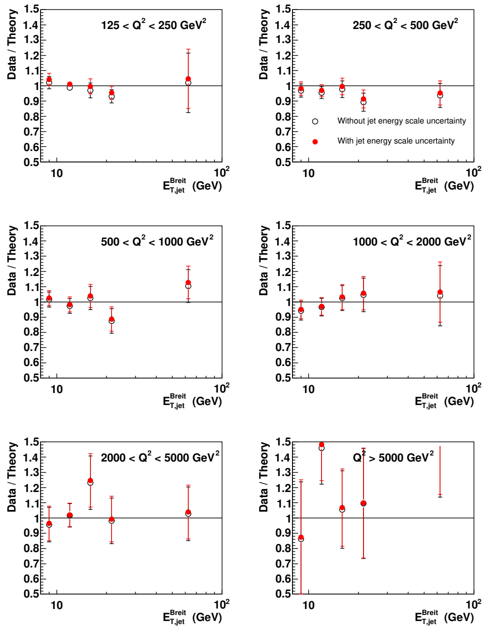


Description of ZEUS jet data using fastNLO

ZEUS 96-97 inclusive jet data, χ^2 = 30/30 pts.



ZEUS 98-00 inclusive jet data, χ^2 = 19/30 pts.



Strangeness in the proton

- NuTeV dimuon data constrain s and $\bar{s}$.
- Preference for a slight positive strange momentum asymmetry, which reduces the NuTeV $sin^2\theta_W$ anomaly.

Inclusion of jet data in PDF fits

- Inclusion of Tevatron Run I jet data in a rigorous way via "**fastNLO**" package gives slightly different gluon distribution than the approximate "pseudogluon" approach.
- HERA jet data have little impact on the gluon distribution.

Outlook

- Include Tevatron Run II jet data.
- Extend to NNLO analysis.
- Produce parton distributions with errors.